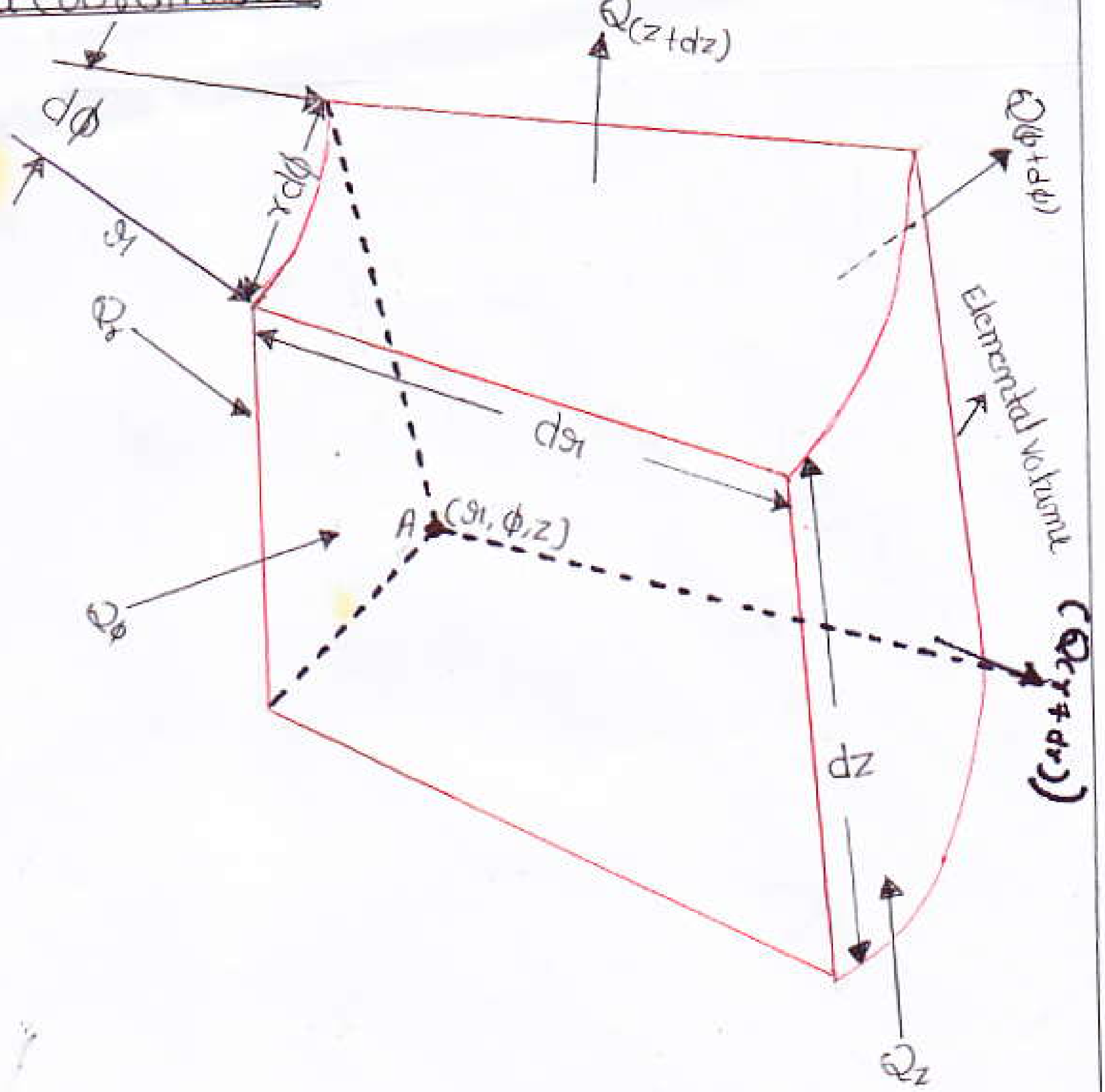
Q1} Drive the general heat conduction equation in cylinderical coordinates.

Ans



Consider a elemental volume having the coordinates (r, ϕ, z), for three - dimensional heal conduction.

A. Net heat accumulated in the element due to conduction of heat from all the coordinates directions considered.

Heat flow in (z, ϕ) plane :-

Heat influx , $Q'_r = -K\,(r\,d\phi\,dz)\frac{\partial t}{\partial r}\cdot dz$ --(i)

Heat efflux , $Q'_{(r+dr)} = Q'_r + \frac{\partial}{\partial r}(Q_r)\,dr$ ---(ii)

Heat accumulated in the element due to heat flow in radial direction.

$$d Q'_r = Q'_r - Q'_{(r+dr)} \quad \text{[subtracting (ii) from (i)]}$$

$$= -\frac{\partial}{\partial r}(Q'_r)\, dr$$

$$= -\frac{\partial}{\partial r}\left[-k(r\, d\phi \cdot dz)\frac{\partial t}{\partial r} \cdot d\tau\right] dr$$

$$= k(dr \cdot d\phi \cdot dz)\frac{\partial}{\partial r}\left(r \cdot \frac{\partial t}{\partial r}\right) d\tau$$

$$= k(dr \cdot d\phi \cdot dz)\left[r\frac{\partial^2 t}{\partial r^2} + \frac{\partial t}{\partial r}\right] d\tau$$

$$= k(dr \cdot r\, d\phi \cdot dz)\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right] d\tau$$

Heat flow in tangential direction ($r-z$) plane :-

Heat influx, $$Q_\phi = -k(dr \cdot dz)\frac{\partial t}{r\,\partial \phi}\, d\tau \quad \text{---(iii)}$$

Heat efflux, $$Q'_{(\phi+d\phi)} = Q'_\phi + \frac{\partial}{r\,\partial\phi}(Q'_\phi)\, r\, d\phi \quad \text{---(iv)}$$

Heat accumulated in the element due to heat flow in tangential direction,

$$dQ'_\phi = Q'_\phi - Q'_{(\phi+d\phi)} \quad \text{[subtracting (iv) from (iii)]}$$

$$= -\frac{\partial}{r\,\partial\phi}(Q_\phi)\, r\, d\phi$$

$$= -\frac{\partial}{r\,\partial\phi}\left[-k(dr \cdot dz)\frac{\partial t}{r\,\partial\phi} \cdot d\tau\right] r\, d\phi$$

$$= k(dr \cdot d\phi\, dz)\frac{\partial}{\partial\phi}\left(\frac{1}{r}\frac{\partial t}{\partial\phi}\right) d\tau$$

$$= k(dr \cdot r\, d\phi \cdot dz)\frac{1}{r^2}\frac{\partial^2 t}{\partial\phi^2} \cdot d\tau$$

Heat flow in axial direction ($r-\phi$) plane :-

Heat influx, $$Q'_z = -k(r \cdot d\phi \cdot dr)\frac{\partial t}{\partial z}\, d\tau \quad \text{--(v)}$$

Heat efflux, $$Q'_{(z+dz)} = Q'_z + \frac{\partial}{\partial z}(Q'_z)\, dz \quad \text{--(vi)}$$

Heat accumulated in the ~~direction~~ elemental due to heat flow in axial direction.

$$dQ'_z = Q'_z - Q'_{(z+dz)} \quad \text{[subtracting (vi) from (v)]}$$

$$= -\frac{\partial}{\partial z}\left[-k(r \cdot d\phi \cdot dr)\frac{\partial t}{\partial z} \cdot d\tau\right] dz$$

$$= k\,(dr \cdot r\,d\phi \cdot dz)\,\frac{\partial^2 t}{\partial z^2} \cdot d\tau$$

Net heat accumulated in the element

$$= k \cdot dr\,r\,d\phi \cdot dz \left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}\right] d\tau$$

B. Heat generated within the element (Q_g) :-

The total heat generated within the element is given by

$$Q_g = q_g\,(dr \cdot r\,d\phi \cdot dz)\,d\tau$$

C. Energy stored in the element :-

The increase in internal energy in the element is equal to

$$= \rho\,(dr \cdot r\,d\phi \cdot dz) \cdot c \cdot \frac{\partial t}{\partial \tau} \cdot d\tau$$

Now $(A) + (B) = (C)$ [Energy balance/equ^n]

Now ∴

$$k \cdot dr \cdot r\,d\phi \cdot dz \left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial t}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}\right] d\tau + q_g \cdot (dr \cdot r\,d\phi\,dz)\,d\tau = \rho\,(dr \cdot r\,d\phi\,dz) \cdot c \cdot \frac{\partial t}{\partial \tau}\,d\tau$$

Dividing both sides by $dr \cdot r\,d\phi \cdot dz \cdot d\tau$, we have

$$k\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}\right] + q_g = \rho \cdot c\,\frac{\partial t}{\partial \tau}$$

or,

$$\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}\right] + \frac{q_g}{k} = \frac{\rho \cdot c}{k}\frac{\partial t}{\partial \tau} = \frac{1}{\alpha}\frac{\partial t}{\partial \tau} \quad \text{.(vii)}$$

Equation (vii) is the general heat conduction equation in cylindrical coordinates

In case there are no heat sources present and heat flow is steady and one-dimensional, then eqn. (vii) reduces to

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} = 0$$

or,

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{dt}{dr} = 0$$

or,

$$\frac{1}{r}\frac{d}{dr}\left(r \cdot \frac{dt}{dr}\right) = 0$$

since $\frac{1}{r} \neq 0$, therefore, $\frac{d}{dr}\left(r \cdot \frac{dt}{dr}\right)$ or $r \cdot \frac{dt}{dr} = \text{constant}$

Equation (vii) can also be derived by transformation of coordinates, as follows:

$$x = r\cos\phi$$
$$y = r\sin\phi$$
$$z = z$$

Now, by chain rule:

$$\frac{\partial t}{\partial r} = \frac{\partial t}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial t}{\partial y}\frac{\partial y}{\partial r} = \frac{\partial t}{\partial x}\cos\phi + \frac{\partial t}{\partial y}\sin\phi$$

or, $$\cos\phi\frac{\partial t}{\partial r} = \cos^2\phi\frac{\partial t}{\partial x} + \sin\phi.\cos\phi.\frac{\partial t}{\partial y} \quad \text{---(1)}$$

Multiplying both sides by $\cos\phi$

Also $$\frac{\partial t}{\partial \phi} = \frac{\partial t}{\partial x}\frac{\partial x}{\partial \phi} + \frac{\partial t}{\partial y}.\frac{\partial y}{\partial \phi} = \frac{\partial t}{\partial x}(-r\sin\phi) + \frac{\partial t}{\partial y}(r\cos\phi)$$

or, $$\frac{\sin\phi}{r}.\frac{\partial t}{\partial \phi} = -\sin^2\phi\frac{\partial t}{\partial x} + \sin\phi.\cos\phi.\frac{\partial t}{\partial y} \quad \text{---(2)}$$

Multiplying both sides by $\frac{\sin\phi}{r}$

From equation (1) and (2) we have

$$\frac{\sin\phi}{r}\frac{\partial t}{\partial \phi} = -\sin^2\phi\frac{\partial t}{\partial x} + \left[\cos\phi\frac{\partial t}{\partial r} - \cos^2\phi\frac{\partial t}{\partial x}\right]$$

$$= -\frac{\partial t}{\partial x} + \cos\phi\frac{\partial t}{\partial r}$$

$$\frac{\partial t}{\partial x} = \cos\phi\frac{\partial t}{\partial r} - \frac{\sin\phi}{r}\frac{\partial t}{\partial \phi}$$

Differentiating both sides with respect to x, we have

$$\frac{\partial}{\partial x}\left(\frac{\partial t}{\partial x}\right) = \frac{\partial}{\partial x}\left[\cos\phi\frac{\partial t}{\partial r} - \frac{\sin\phi}{r}\frac{\partial t}{\partial \phi}\right]$$

or $$\frac{\partial^2 t}{\partial x^2} = \cos\phi\frac{\partial}{\partial r}\left(\frac{\partial t}{\partial x}\right) - \frac{\sin\phi}{r}\frac{\partial}{\partial \phi}\left(\frac{\partial t}{\partial x}\right) \quad \text{---(3)}$$

$$= \cos\phi\frac{\partial}{\partial r}\left(\cos\phi\frac{\partial t}{\partial r} - \frac{\sin\phi}{r}\frac{\partial t}{\partial \phi}\right) - \frac{\sin\phi}{r}\frac{\partial}{\partial \phi}\left(\cos\phi\frac{\partial t}{\partial r} - \frac{\sin\phi}{r}\frac{\partial t}{\partial \phi}\right)$$

[Substituting the value of $\frac{\partial t}{\partial x}$ from (3)]

$$= \cos^2\phi \frac{\partial^2 t}{\partial r^2} - \frac{\cos\phi . \sin\phi}{r^2}\frac{\partial t}{\partial \phi} + \frac{\sin^2\phi}{r}\frac{\partial t}{\partial r} + \frac{\sin^2\phi}{r^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\sin\phi \cos\phi}{r^2}\frac{\partial t}{\partial \phi}$$

Similarly $\frac{\partial^2 t}{\partial y^2} = \sin^2\phi \frac{\partial^2 t}{\partial r^2} + \frac{\cos^2\phi}{r}\frac{\partial t}{\partial r} - \frac{\cos\phi \sin\phi}{r^2}\frac{\partial t}{\partial \phi} + \frac{\cos^2\phi}{r^2}\frac{\partial^2 t}{\partial \phi^2} - \frac{\cos\phi \sin\phi}{r^2}\frac{\partial t}{\partial \phi}$ --- (5)

By adding (3) and (4), we get

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = \frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2}$$

Substituting it in equation

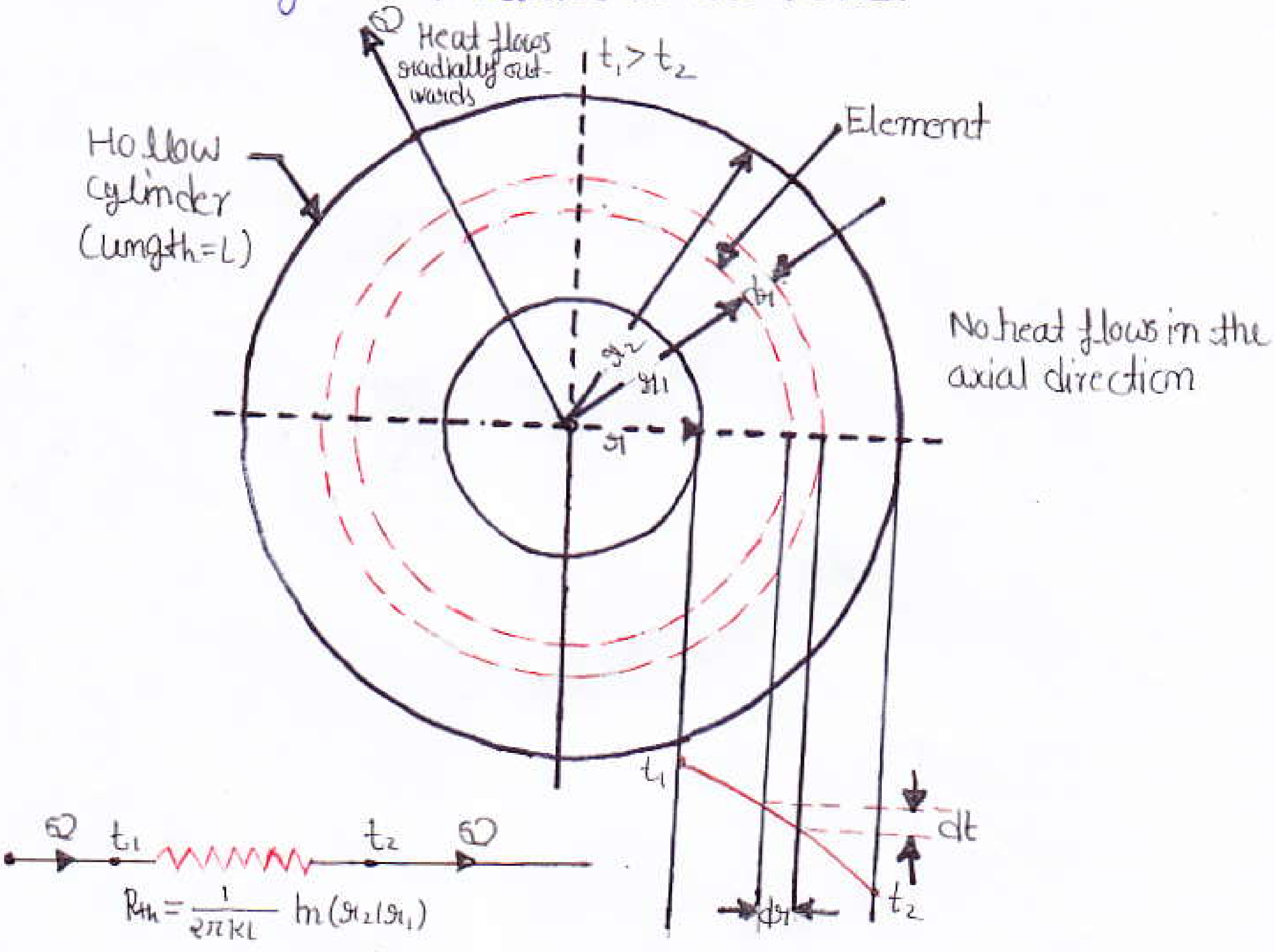
$$\left[\frac{\partial^2 t}{\partial x^2} + \frac{1}{r}\frac{\partial^2 t}{\partial y^2} + \frac{1}{r^2} \cdot \frac{\partial^2 t}{\partial \phi} + \frac{\partial^2 t}{\partial z^2}\right] + \frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial t}{\partial \tau}$$

which is the same as equation (vii)

Q2) Drive the equation for heat conduction for a hollow cylinder.

Case I - Uniform Conductivity :-

Consider a hollow cylinder made of material having thermal conductivity and insulated at both end.



Let, r_1, r_2 = Inner and outer radii.

t_1, t_2 = Temperature of inner and outer surfaces,

K = Constant thermal conductivity within the given temperature range.

The general heat conduction equation in thermal coordinates is given by,

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}\right] + \frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial t}{\partial \tau}$$

for steady state $\left(\frac{\partial t}{\partial \tau}=0\right)$, unidirectional $[t \neq f(\phi, z)]$ heat flow in radial direction and with no internal heat generation $(q_g = 0)$, the above equation reduces to

$$\frac{d^2t}{dr^2}+\frac{1}{r}\frac{dt}{dr}=0$$

or

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{dt}{dr}\right]=0$$

since, $\frac{1}{r} \neq 0$ therefore, $\frac{d}{dr}\left(r\frac{dt}{dr}\right)=0$

or,

$$r \cdot \frac{dt}{dr}=C \quad \text{(a constant)}$$

Integrating the above equation, we get

$$t = C\ln(r)+C_1 \qquad \text{- - - - - (1)}$$

(where C_1 = constant of Integeration)

Using the following boundary coordinates, we have

At $r = r_1$, $t = t_1$, At $r = r_2$, $t = t_2$

$$t_1 = C\ln(r_1)+C_1 \qquad \text{- - - (i)}$$

$$t_2 = C\ln(r_2)+C_1 \qquad \text{- - - - (ii)}$$

from (i) and (ii) we have,

$$C = -\frac{(t_1-t_2)}{\ln(r_2/r_1)}$$

and

$$C_1 = t_1+\frac{t_1-t_2}{\ln(r_2/r_1)}\ln(r_1)$$

Substituting the values of these constants in eq (1), we have

$$\boxed{t = t_1+\frac{t_1-t_2}{\ln(r_2/r_1)}\ln(r_1)-\frac{(t_1-t_2)}{\ln(r_2/r_1)}\ln(r)}$$

This equation is the expression for temperature distribution in a hollow cylinder.

or,

$$(t-t_1)\ln(r_2/r_1) = (t_1-t_2)\ln(r_1)-(t_1-t_2)\ln(r)$$

$$= (t_2-t_1)\ln(r)-(t_2-t_1)\ln(r_1)$$

$$= (t_2-t_1)\ln(r/r_1)$$

or,

$$\frac{t_1-t_1}{t_2-t_1}=\frac{\ln(r/r_1)}{\ln(r_2/r_1)} \qquad \text{[Dimensionless form]}$$

From the above equation, the following points are worth noting

(i) The temperature distribution is logarithmic (not linear as in the case of plane wall)

(ii) Temperature at any point in the cylinder can be expressed as a function of radius only. Isotherms (or lines of constant temperature) are then concentric circle lying between the inner and outer boundaries of the hollow cylinder.

(iii) The temperature profile [equation (a)] is nearly linear for values of (r_2/r_1) of the order of unity, but decidedly non-linear for large values of (r_2/r_1).

Determination of conduction heat transfer rate (Q)

The conduction heat transfer rate is determined by utilizing the temperature distribution [equation(a)] in conjuction with fourier's equation as follows:

$$Q = -kA\frac{dt}{dr}$$

$$= -kA\frac{d}{dr}\left[t_1 + \frac{t_1 - t_2}{\ln(r_2/r_1)}\ln(r_1) - \frac{t_1 - t_2}{\ln(r_2/r_1)}\ln(r)\right]$$

[Substituting the value of t from equation (a)]

$$= -k(2\pi r.L)\left[\frac{-(t_1 - t_2)}{r\ln(r_2/r_1)}\right]$$

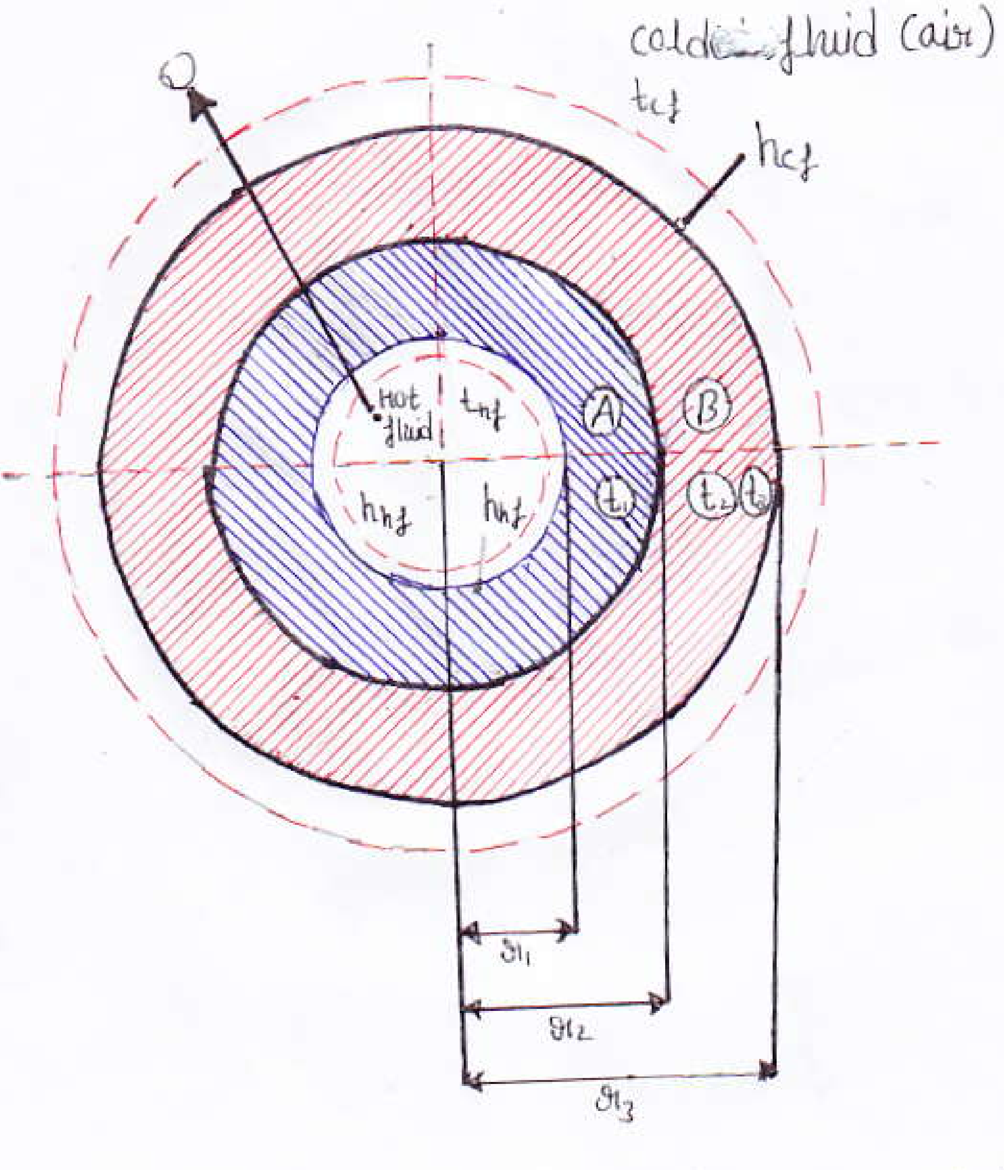
$$= 2\pi kL\frac{(t_1 - t_2)}{\ln(r_2/r_1)} = \frac{t_1 - t_2}{\frac{\ln(r_2/r_1)}{2\pi kL}} \left[= \frac{\Delta t}{R_{th}}\right]$$

where $R_{th} = \dfrac{\ln(r_2/r_1)}{2\pi kL}$

Hence $$\boxed{Q = \frac{(t_1 - t_2)}{\frac{\ln(r_2/r_1)}{2\pi kL}}}$$

Q3) Drive the expression for heat conduction for composite cylinder.

Consider flow of heat through a composite cylinder as shown in fig.



Let,
t_{hf} = The temperature of hot fluid flowing inside the cylinder,
t_{cf} = The temperature of cold fluid (atmospheric air)
K_A = Thermal conductivity of the inside layer A
K_B = Thermal conductivity of the layer (outside) B.
t_1, t_2, t_3 = Temperature at the point 1, 2 and 3 [see fig.]
L = length of the composite cylinder and,
h_{hf}, h_{cf} = Inside and outside heat transfer coefficients

The rate of heat transfer is given by

$$Q = h_{hf} \cdot 2\pi L r_1 (t_{hf} - t_1) = \frac{K_A 2\pi L (t_1 - t_2)}{\ln(r_2/r_1)}$$

$$= \frac{K_B \cdot 2\pi L (t_2 - t_3)}{\ln(r_3/r_2)} = h_{cf} 2\pi r_3 L (t_3 - t_{cf})$$

Rearranging the above equation expression, we get

$$t_{hf} - t_1 = \frac{Q}{h_{hf} r_1 2\pi L} \quad \text{---(i)}$$

$$t_1 - t_2 = \frac{Q}{\frac{K_A 2\pi L}{\ln(r_2/r_1)}} \quad \text{---(ii)}$$

$$t_2 - t_3 = \frac{Q}{\frac{K_B 2\pi L}{\ln(r_3/r_2)}} \quad \text{---(iii)}$$

$$t_3 - t_{cf} = \frac{Q}{h_{cf} r_3 2\pi L} \quad \text{---(iv)}$$

Adding (i), (ii), (iii) and (iv), we have

$$\frac{Q}{2\pi L}\left[\frac{1}{h_{hf} r_1} + \frac{1}{\frac{K_A}{\ln(r_2/r_1)}} + \frac{1}{\frac{K_B}{\ln(r_3/r_2)}} + \frac{1}{h_{cf} r_3}\right] = t_{hf} - t_{cf}$$

$$Q = \frac{2\pi L (t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} r_1} + \frac{1}{\frac{K_A}{\ln(r_2/r_1)}} + \frac{1}{\frac{K_B}{\ln(r_3/r_2)}} + \frac{1}{h_{cf} r_3}\right]}$$

or

$$Q = \frac{2\pi L (t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} r_1} + \frac{\ln(r_2/r_1)}{K_A} + \frac{\ln(r_3/r_2)}{K_B} + \frac{1}{h_{cf} r_3}\right]}$$

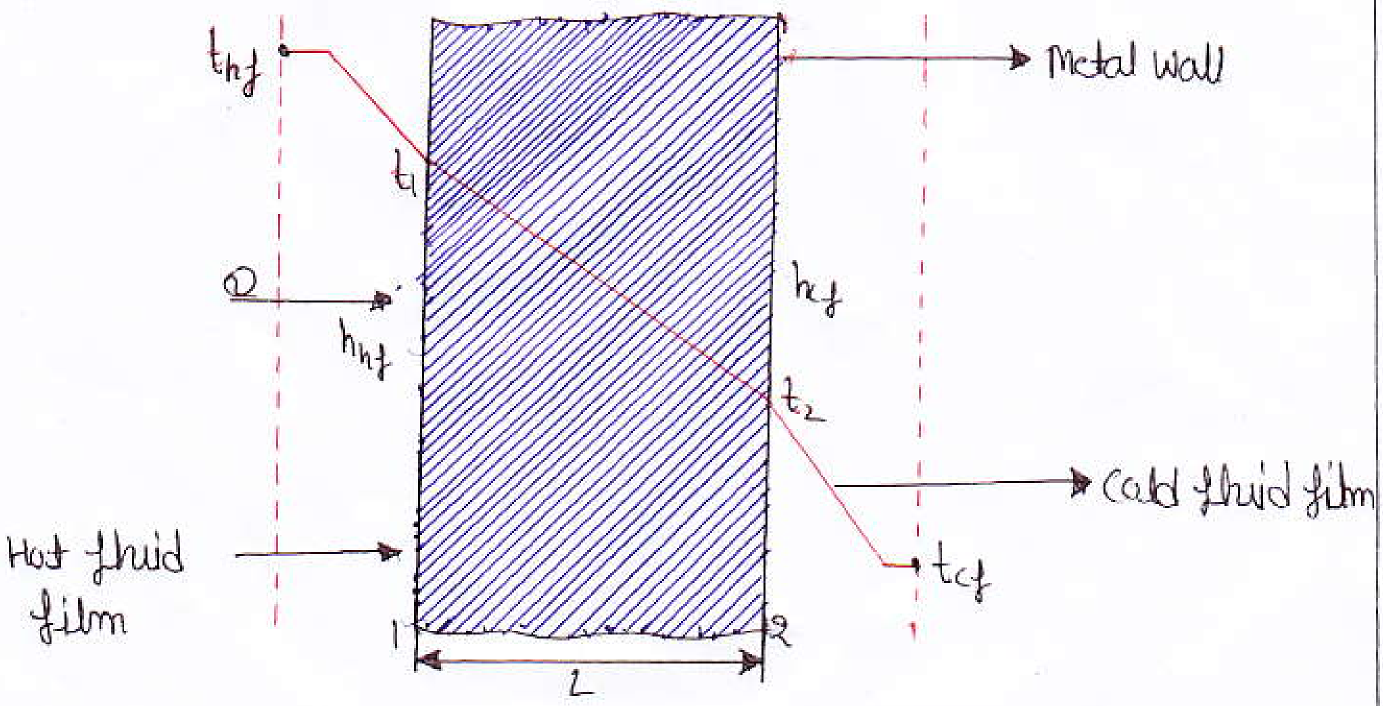
If there are 'n' concentric cylinder then,

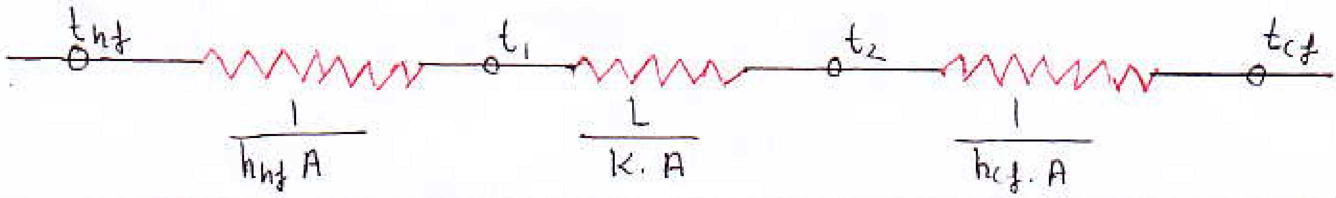
$$Q = \frac{2\pi L (t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} r_1} + \sum_{n=1}^{n=n} \frac{1}{K_n} \ln\{r_{(n+1)}/r_n\} + \frac{1}{h_{cf} r_{(n+1)}}\right]}$$

If inside and outside heat transfer coefficient are not considered then the above equation can be written as.

$$\boxed{Q = \frac{2\pi L\left[t_1 - t_{n+1}\right]}{\sum_{n=1}^{n=n} \frac{1}{K_n} \ln\left[r_{(n+1)} / r_n\right]}}$$

Q4) Drive the expression for overall heat transfer cofficient for cylinder (for Plane Wall.





While dealing with the problems of fluid to fluid heat transfer across a metal boundary, it is usual to adopt an overall heat transfer coefficient, U which gives the heat transmitted per unit area per unit time per degree temperature difference the bulk fluids on each side of the metal.

let.

L = Thickness of the metal wall,

K = Thermal conductivity of the wall material,

t_1 = Temperature of the surface - 1,

t_2 = Temperature of the surface - 2,

t_{hf} = Temperature of the hot fluid,

t_{cf} = Temperature of the cold fluid,

h_{hf} = Heat transfer coefficient from hot fluid to metal surface and,

h_{cf} = Heat transfer coefficient from metal surface to cold fluid.

The surfaces hf and cf stand for hot fluid and cold fluid respectively.

The equation of heat flow through the fluid and the metal surface are given by

$$Q = h_{hf} A (t_{hf} - t_1) \quad \text{---(i)}$$

$$Q = \frac{K \cdot A (t_1 - t_2)}{L} \quad \text{---(ii)}$$

$$Q = h_{cf} A (t_2 - t_{cf}) \quad \text{---(iii)}$$

By rearranging (i), (ii) and (iii), we get

$$t_{hf} - t_1 = \frac{Q}{h_{hf} A} \quad \text{---(iv)}$$

$$t_1 - t_2 = \frac{Q L}{K \cdot A} \quad \text{---(v)}$$

$$t_2 - t_{cf} = \frac{Q}{h_{cf} A} \quad \text{---(vi)}$$

Adding (iv), (v) and (vi) we get

$$t_{hf} - t_{cf} = Q \left[\frac{1}{h_{hf} A} + \frac{L}{K \cdot A} + \frac{1}{h_{cf} A} \right]$$

or $$Q = \frac{A(t_{hf} - t_{cf})}{\frac{1}{h_{hf}} + \frac{L}{K} + \frac{1}{h_{cf}}}$$

If U is the overall coefficient of heat transfer, then

$$Q = U \cdot A(t_{hf} - t_{cf}) = \frac{A(t_{hf} - t_{cf})}{\frac{1}{h_{hf}} + \frac{L}{K} + \frac{1}{h_{cf}}}.$$

or $$\boxed{U = \frac{1}{\frac{1}{h_{hf}} + \frac{L}{K} + \frac{1}{h_{cf}}}}$$

It may be noticed from the above equation that if the individual coefficient differ greatly in magnitude only a change in the least will have any significant effect on the rate of heat transfer.

Q5) What is critical thickness of Insulating / Insulation ? Drive a relation for critical radius of Cylinder.

Ans

Insulation :⇒

A material which retards the flow of heat with reasonable effectiveness is known as Insulation. Insulation serves the following two purpose :-

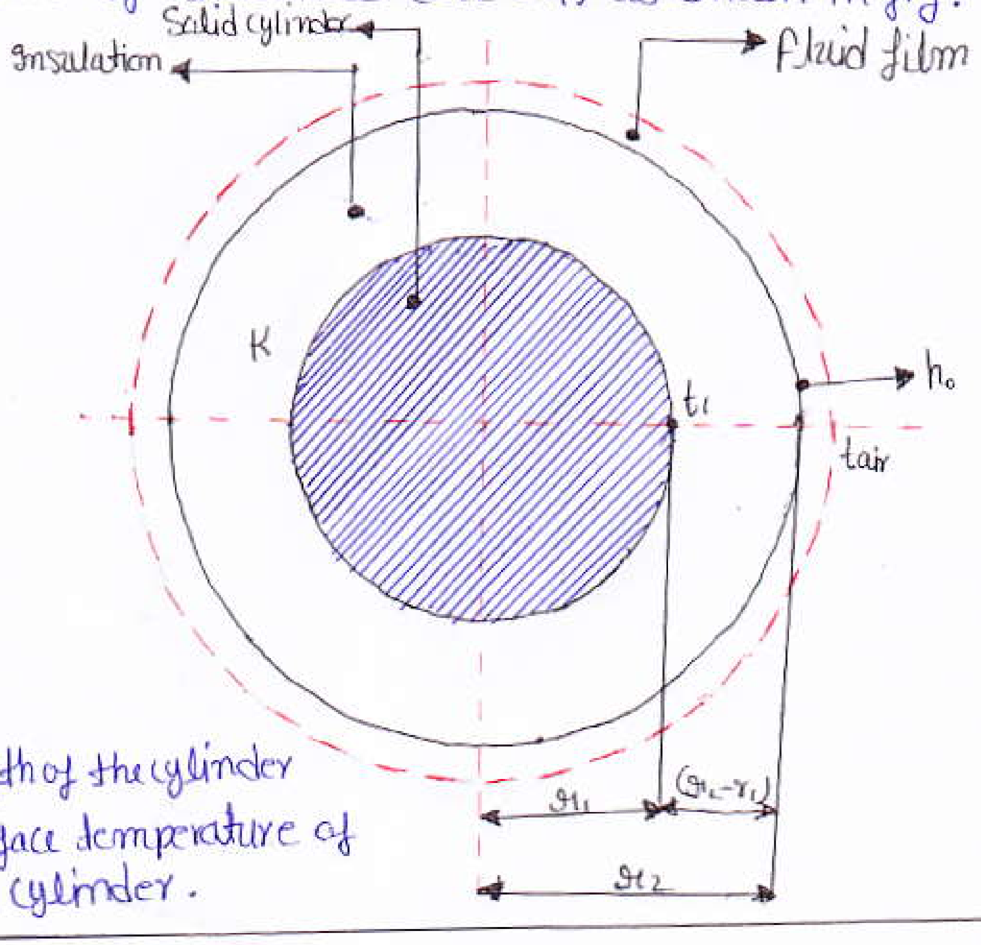
(i) It prevent the heat flow from the system to the surrounding.
(ii) It prevent the heat flow from the surrounding to the system

Critical thickness ⇒

The thickness upto which heat flow increase and after which heat flow decrease is termed as Critical thickness. In case of sphere and cylinder it is called critical radius.

Drive a relation for critical radius of cylinder :-

Consider a solid cylinder of radius r_1 insulated with an insulation of thickness $(r_2 - r_1)$ as shown in fig.



Let,
L = length of the cylinder
t_1 = Surface temperature of the cylinder.

t_{air} = Temperature of air,
h_o = Heat transfer coefficient at outer surface of the insulation, and
K = Thermal conductivity of insulating material,

Then the rate of heat transfer from the surface of the solid cylinder to the surrounding is given by.

$$Q = \frac{2\pi L(t_1 - t_{air})}{\frac{\ln(r_2/r_1)}{K} + \frac{1}{h_o r_2}} \quad \text{---(i)}$$

From equation (i) it is evident that as r_2 increase, the factor $\ln(r_2/r_1)/K$ increase but the factor $1/h_o r_2$ decrease. Thus Q becomes maximum when the denominator $\left[\frac{\ln(r_2/r_1)}{K} + \frac{1}{h_o r_2}\right]$ becomes minimum.

The required condition is

$$\frac{d}{dr_2}\left[\frac{\ln(r_2/r_1)}{K} + \frac{1}{h_o r_2}\right] = 0 \quad [r_2 \text{ being the only variable}]$$

$$\frac{1}{K}\cdot\frac{1}{r_2} + \frac{1}{h_o}\left(-\frac{1}{r_2^2}\right) = 0$$

or

$$\frac{1}{K} - \frac{1}{h_o r_2} = 0$$

$$h_o \cdot r_2 = K$$

$$\boxed{r_2 (= r_c) = \frac{K}{h_o}}$$

The above relation represents the condition for minimum resistance and consequently maximum heat flow rate. The insulation radius on which resistance to heat flow is minimum is called critical radius (r_c).

The critical radius r_c is dependent on the thermal quantity K and h_o is independent of r_1 (i.e cylinder radius).

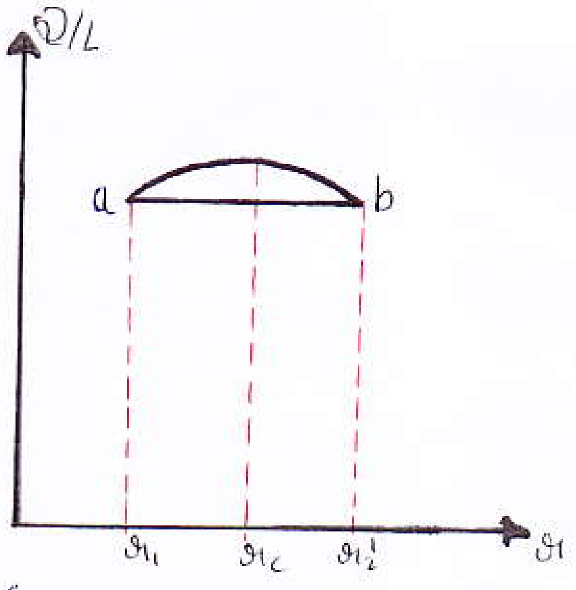
It may be noted that if the second derivative of the denominator is evaluated, it will come out to be possitive, this would verify that heat flow rate will be maximum, when $r_2 = r_c$.

Case I :-

For cylinderical bodies with $r_1 < r_c$, the heat transfer increase by adding insulation till $r_2 = r_c$ as shown in fig. (1) If insulation thickness is further increased, the rate of heat loss will decreases from this peak value, but until a certain amount of insulation denoted by r_2' at b is added the heat loss rate is still greater for the solid cylinder. This happenes when r_1 is small and r_c is large, viz, the thermal conductivity of the insulation k is high and h_o is low. A paractical application would be the insulation of electric cables which should be a good insulator for current but poor for heat.

Case II

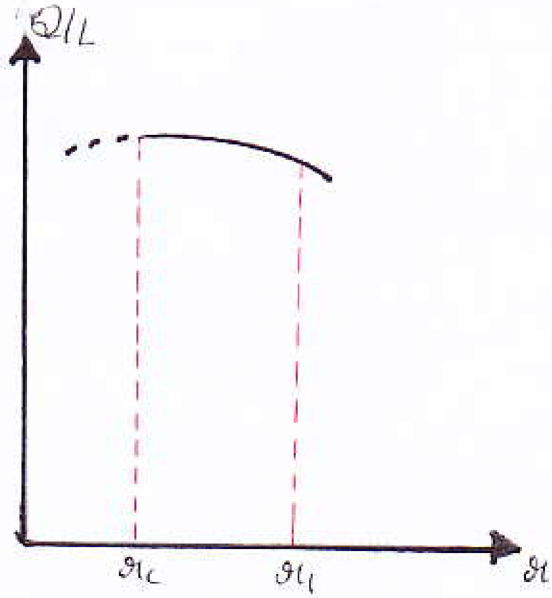
For cylinderical bodies, with $r_1 > r_c$, the heat transfer decreases by adding insulation [fig. 2]. This happenes when r_1 is large and r_c is small, viz, a good insulating material is used with low K and h_o is high. In steam and refrigeration pipes heat insulation is the main objective. For insulation to to be properly effective in restricting heat transmission the outer radius must be greater than or equal to the critical radius.



(cylinder radius)

$r_1 \leq r_c = \frac{K}{h_o}$

(1)



(cylinder radius)

$r_1 > r_c = \frac{K}{h_o}$

(2)

Q6} Give the significance of critical thickness radius.

Case I :-

For cylinderical bodies with $r_1 < r_c$, the heat transfer increase by adding till $r_2 = r_c$ as shown in fig. (1) If insulation thickness is further increased, the rate of heat loss will decrease from this peak value, but until a certain amount of insulation denoted by r_2' at b is added the heat loss rate is still greater for the solid cylinder. This happenes when r_1 is small and r_c is large, viz, the thermal conductivity of the insulation K is high and h_0 is low. A paractical application would be the insulation of electric cables which should be a good insulator for current but poor for heat.

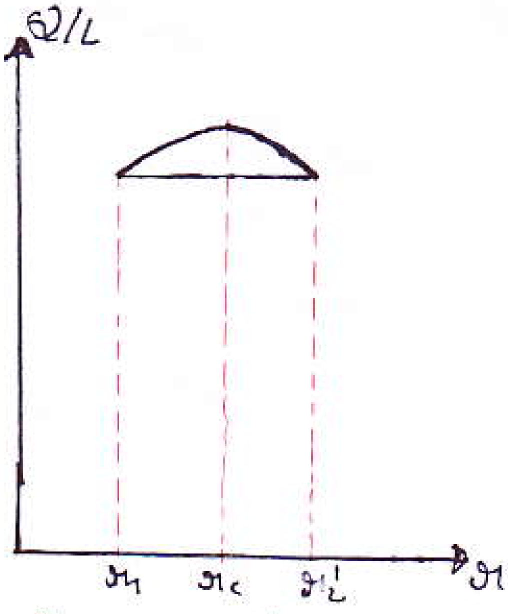
When $r_1 < r_c$ = Minimum.

Case II :-

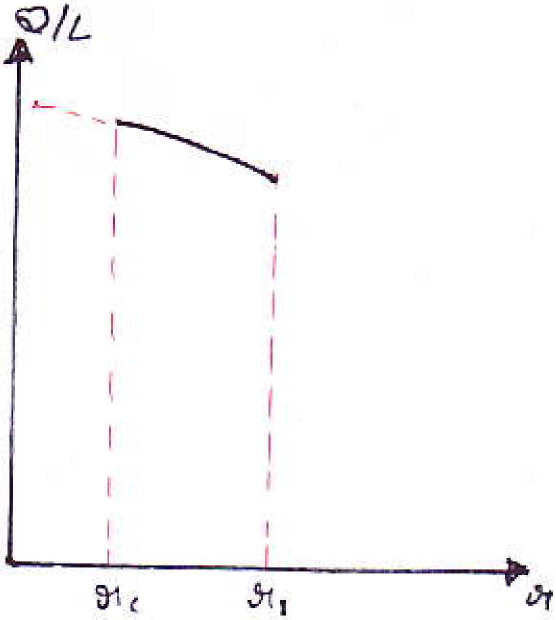
For cylinder bodies, with $r_1 > r_c$, the heat transfer decrease by adding insulation [fig 2].

When $r_1 < r_c$ = Minimum

$r_1 > r_c$ = Maximum



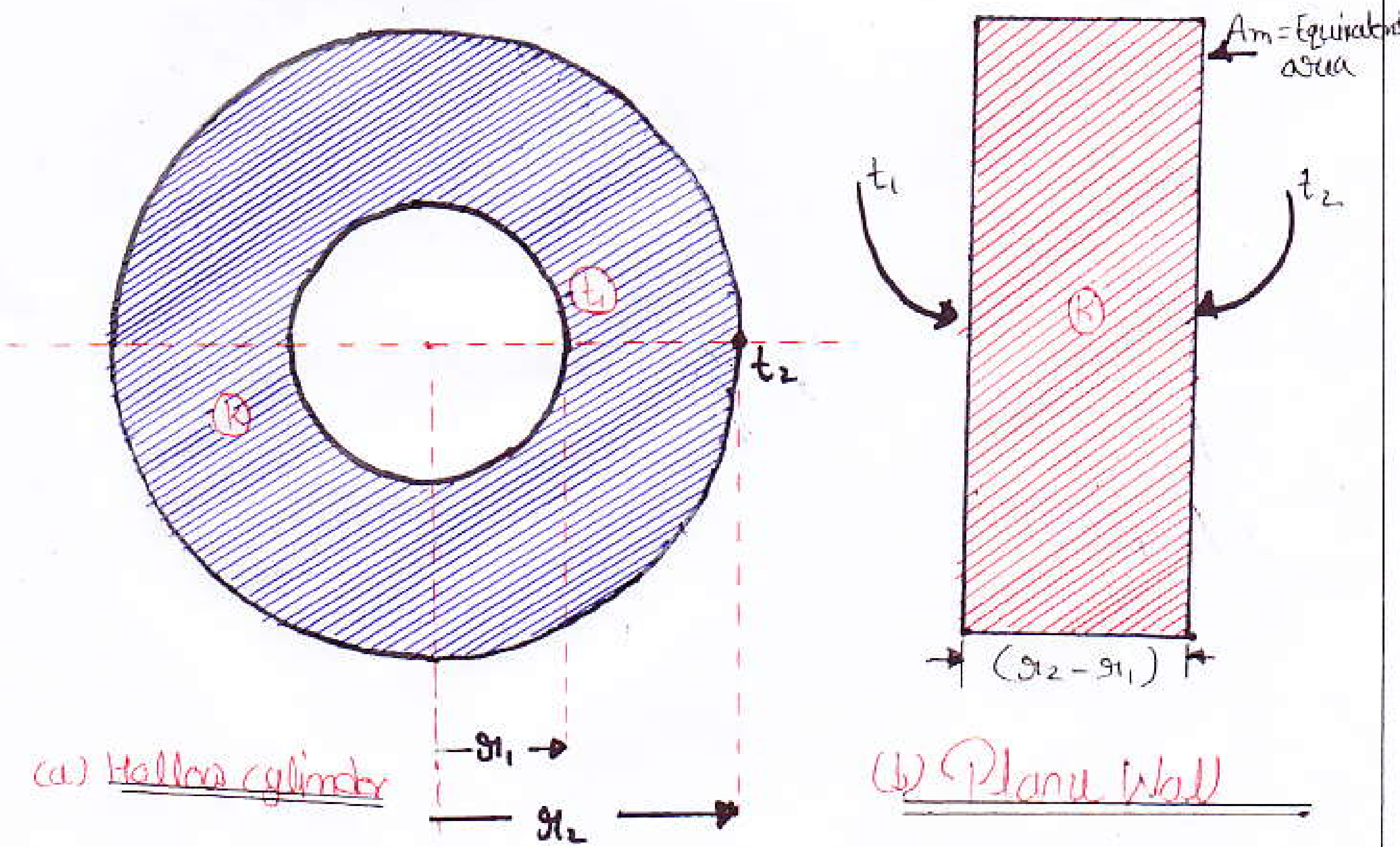
$r_1 \leq r_c = K/h_0$
(1)



$r_1 > r_c = K/h_0$

Q7] What is equivalent log mean area for a cylinder? Derive the relation for it.

Invariably it is considered convenient to have an expression for the heat flow through a hollow cylinder of the same form as that for a plane wall. Then thickness will be equal to $(r_2 - r_1)$ and the area A will be an equivalent area Am as shown in fig.. Now expressions for heat flow through the hollow cylinder and plane wall will be as fallows.



(a) Hollow cylinder

(b) Plane Wall

$$Q = \frac{\cancel{r_2 - r_1}\,(t_1 - t_2)}{\dfrac{\ln(r_2/r_1)}{2\pi K L}}$$ --- Heat flow through cylinder

$$Q = \frac{(t_1 - t_2)}{\dfrac{(r_2 - r_1)}{K A_m}}$$ ---- Heat flow through plane wall

A_m is so chosen that heat flow through cylinder and plane wall be equal for the same thermal potential.

$$\therefore \quad \frac{t_1 - t_2}{\frac{\ln(r_2/r_1)}{2\pi KL}} = \frac{(t_1 - t_2)}{\frac{(r_2 - r_1)}{KA_m}}$$

or

$$\frac{\ln(r_2/r_1)}{2\pi KL} = \frac{(r_2 - r_1)}{K A_m}$$

or

$$A_m = \frac{2\pi L(r_2 - r_1)}{\ln(r_2/r_1)}$$

$$A_m = \frac{2\pi L r_2 - 2\pi L r_1}{\ln(2\pi L r_2 / 2\pi L r_1)}$$

$$A_m = \frac{A_o - A_i}{\ln(A_o - A_i)}$$

Where A_i and A_o are inside and outside surfaces area of the cylinder.

The expression is known as logarithmic mean area of plane wall and hollow cylinder by the use of this expression a cylinder can be transformed into a plane wall and problem can be solved easily.

If $\frac{A_o}{A_i} < 2$ then we take

$A_{av} = \frac{A_i + A_o}{2}$ which is within 4% of A_m

Further $A_m = 2\pi r_m L = \frac{2\pi L(r_2 - r_1)}{\ln(r_2/r_1)}$

obviously, logarithmic mean radius of the hollow cylinder is

$$r_m = (r_2 - r_1)/\ln(r_2/r_1)$$

Q8) A pipe ($K = 180\ W/m°C$) having inner and outer diameter 80 mm and 100 mm respectively is located in a space at 25°C. Hot gases at temperature 160°C flow through the pipe. Neglecting surface heat transfer coefficient, calculate:

(i) The heat loss through the pipe per unit length.

(ii) The temperature at a point halfway between the inner and outer surface and.

(iii) The surfaces area normal to the direction of heat flow so that the heat transfer through the pipe can be determined by considering material of pipe as a plane wall of the same thickness.

Ans Inner diameter of the pipe, $r_i = \frac{80}{2} = 40mm = 0.04m$

Outer diameter of the pipe, $r_o = \frac{100}{2} = 50\ mm = 0.05m$

Temperature of hot gases, $t_i = 160°C$

Temperature of space in which the pipe is located, $t_o = 25°C$

Thermal conductivity of pipe material, $K = 180\ W/m°C$

(i) The heat loss through the pipe per unit length Q :-

$$Q = \frac{\Delta t}{R_{th}} = \frac{(160-25)}{\left[\frac{\ln(r_o/r_i)}{2\pi KL}\right]} = \frac{135}{\frac{\ln(0.05/0.04)}{2\pi \times 180 \times 1}}$$

$$\boxed{Q = 684229\ W}$$

(ii) The temperature at a point halfway between the inner and outer surfaces, t :-

Radius at halfway through the pipe wall

$$r = \frac{r_i + r_o}{2} = \frac{40+50}{2} = 45 = 0.045m$$

Thermal resistance of the pipe upto its mid plane

$$= \frac{\ln(r/r_i)}{2\pi KL} = \frac{\ln(0.045/0.04)}{2\pi \times 180 \times 1} = 0.0414 \times 10^{-4}\ °C/W$$

As the same heat flows through each section

$$684229 = \frac{(t_i - t)}{1.0414 \times 10^{-4}} = \frac{(160 - t)}{1.0414 \times 10^{-4}}$$

$$t = 160 - 684229 \times 1.0414 \times 10^{-4}$$

$$\boxed{t = 88.74°C}$$

(iii) Equivalent log-mean area, Am :

$$A_m = \frac{A_o - A_i}{\ln(A_o/A_i)} = \frac{2\pi L(r_o - r_i)}{\ln(r_o/r_i)}$$

$$= \frac{2\pi \times 1 \times (0.05 - 0.04)}{\ln(0.05/0.04)}$$

$$\boxed{A_m = 0.2816 \, m^2}$$

Check :-

$$Q = \frac{K A_m (t_i - t_o)}{(r_o - r_i)} = \frac{180 \times 0.2616 (160 - 25)}{(0.05 - 0.04)}$$

$$\boxed{Q = 684288 \, W}$$

Q9) Hot air at a temperature of 60°C is flowing through a steel pipe of 100 mm diameter. The pipe is covered with two layer of different insulating layer of thickness 50 and 30 mm and their thermal conductivities are 0.23 and 0.37 W/m°C. The inside and outside heat transfer coefficient are 58 and 12 W/m²°C the atm temperature is at 25°C. Find the rate of heat loss from a 50m length of pipe.

Ans From fig.

$$r_1 = 100/2 = 50 mm = 0.05 m$$

$$r_2 = 50 + 50 = 100 mm = 0.1 m$$

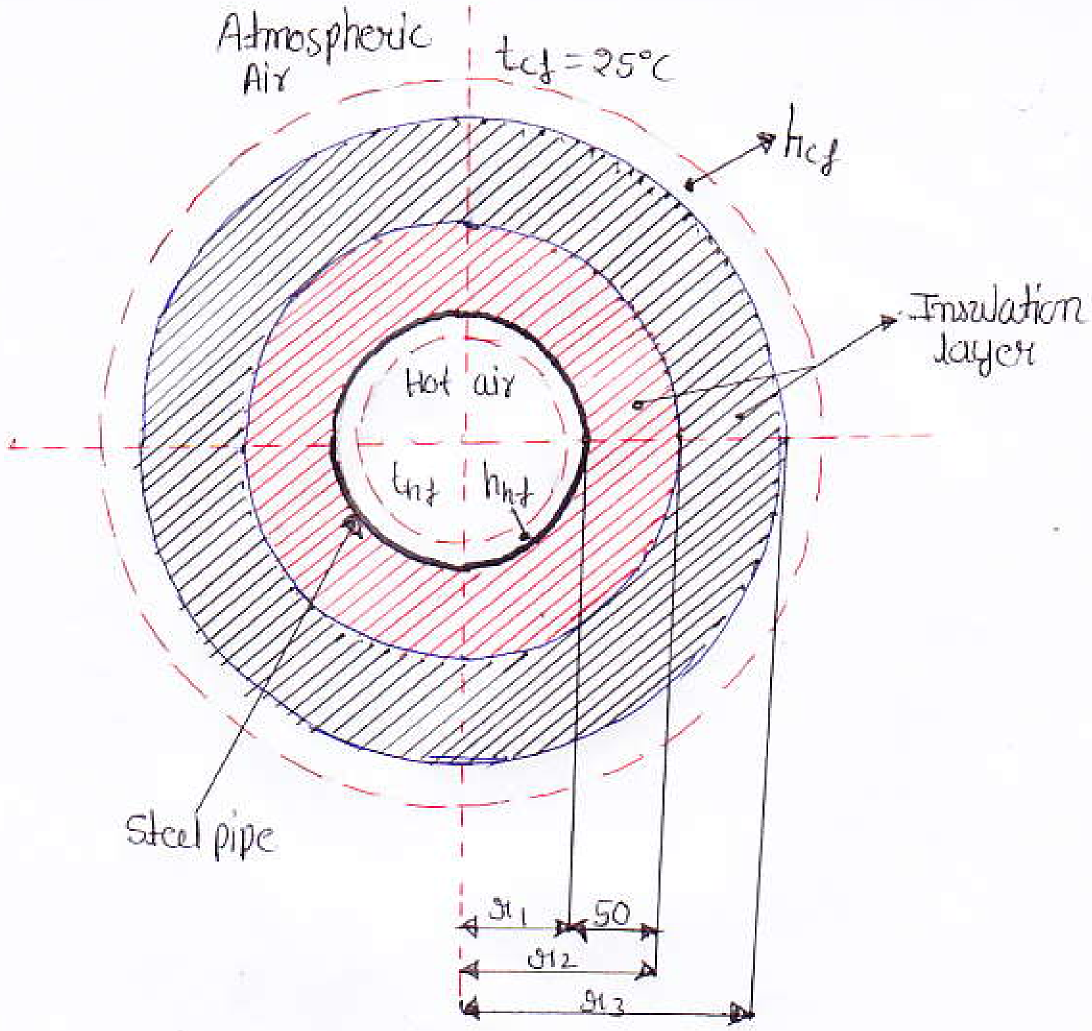
$$r_3 = 50 + 50 + 30 = 130 mm = 0.13 m$$

$$K_A = 0.23 \, W/m°C, \quad K_B = 0.37 \, W/m°C$$

$$h_{hf} = 58 \, W/m^2°C, \quad h_{cf} = 12 \, W/m^2°C$$

$$t_{hf} = 60°C, \quad t_{cf} = 25°C$$

Length of pipe $L = 50 \, m$



Rate of heat loss Q :-

Rate of heat loss is given by

$$Q = \frac{2\pi (t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} r_1} + \frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B} + \frac{1}{h_{cf} r_3}\right]}$$

$$= \frac{2 \times \pi \times 50 (60 - 25)}{\left[\frac{1}{58 \times 0.05} + \frac{\ln(0.1/0.05)}{0.23} + \frac{\ln(0.13/0.1)}{0.37} + \frac{1}{12 \times 0.13}\right]}$$

$$= \frac{10995.5}{0.3448 + 3.0136 + 0.709 + 0.641} = \frac{10995.5}{4.708}$$

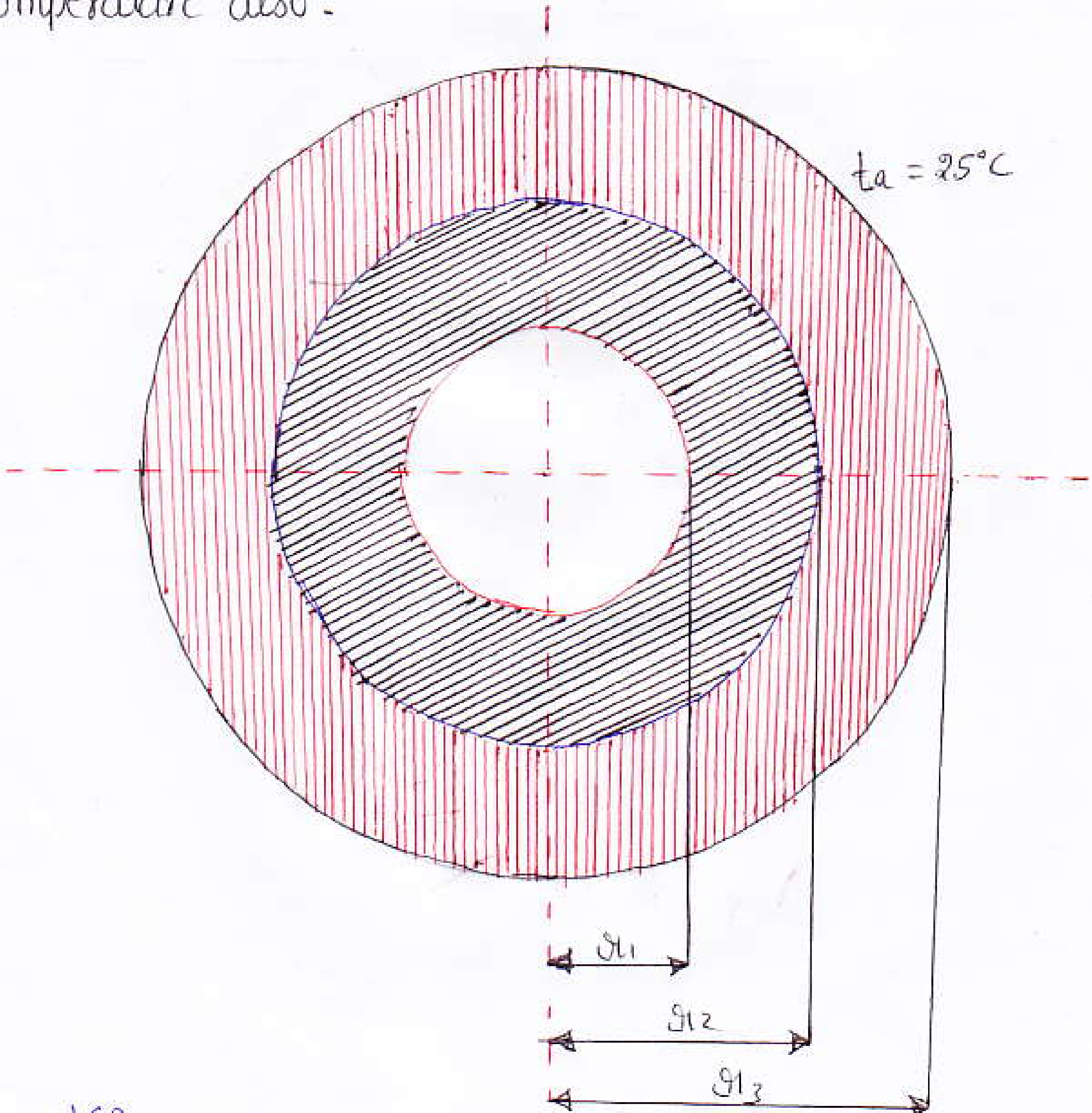
$$\boxed{Q = 2.3345 \text{ KW}}$$

Q19 An insulated steam pipe 160 mm inner and 180 mm outer diameter is covered with insulation of 40 mm thickness and carries steam at 200°C

$K_{(pipe)} = 29\ W/m°C$ and $K_{(insulation)} = 0.23\ W/m°C$

$h_i = 11.6\ W/m^2°C$ and $h_o = 23.2\ W/m^2°C$.

If is the temperature of air surrounding the pipe is 25°C. Calculate the rate of heat loss to surrounding from the pipe of 5 m length. Also find the interface temperature also.



$r_1 = \frac{160}{2} = 80\ mm = 0.08\ m$,

$r_2 = \frac{180}{2} = 90\ mm = 0.09\ m$,

$r_3 = 90 + 40 = 130\ mm = 0.13\ m$,

$t_o = 200°C$

$t_a = 25°C$

$h_i = 11.6\ W/m^2°C$

$h_o = 23.2\ W/m^2°C$

$K_p = 29\ W/m°C$

$K_i = 0.23\ W/m°C$

$L =$

(1) <u>Rate of heat loss</u> :→

$$Q = \frac{2\pi L (t_o - t_a)}{\frac{1}{h_i r_1} + \frac{\ln(r_2/r_1)}{K_p} + \frac{\ln(r_3/r_2)}{K_i} + \frac{1}{h_o r_3}}$$

$$Q = \frac{2 \times 3.14 \times 5 (200-25)}{\frac{1}{11.2 \times 0.08} + \frac{\ln(0.09/0.08)}{29} + \frac{\ln(0.13/0.09)}{0.23} + \frac{1}{23.2 \times 0.13}}$$

$$= \frac{31.4 \times 1750}{1.077 + 0.000406 + 1.5988 + 0.3315}$$

$$= \frac{5495}{3.014}$$

$$= 1823.15$$

$$\boxed{Q = 1823\ W}$$

(2) <u>Temperature at interface 1</u> :⇒

$$Q = \frac{2\pi L (t_o - t_1)}{\frac{1}{h_i r_1}} = \frac{2 \times 3.14 \times 5 (200 - t_1)}{1.077}$$

$$1823 = \frac{31.4 (200 - t_1)}{1.077}$$

$$62.52 = 200 - t_1$$

$$t_1 = 200 - 62.52$$

$$t_1 = 137.47\ ^\circ C$$

3) Temperature at interface 2 :⇒

$$Q = \frac{2\pi L(t_0 - t_2)}{\frac{1}{h_i r_1} + \frac{\ln(r_2/r_1)}{k_p}} = \frac{2 \times 3.14 \times 5(200 - t_2)}{1.077 + 0.000406}$$

$$1823 = \frac{31.4(200 - t_2)}{1.081}$$

$$\boxed{t_2 = 137.24°C}$$

4) Temperature at interface 3 :⇒

$$Q = \frac{2\pi L(t_0 - t_3)}{\frac{1}{h_i r_1} + \frac{\ln(r_2/r_1)}{k_p} + \frac{\ln(r_3/r_2)}{k_i}}$$

$$1823 = \frac{2\pi \times 5(200 - t_3)}{1.077 + 0.000406 + 1.5988}$$

$$200 - t_3 = \frac{4878.72}{31.4}$$

$$200 - t_3 = 155.37$$

$$\boxed{t_3 = 44.6°C}$$

Q14 A 160 mm diameter pipe carrying saturated steam is covered by a layer of lagging of thickness of 40 mm ($K = 0.8\ W/m^\circ C$). later an extra layer of lagging 10 mm thickness ($K = 1.2\ W/m^\circ C$) is added. If the surrounding temperature remains constant and heat transfer coefficient for both lagging material is $10\ W/m^2 {}^\circ C$. Determine the percentage change in the rate of heat loss due to extra lagging layer.

Ans

Case I :- Without extra layer of lagging

from fig (a)

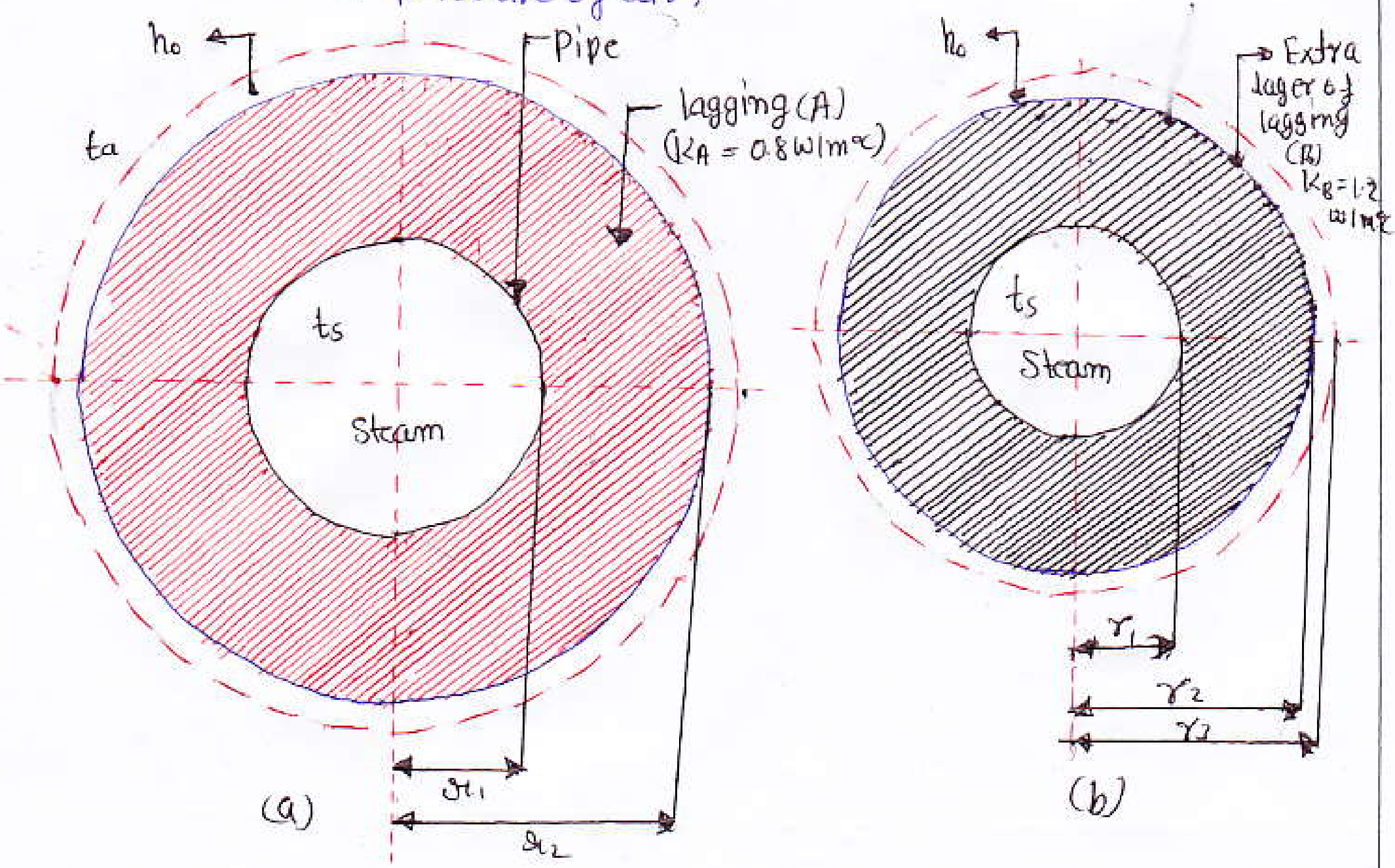
$$r_1 = \frac{160}{2} = 80\,mm = 0.8\,m$$

$$r_2 = 80 + 40 = 120\,mm = 0.12\,m$$

$$K_A = 0.8\ W/m^\circ C,\ h_o = 10\ W/m^2 {}^\circ C$$

let, t_s = temperature of steam, and

t_a = Temperature of air.



consider unit length of the pipe in both the cases (case I and case II) and neglected internal heat transfer coefficient (being not given) and also neglecting the resistance of the pipe (as thickness and conductivity of the pipe are not given), the heat flow rate is given as.

$$Q_1 = \frac{2\pi(t_s - t_a)}{\frac{\ln(r_2/r_1)}{K_A} + \frac{1}{h_o r_2}} = \frac{2\pi(t_s - t_a)}{\frac{\ln(0.12/0.08)}{0.8} + \frac{1}{10\times 0.12}}$$

$$= \frac{2\pi(t_s - t_a)}{1.340} \quad \text{----(i)}$$

Case II With extra layer of lagging :⇒

$r_3 = 120 + 10 = 130\text{ mm} = 0.13\text{ m}$

$K_B = 1.2\ W/m^\circ C, \quad h_o = 10\ W/m^2\,^\circ C$

$$Q_2 = \frac{2\pi(t_s - t_a)}{\frac{\ln(r_2/r_1)}{K_A} + \frac{\ln(r_3/r_2)}{K_B} + \frac{1}{h_o r_3}}$$

$$= \frac{2\pi(t_s - t_a)}{\frac{\ln(0.12/0.08)}{0.8} + \frac{\ln(0.13/0.12)}{1.2} + \frac{1}{10\times 0.13}}$$

$$= \frac{2\pi(t_s - t_a)}{1.343} \quad \text{---(ii)}$$

The percentage decrease in heat flow due to extra addition of insulation can be calculated using eq^n (i) and (ii) as follows:-

$$\frac{Q_1 - Q_2}{Q_1} = \left[\frac{(1/1.34) - (1/1.343)}{1/1.343}\right] = 0.00223$$

$$= 0.223\%$$

Q12) A steam pipe 10 cm ID, and 11 cm OD, is covered with an insulating substance ($k = 1 W/mK$). The steam temperature and ambient temperature are 200°C and 20°C. If the convective heat transfer coefficient between the insulation surface and air is $8 W/m^2K$. Find the critical radius of insulation, heat loss per meter of pipe and the outer temperature for the same.

Ans.

$$Q = \frac{2\pi L (T_2 - T_1)}{\frac{1}{h r_0} + \frac{\ln(r_0/r_i)}{K}}$$

$$r_c = r_0 = \frac{K}{h}$$

$$r_c = \frac{1}{8} = 0.125 m$$

$$\boxed{r_c = 12.5 cm}$$

Heat loss per meter of pipe :-

$$\frac{Q}{L} = \frac{2\pi (T_2 - T_1)}{\frac{1}{8 \times 0.125} + \frac{\ln(0.125/0.055)}{1}}$$

$$= \frac{2\pi (200-20)}{1 + 0.8209}$$

$$= \frac{1130.4}{1.82089}$$

$$\boxed{\frac{Q}{L} = 620 W}$$

Outer temperature :-

$$\frac{Q}{L} = \frac{2\pi(T_3 - T_2)}{\frac{1}{h r_c}}$$

$$620 = 2\pi(T_3 - 20)$$

$$T_3 - 20 = \frac{620}{2\pi}$$

$$T_3 - 20 = 98.67$$

$$\boxed{T_3 = 118.67°C}$$

Q13 An electrical conductor of copper with diameter of 1mm is covered with a plastic insulation of thickness 1mm. The temperature of its surrounding is 20°c. find the maximum current carried by the conductor so that no part of the plastic is above 80°c. The following data is given :-

K of copper = 400 W/mK, K of plastic 0.5 W/mK, h = 8 W/m²K

ρ = (Specific electric resistance of copper) = 3×10^{-8} ohm-m.

Discuss the effect of increase or decrease of insulation on the current carrying capacity of the conductor.

Ans

The electrical resistance / meter length = $\frac{\text{Specific resistance}}{\text{Cross-sectional area}}$

$$= \frac{3 \times 10^{-8}}{\pi(0.05 \times 10^{-3})^2}$$

$$= 3.8197\ \Omega/m$$

$$Q = I^2 R$$

$$= 3.81971\ I^2\ W$$

The thermal resistance of the convection film insulation per meter length is

$$R_{th} = \frac{1}{2\pi r_o h} + \frac{\ln(r_o/r_i)}{2\pi k}$$

$$= \frac{1}{(6.28)(0.0015)(8)} + \frac{\ln(0.0015/0.0005)}{2\pi(0.5)}$$

$$\boxed{R_{th} = 13.60}$$

$$Q = \frac{T_i - T_\infty}{R_{th}} = \frac{80 - 20}{13.60}$$

$$Q = 4.41\ W$$

And $$Q = I^2 R$$

$$I^2 = \frac{Q}{R}$$

Or $$4.41 = 3.81971\ I^2$$

$$I^2 = \frac{4.41}{3.81971}$$

$$I = 1.074\ A$$

The maximum safe current limit is 1.074 A for plastic temperature not to exceed 80°C

The critical radius, $r_c = \frac{k}{h} = \frac{0.5}{8} = 0.062\,m$

$$\boxed{r_c = 6.2\,cm}$$