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Ph : 0141 - 2101150.
www.nodia.co.in
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SYLLABUS

ENGINEERING MATHEMATICS

Linear Algebra:
Matrix algebra, Systems of linear equations, Eigen values and eigen vectors.

Calculus:
Functions of single variable, Limit, continuity and differentiability, Mean value theorems, Evaluation of definite and improper integrals, Partial derivatives, Total derivative,Maxima and minima, Gradient, Divergence and Curl, Vector identities, Directional derivatives, Line, Surface and Volume integrals, Stokes, Gauss and Green's theorems.

Differential equations:
First order equations (linear and nonlinear), Higher order linear differential equationswith constant coefficients, Cauchy's and Euler's equations, Initial and boundary value problems, Laplace transforms, Solutions of one dimensional heat and wave equations and Laplace equation.

Complex variables:
Analytic functions, Cauchy's integral theorem, Taylor and Laurent series.
Probability and Statistics:
Definitions of probability and sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Random variables, Poisson, Normal and Binomial distributions.

Numerical Methods:
Numerical solutions of linear and non-linear algebraic equations Integration by trapezoidal and Simpson's rule, single and multi-step methods for differential equations.

APPLIED MECHANICS AND DESIGN

Engineering Mechanics:
Free body diagrams and equilibrium; trusses and frames; virtual work; kinematics and dynamics of particles and of rigid bodies in plane motion, including impulse and momentum (linear and angular) and energy formulations; impact.

Strength of Materials:
Stress and strain, stress-strain relationship and elastic constants, Mohr's circle for plane stress and plane strain, thin cylinders; shear force and bending moment diagrams; bending and shear stresses; deflection of beams; torsion of circular shafts; Euler's theory of columns; strain energy methods; thermal stresses.

Theory of Machines:
Displacement, velocity and acceleration analysis of plane mechanisms; dynamic analysis of slider-crank mechanism; gear trains; flywheels.

Vibrations:
Free and forced vibration of single degree of freedom systems; effect of damping; vibration isolation; resonance, critical speeds of shafts.

Design:
Design for static and dynamic loading; failure theories; fatigue strength and the S-N diagram; principles of the design of machine elements such as bolted, riveted and welded joints, shafts, spur gears, rolling and sliding contact bearings, brakes and clutches.

FLUID MECHANICS AND THERMAL SCIENCES

Fluid Mechanics:
Fluid properties; fluid statics, manometry, buoyancy; control-volume analysis of mass, momentum and energy; fluid acceleration; differential equations of continuity and momentum; Bernoulli's equation; viscous flow of incompressible fluids; boundary layer; elementary turbulent flow; flow through pipes, head losses in pipes, bends etc.

Heat-Transfer:
Modes of heat transfer; one dimensional heat conduction, resistance concept, electrical analogy, unsteady heat conduction, fins; dimensionless parameters in free and forced convective heat transfer, various correlations for heat transfer in flow over flat plates and through pipes; thermal boundary layer; effect of turbulence; radiative heat transfer, black and grey surfaces, shape factors, network analysis; heat exchanger performance,
LMTD and NTU methods.

Thermodynamics:
Zeroth, First and Second laws of thermodynamics; thermodynamic system and processes; Carnot cycle. irreversibility and availability; behaviour of ideal and real gases, properties of pure substances, calculation of work and heat in ideal processes; analysis of thermodynamic cycles related to energy conversion.

Applications:
Power Engineering: Steam Tables, Rankine, Brayton cycles with regeneration and reheat. I.C. Engines: air-standard Otto, Diesel cycles. Refrigeration and air-conditioning: Vapour refrigeration cycle, heat pumps, gas refrigeration, Reverse Brayton cycle; moist air: psychrometric chart, basic psychrometric processes. Turbomachinery: Peltonwheel,Francis and Kaplan turbines - impulse and reaction principles, velocity diagrams.

MANUFACTURING AND INDUSTRIAL ENGINEERING

Engineering Materials :
Structure and properties of engineering materials, heat treatment, stress-strain diagrams for engineering materials.

Metal Casting:
Design of patterns, moulds and cores; solidification and cooling; riser and gating design, design considerations.

Forming:
Plastic deformation and yield criteria; fundamentals of hot and cold working processes; load estimation for bulk (forging, rolling, extrusion, drawing) and sheet (shearing, deep drawing, bending) metal forming processes; principles of powder metallurgy.

Joining:
Physics of welding, brazing and soldering; adhesive bonding; design considerations in welding.

Machining and Machine Tool Operations:
Mechanics of machining, single and multi-point cutting tools, tool geometry and materials, tool life and wear; economics of machining; principles of non-traditional machining processes; principles of work holding, principles of design of jigs and fixtures

Metrology and Inspection:
Limits, fits and tolerances; linear and angular measurements; comparators; gauge design; interferometry; form and finish measurement; alignment and testing methods; tolerance analysis in manufacturing and assembly.

Computer Integrated Manufacturing:
Basic concepts of CAD/CAM and their integration tools.

Production Planning and Control:
Forecasting models, aggregate production planning, scheduling, materials requirement planning.

Inventory Control:
Deterministic and probabilistic models; safety stock inventory control systems.

Operations Research:
Linear programming, simplex and duplex method, transportation, assignment, network flow models, simple queuing models, PERT and CPM.

GENERAL APTITUDE (GA)

Verbal Ability:
English grammar, sentence completion, verbal analogies, word groups, instructions, critical reasoning and verbal deduction.
Numerical Ability:
Numerical computation, numerical estimation, numerical reasoning and data interpretation.

CONTENTS

CHAPTER 1

ENGINEERING MATHEMATICS

YEAR 2012 ONE MARK

MCQ 1.1 The area enclosed between the straight line $y = x$ and the parabola $y = x^2$ in the x-y plane is

(A) 1/6 (B) 1/4

(C) 1/3 (D) 1/2

MCQ 1.2 Consider the function $f(x) = |x|$ in the interval $-1 \le x \le 1$. At the point $x = 0$, $f(x)$ is

(A) continuous and differentiable

(B) non-continuous and differentiable

(C) continuous and non-differentiable

(D) neither continuous nor differentiable

MCQ 1.3 $\lim_{x \to 0}\left(\frac{1 - \cos x}{x^2}\right)$ is

(A) 1/4 (B) 1/2

(C) 1 (D) 2

MCQ 1.4 At $x = 0$, the function $f(x) = x^3 + 1$ has

(A) a maximum value (B) a minimum value

(C) a singularity (D) a point of inflection

MCQ 1.5 For the spherical surface $x^2 + y^2 + z^2 = 1$, the unit outward normal vector at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ is given by

(A) $\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j$ (B) $\frac{1}{\sqrt{2}}i - \frac{1}{\sqrt{2}}j$

(C) k (D) $\frac{1}{\sqrt{3}}i + \frac{1}{\sqrt{3}}j + \frac{1}{\sqrt{3}}k$

YEAR 2012 TWO MARKS

MCQ 1.6 The inverse Laplace transform of the function $F(s) = \frac{1}{s(s+1)}$ is given by

(A) $f(t) = \sin t$

(B) $f(t) = e^{-t}\sin t$

(C) $f(t) = e^{-t}$

(D) $f(t) = 1 - e^{-t}$

MCQ 1.7 For the matrix $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$, ONE of the normalized eigen vectors given as

(A) $\begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$

(B) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

(C) $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} \end{pmatrix}$

(D) $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$

MCQ 1.8 A box contains 4 red balls and 6 black balls. Three balls are selected randomly from the box one after another, without replacement. The probability that the selected set contains one red ball and two black balls is

(A) 1/20

(B) 1/12

(C) 3/10

(D) 1/2

MCQ 1.9 Consider the differential equation $x^2(d^2y/dx^2) + x(dy/dx) - 4y = 0$ with the boundary conditions of $y(0) = 0$ and $y(1) = 1$. The complete solution of the differential equation is

(A) x^2

(B) $\sin\left(\frac{\pi x}{2}\right)$

(C) $e^x \sin\left(\frac{\pi x}{2}\right)$

(D) $e^{-x}\sin\left(\frac{\pi x}{2}\right)$

MCQ 1.10

$$\begin{aligned} x + 2y + z &= 4 \\ 2x + y + 2z &= 5 \\ x - y + z &= 1 \end{aligned}$$

The system of algebraic equations given above has

(A) a unique solution of $x = 1, y = 1$ and $z = 1$.

(B) only the two solutions of $(x = 1, y = 1, z = 1)$ and $(x = 2, y = 1, z = 0)$

(C) infinite number of solutions

(D) no feasible solution

YEAR 2011 ONE MARK

MCQ 1.11 A series expansion for the function $\sin\theta$ is

(A) $1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}-...$
(B) $\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-...$
(C) $1+\theta+\frac{\theta^2}{2!}+\frac{\theta^3}{3!}+...$
(D) $\theta+\frac{\theta^3}{3!}+\frac{\theta^5}{5!}+...$

MCQ 1.12 What is $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}$ equal to ?

(A) θ
(B) $\sin\theta$
(C) 0
(D) 1

MCQ 1.13 Eigen values of a real symmetric matrix are always

(A) positive
(B) negative
(C) real
(D) complex

MCQ 1.14 The product of two complex numbers $1+i$ and $2-5i$ is

(A) $7-3i$
(B) $3-4i$
(C) $-3-4i$
(D) $7+3i$

MCQ 1.15 If $f(x)$ is an even function and a is a positive real number, then $\int_{-a}^{a}f(x)\,dx$ equals

(A) 0
(B) a
(C) $2a$
(D) $2\int_0^a f(x)\,dx$

YEAR 2011 TWO MARKS

MCQ 1.16 The integral $\int_1^3 \frac{1}{x}dx$, when evaluated by using Simpson's 1/3 rule on two equal sub-intervals each of length 1, equals

(A) 1.000
(B) 1.098
(C) 1.111
(D) 1.120

MCQ 1.17 Consider the differential equation $\frac{dy}{dx}=(1+y^2)x$. The general solution with constant c is

(A) $y=\tan\frac{x^2}{2}+\tan c$
(B) $y=\tan^2\left(\frac{x}{2}+c\right)$
(C) $y=\tan^2\left(\frac{x}{2}\right)+c$
(D) $y=\tan\left(\frac{x^2}{2}+c\right)$

MCQ 1.18 An unbiased coin is tossed five times. The outcome of each toss is either a head or a tail. The probability of getting at least one head is

(A) $\frac{1}{32}$
(B) $\frac{13}{32}$
(C) $\frac{16}{32}$
(D) $\frac{31}{32}$

MCQ 1.19 Consider the following system of equations

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 0 \\ x_2 - x_3 &= 0 \\ x_1 + x_2 &= 0 \end{aligned}$$

This system has

(A) a unique solution

(B) no solution

(C) infinite number of solutions

(D) five solutions

YEAR 2010 ONE MARK

MCQ 1.20 The parabolic arc $y = \sqrt{x}$, $1 \leq x \leq 2$ is revolved around the x-axis. The volume of the solid of revolution is

(A) $\pi/4$

(B) $\pi/2$

(C) $3\pi/4$

(D) $3\pi/2$

MCQ 1.21 The Blasius equation, $\frac{d^3 f}{d\eta^3} + \frac{f}{2}\frac{d^2 f}{d\eta^2} = 0$, is a

(A) second order nonlinear ordinary differential equation

(B) third order nonlinear ordinary differential equation

(C) third order linear ordinary differential equation

(D) mixed order nonlinear ordinary differential equation

MCQ 1.22 The value of the integral $\int_{-\infty}^{\infty} \frac{dx}{1 + x^2}$ is

(A) $-\pi$ (B) $-\pi/2$

(C) $\pi/2$ (D) π

MCQ 1.23 The modulus of the complex number $\left(\frac{3 + 4i}{1 - 2i}\right)$ is

(A) 5 (B) $\sqrt{5}$

(C) $1/\sqrt{5}$ (D) $1/5$

MCQ 1.24 The function $y = |2 - 3x|$

(A) is continuous $\forall x \in R$ and differentiable $\forall x \in R$

(B) is continuous $\forall x \in R$ and differentiable $\forall x \in R$ except at $x = 3/2$

(C) is continuous $\forall x \in R$ and differentiable $\forall x \in R$ except at $x = 2/3$

(D) is continuous $\forall x \in R$ except $x = 3$ and differentiable $\forall x \in R$

YEAR 2010 TWO MARKS

MCQ 1.25 One of the eigen vectors of the matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ is

(A) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$
(B) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$
(C) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$
(D) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

MCQ 1.26 The Laplace transform of a function $f(t)$ is $\frac{1}{s^2(s+1)}$. The function $f(t)$ is

(A) $t - 1 + e^{-t}$
(B) $t + 1 + e^{-t}$
(C) $-1 + e^{-t}$
(D) $2t + e^{t}$

MCQ 1.27 A box contains 2 washers, 3 nuts and 4 bolts. Items are drawn from the box at random one at a time without replacement. The probability of drawing 2 washers first followed by 3 nuts and subsequently the 4 bolts is

(A) 2/315
(B) 1/630
(C) 1/1260
(D) 1/2520

MCQ 1.28 Torque exerted on a flywheel over a cycle is listed in the table. Flywheel energy (in J per unit cycle) using Simpson's rule is

Angle (Degree)	0	60°	120°	180°	240°	300°	360°
Torque (N-m)	0	1066	-323	0	323	-355	0

(A) 542
(B) 993
(C) 1444
(D) 1986

YEAR 2009 ONE MARK

MCQ 1.29 For a matrix $[M] = \begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix}$, the transpose of the matrix is equal to the inverse of the matrix, $[M]^T = [M]^{-1}$. The value of x is given by

(A) $-\frac{4}{5}$
(B) $-\frac{3}{5}$
(C) $\frac{3}{5}$
(D) $\frac{4}{5}$

MCQ 1.30 The divergence of the vector field $3xzi + 2xyj - yz^2k$ at a point $(1,1,1)$ is equal to

(A) 7
(B) 4
(C) 3
(D) 0

MCQ 1.31 The inverse Laplace transform of $1/(s^2+s)$ is

(A) $1+e^t$ (B) $1-e^t$

(C) $1-e^{-t}$ (D) $1+e^{-t}$

MCQ 1.32 If three coins are tossed simultaneously, the probability of getting at least one head is

(A) 1/8 (B) 3/8

(C) 1/2 (D) 7/8

YEAR 2009 TWO MARKS

MCQ 1.33 An analytic function of a complex variable $z=x+iy$ is expressed as $f(z)=u(x,y)+iv(x,y)$ where $i=\sqrt{-1}$. If $u=xy$, the expression for v should be

(A) $\frac{(x+y)^2}{2}+k$ (B) $\frac{x^2-y^2}{2}+k$

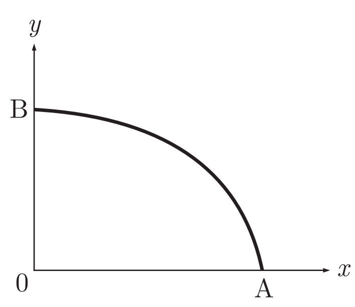
(C) $\frac{y^2-x^2}{2}+k$ (D) $\frac{(x-y)^2}{2}+k$

MCQ 1.34 The solution of $x\frac{dy}{dx}+y=x^4$ with the condition $y(1)=\frac{6}{5}$ is

(A) $y=\frac{x^4}{5}+\frac{1}{x}$ (B) $y=\frac{4x^4}{5}+\frac{4}{5x}$

(C) $y=\frac{x^4}{5}+1$ (D) $y=\frac{x^5}{5}+1$

MCQ 1.35 A path AB in the form of one quarter of a circle of unit radius is shown in the figure. Integration of $(x+y)^2$ on path AB traversed in a counter-clockwise sense is



(A) $\frac{\pi}{2}-1$ (B) $\frac{\pi}{2}+1$

(C) $\frac{\pi}{2}$ (D) 1

MCQ 1.36 The distance between the origin and the point nearest to it on the surface $z^2=1+xy$ is

(A) 1 (B) $\frac{\sqrt{3}}{2}$

(C) $\sqrt{3}$ (D) 2

MCQ 1.37 The area enclosed between the curves $y^2 = 4x$ and $x^2 = 4y$ is

(A) $\frac{16}{3}$ (B) 8

(C) $\frac{32}{3}$ (D) 16

MCQ 1.38 The standard deviation of a uniformly distributed random variable between 0 and 1 is

(A) $\frac{1}{\sqrt{12}}$ (B) $\frac{1}{\sqrt{3}}$

(C) $\frac{5}{\sqrt{12}}$ (D) $\frac{7}{\sqrt{12}}$

YEAR 2008 ONE MARK

MCQ 1.39 In the Taylor series expansion of e^x about $x = 2$, the coefficient of $(x-2)^4$ is

(A) $1/4\,!$ (B) $2^4/4!$

(C) $e^2/4!$ (D) $e^4/4!$

MCQ 1.40 Given that $\ddot{x} + 3x = 0$, and $x(0) = 1$, $\dot{x}(0) = 0$, what is $x(1)$?

(A) -0.99 (B) -0.16

(C) 0.16 (D) 0.99

MCQ 1.41 The value of $\lim_{x \to 8} \frac{x^{1/3} - 2}{(x-8)}$

(A) $\frac{1}{16}$ (B) $\frac{1}{12}$

(C) $\frac{1}{8}$ (D) $\frac{1}{4}$

MCQ 1.42 A coin is tossed 4 times. What is the probability of getting heads exactly 3 times ?

(A) $\frac{1}{4}$ (B) $\frac{3}{8}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

MCQ 1.43 The matrix $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$ has one eigen value equal to 3. The sum of the other two eigen value is

(A) p (B) $p-1$

(C) $p-2$ (D) $p-3$

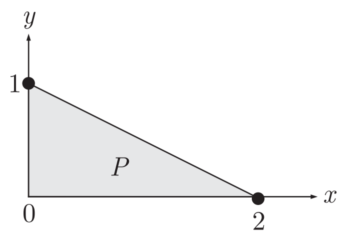
MCQ 1.44 The divergence of the vector field $(x-y)i+(y-x)j+(x+y+z)k$ is

(A) 0 (B) 1

(C) 2 (D) 3

YEAR 2008 TWO MARKS

MCQ 1.45 Consider the shaded triangular region P shown in the figure. What is $\iint_P xydxdy$?



(A) $\frac{1}{6}$ (B) $\frac{2}{9}$

(C) $\frac{7}{16}$ (D) 1

MCQ 1.46 The directional derivative of the scalar function $f(x,y,z)=x^2+2y^2+z$ at the point

$P=(1,1,2)$ in the direction of the vector $a=3i-4j$ is

(A) -4 (B) -2

(C) -1 (D) 1

MCQ 1.47 For what value of a, if any will the following system of equation in x,y and z have a solution ?

$$2x+3y=4$$
$$x+y+z=4$$
$$3x+2y-z=a$$

(A) Any real number (B) 0

(C) 1 (D) There is no such value

MCQ 1.48 Which of the following integrals is unbounded ?

(A) $\int_0^{\pi/4}\tan x dx$ (B) $\int_0^{\infty}\frac{1}{x^2+1}dx$

(C) $\int_0^{\infty}xe^{-x}dx$ (D) $\int_0^1\frac{1}{1-x}dx$

MCQ 1.49 The integral $\oint f(z)dz$ evaluated around the unit circle on the complex plane for $f(z)=\frac{\cos z}{z}$ is

(A) $2\pi i$ (B) $4\pi i$

(C) $-2\pi i$ (D) 0

MCQ 1.50 The length of the curve $y=\frac{2}{3}x^{3/2}$ between $x=0$ and $x=1$ is

(A) 0.27 (B) 0.67

(C) 1 (D) 1.22

MCQ 1.51 The eigen vector of the matrix $\begin{bmatrix}1 & 2\\0 & 2\end{bmatrix}$ are written in the form $\begin{bmatrix}1\\a\end{bmatrix}$ and $\begin{bmatrix}1\\b\end{bmatrix}$. What is $a+b$?

(A) 0 (B) $\frac{1}{2}$

(C) 1 (D) 2

MCQ 1.52 Let $f=y^x$. What is $\frac{\partial^2 f}{\partial x \partial y}$ at $x=2, y=1$?

(A) 0 (B) $\ln 2$

(C) 1 (D) $\frac{1}{\ln 2}$

MCQ 1.53 It is given that $y''+2y'+y=0,\ y(0)=0, y(1)=0$. What is $y(0.5)$?

(A) 0 (B) 0.37

(C) 0.62 (D) 1.13

YEAR 2007 ONE MARK

MCQ 1.54 The minimum value of function $y=x^2$ in the interval [1, 5] is

(A) 0 (B) 1

(C) 25 (D) undefined

MCQ 1.55 If a square matrix A is real and symmetric, then the eigen values

(A) are always real (B) are always real and positive

(C) are always real and non-negative (D) occur in complex conjugate pairs

MCQ 1.56 If $\varphi(x,y)$ and $\psi(x,y)$ are functions with continuous second derivatives, then $\varphi(x,y)+i\psi(x,y)$ can be expressed as an analytic function of $x+i\psi(i=\sqrt{-1})$, when

(A) $\frac{\partial\varphi}{\partial x}=-\frac{\partial\psi}{\partial x},\frac{\partial\varphi}{\partial y}=\frac{\partial\psi}{\partial y}$ (B) $\frac{\partial\varphi}{\partial y}=-\frac{\partial\psi}{\partial x},\frac{\partial\varphi}{\partial x}=\frac{\partial\psi}{\partial y}$

(C) $\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=1$ (D) $\frac{\partial\varphi}{\partial x}+\frac{\partial\varphi}{\partial y}=\frac{\partial\psi}{\partial x}+\frac{\partial\psi}{\partial y}=0$

MCQ 1.57 The partial differential equation $\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial\varphi}{\partial x}+\frac{\partial\varphi}{\partial y}=0$ has

(A) degree 1 order 2 (B) degree 1 order 1

(C) degree 2 order 1 (D) degree 2 order 2

YEAR 2007 TWO MARKS

MCQ 1.58 If $y=x+\sqrt{x+\sqrt{x+\sqrt{x+......\infty}}}$, then $y(2)=$

(A) 4 or 1 (B) 4 only

(C) 1 only (D) undefined

MCQ 1.59 The area of a triangle formed by the tips of vectors $\bar{a},\bar{b}$ and $\bar{c}$ is

(A) $\frac{1}{2}(a-b)\cdot(a-c)$ (B) $\frac{1}{2}|(a-b)\times(a-c)|$

(C) $\frac{1}{2}|a\times b\times c|$ (D) $\frac{1}{2}(a\times b)\cdot c$

MCQ 1.60 The solution of $\frac{dy}{dx}=y^2$ with initial value $y(0)=1$ bounded in the interval

(A) $-\infty\le x\le\infty$ (B) $-\infty\le x\le 1$

(C) $x<1, x>1$ (D) $-2\le x\le 2$

MCQ 1.61 If $F(s)$ is the Laplace transform of function $f(t)$, then Laplace transform of $\int_0^t f(\tau)d\tau$ is

(A) $\frac{1}{s}F(s)$ (B) $\frac{1}{s}F(s)-f(0)$

(C) $sF(s)-f(0)$ (D) $\int F(s)ds$

MCQ 1.62 A calculator has accuracy up to 8 digits after decimal place. The value of $\int_0^{2\pi}\sin x dx$

when evaluated using the calculator by trapezoidal method with 8 equal intervals, to 5 significant digits is

(A) 0.00000 (B) 1.0000

(C) 0.00500 (D) 0.00025

MCQ 1.63 Let X and Y be two independent random variables. Which one of the relations between expectation (E), variance (Var) and covariance (Cov) given below is FALSE ?

(A) $E(XY)=E(X)E(Y)$ (B) $\mathrm{Cov}(X,Y)=0$

(C) $\mathrm{Var}(X+Y)=\mathrm{Var}(X)+\mathrm{Var}(Y)$ (D) $E(X^2Y^2)=(E(X))^2(E(Y))^2$

MCQ 1.64 $\lim_{x \to 0} \frac{e^x - \left(1 + x + \frac{x^2}{2}\right)}{x^3} =$

(A) 0 (B) 1/6

(C) 1/3 (D) 1

MCQ 1.65 The number of linearly independent eigen vectors of $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is

(A) 0 (B) 1

(C) 2 (D) infinite

YEAR 2006 ONE MARK

MCQ 1.66 Match the items in column I and II.

	Column I		Column II
P.	Gauss-Seidel method	1.	Interpolation
Q.	Forward Newton-Gauss method	2.	Non-linear differential equations
R.	Runge-Kutta method	3.	Numerical integration
S.	Trapezoidal Rule	4.	Linear algebraic equations

(A) P-1, Q-4, R-3, S-2 (B) P-1, Q-4, R-2, S-3

(C) P-1. Q-3, R-2, S-4 (D) P-4, Q-1, R-2, S-3

MCQ 1.67 The solution of the differential equation $\frac{dy}{dx} + 2xy = e^{-x^2}$ with $y(0) = 1$ is

(A) $(1 + x)e^{+x^2}$ (B) $(1 + x)e^{-x^2}$

(C) $(1 - x)e^{+x^2}$ (D) $(1 - x)e^{-x^2}$

MCQ 1.68 Let x denote a real number. Find out the INCORRECT statement.

(A) $S = \{x : x > 3\}$ represents the set of all real numbers greater than 3

(B) $S = \{x : x^2 < 0\}$ represents the empty set.

(C) $S = \{x : x \in A \text{ and } x \in B\}$ represents the union of set A and set B.

(D) $S = \{x : a < x < b\}$ represents the set of all real numbers between a and b, where a and b are real numbers.

MCQ 1.69 A box contains 20 defective items and 80 non-defective items. If two items are selected at random without replacement, what will be the probability that both items are defective ?

(A) $\frac{1}{5}$ (B) $\frac{1}{25}$

(C) $\frac{20}{99}$ (D) $\frac{19}{495}$

YEAR 2006 TWO MARKS

MCQ 1.70 Eigen values of a matrix $S = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ are 5 and 1. What are the eigen values of the matrix $S^2 = SS$?

(A) 1 and 25 (B) 6 and 4

(C) 5 and 1 (D) 2 and 10

MCQ 1.71 Equation of the line normal to function $f(x) = (x-8)^{2/3} + 1$ at $P(0,5)$ is

(A) $y = 3x - 5$ (B) $y = 3x + 5$

(C) $3y = x + 15$ (D) $3y = x - 15$

MCQ 1.72 Assuming $i = \sqrt{-1}$ and t is a real number, $\int_0^{\pi/3} e^{it}\,dt$ is

(A) $\frac{\sqrt{3}}{2} + i\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2} - i\frac{1}{2}$

(C) $\frac{1}{2} + i\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2} + i\left(1 - \frac{\sqrt{3}}{2}\right)$

MCQ 1.73 If $f(x) = \frac{2x^2 - 7x + 3}{5x^2 - 12x - 9}$, then $\lim_{x \to 3} f(x)$ will be

(A) $-1/3$ (B) $5/18$

(C) 0 (D) $2/5$

MCQ 1.74 Match the items in column I and II.

	Column I		Column II
P.	Singular matrix	1.	Determinant is not defined
Q.	Non-square matrix	2.	Determinant is always one
R.	Real symmetric	3.	Determinant is zero
S.	Orthogonal matrix	4.	Eigenvalues are always real
		5.	Eigenvalues are not defined

(A) P-3, Q-1, R-4, S-2

(B) P-2, Q-3, R-4, S-1

(C) P-3, Q-2, R-5, S-4

(D) P-3, Q-4, R-2, S-1

MCQ 1.75 For $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 3e^{2x}$, the particular integral is

(A) $\frac{1}{15}e^{2x}$ (B) $\frac{1}{5}e^{2x}$

(C) $3e^{2x}$ (D) $C_1e^{-x} + C_2e^{-3x}$

MCQ 1.76 Multiplication of matrices E and F is G. matrices E and G are

$$E = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

What is the matrix F ?

(A) $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} \cos\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} \sin\theta & -\cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

MCQ 1.77 Consider the continuous random variable with probability density function

$$\begin{aligned} f(t) &= 1 + t \text{ for } -1 \le t \le 0 \\ &= 1 - t \text{ for } 0 \le t \le 1 \end{aligned}$$

The standard deviation of the random variable is

(A) $\frac{1}{\sqrt{3}}$

(B) $\frac{1}{\sqrt{6}}$

(C) $\frac{1}{3}$

(D) $\frac{1}{6}$

YEAR 2005 ONE MARK

MCQ 1.78 Stokes theorem connects

(A) a line integral and a surface integral

(B) a surface integral and a volume integral

(C) a line integral and a volume integral

(D) gradient of a function and its surface integral

MCQ 1.79 A lot has 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly 2 of the chosen items are defective is

(A) 0.0036

(B) 0.1937

(C) 0.2234

(D) 0.3874

MCQ 1.80 $\int_{-a}^{a} (\sin^6 x + \sin^7 x)\, dx$ is equal to

(A) $2\int_0^a \sin^6 x\, dx$

(B) $2\int_0^a \sin^7 x\, dx$

(C) $2\int_0^a (\sin^6 x + \sin^7 x)\, dx$

(D) zero

MCQ 1.81 A is a 3×4 real matrix and $Ax = b$ is an inconsistent system of equations. The highest possible rank of A is

(A) 1 (B) 2

(C) 3 (D) 4

MCQ 1.82 Changing the order of the integration in the double integral $I = \int_0^8 \int_{\frac{x}{4}}^2 f(x,y)\,dydx$ leads to $I = \int_r^s \int_p^q f(x,y)\,dxdy$ What is q ?

(A) $4y$ (B) $16\,y^2$

(C) x (D) 8

YEAR 2005 TWO MARKS

MCQ 1.83 Which one of the following is an eigen vector of the matrix

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

(A) $\begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -2 \end{bmatrix}$ (D) $\begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}$

MCQ 1.84 With a 1 unit change in b, what is the change in x in the solution of the system of equations $x + y = 2, 1.01x + 0.99y = b$?

(A) zero (B) 2 units

(C) 50 units (D) 100 units

MCQ 1.85 By a change of variable $x(u,v) = uv, y(u,v) = v/u$ is double integral, the integrand $f(x,y)$ changes to $f(uv, v/u)\phi(u,v)$. Then, $\phi(u,v)$ is

(A) $2v/u$ (B) $2uv$

(C) v^2 (D) 1

MCQ 1.86 The right circular cone of largest volume that can be enclosed by a sphere of 1 m radius has a height of

(A) 1/3 m (B) 2/3 m

(C) $\frac{2\sqrt{2}}{3}$ m (D) 4/3 m

MCQ 1.87 If $x^2\frac{dy}{dx}+2xy=\frac{2\ln(x)}{x}$ and $y(1)=0$, then what is $y(e)$?

(A) e (B) 1

(C) $1/e$ (D) $1/e^2$

MCQ 1.88 The line integral $\int V\cdot dr$ of the vector $V\cdot(r)=2xyzi+x^2zj+x^2yk$ from the origin to the point P (1, 1, 1)

(A) is 1

(B) is zero

(C) is – 1

(D) cannot be determined without specifying the path

MCQ 1.89 Starting from $x_0=1$, one step of Newton-Raphson method in solving the equation $x^3+3x-7=0$ gives the next value (x_1) as

(A) $x_1=0.5$ (B) $x_1=1.406$

(C) $x_1=1.5$ (D) $x_1=2$

MCQ 1.90 A single die is thrown twice. What is the probability that the sum is neither 8 nor 9 ?

(A) 1/9 (B) 5/36

(C) 1/4 (D) 3/4

• Common Data For Q. 91 and 92

The complete solution of the ordinary differential equation

$$\frac{d^2y}{dx^2}+p\frac{dy}{dx}+qy=0 \text{ is } y=c_1e^{-x}+c_2e^{-3x}$$

MCQ 1.91 Then p and q are

(A) $p=3,q=3$ (B) $p=3,q=4$

(C) $p=4,q=3$ (D) $p=4,q=4$

MCQ 1.92 Which of the following is a solution of the differential equation

$$\frac{d^2y}{dx^2}+p\frac{dy}{dx}+(q+1)y=0$$

(A) e^{-3x} (B) xe^{-x}

(C) xe^{-2x} (D) x^2e^{-2x}

YEAR 2004 ONE MARK

MCQ 1.93 If $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$, then $\frac{dy}{dx}$ will be equal to

(A) $\sin\left(\frac{\theta}{2}\right)$ (B) $\cos\left(\frac{\theta}{2}\right)$

(C) $\tan\left(\frac{\theta}{2}\right)$ (D) $\cot\left(\frac{\theta}{2}\right)$

MCQ 1.94 The angle between two unit-magnitude coplanar vectors $P(0.866, 0.500, 0)$ and $Q(0.259, 0.966, 0)$ will be

(A) 0° (B) 30°

(C) 45° (D) 60°

MCQ 1.95 The sum of the eigen values of the matrix given below is $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

(A) 5 (B) 7

(C) 9 (D) 18

YEAR 2004 TWO MARKS

MCQ 1.96 From a pack of regular playing cards, two cards are drawn at random. What is the probability that both cards will be Kings, if first card in NOT replaced ?

(A) $\frac{1}{26}$ (B) $\frac{1}{52}$

(C) $\frac{1}{169}$ (D) $\frac{1}{221}$

MCQ 1.97 A delayed unit step function is defined as $U(t-a) = \begin{cases} 0, \text{for } t < a \\ 1, \text{for } t \geq a \end{cases}$ Its Laplace transform is

(A) ae^{-as} (B) $\frac{e^{-as}}{s}$

(C) $\frac{e^{as}}{s}$ (D) $\frac{e^{as}}{a}$

MCQ 1.98 The values of a function $f(x)$ are tabulated below

x	$f(x)$
0	1
1	2
2	1
3	10

Using Newton's forward difference formula, the cubic polynomial that can be fitted to the above data, is

(A) $2x^3 + 7x^2 - 6x + 2$ (B) $2x^3 - 7x^2 + 6x - 2$
(C) $x^3 - 7x^2 - 6x^2 + 1$ (D) $2x^3 - 7x^2 + 6x + 1$

MCQ 1.99 The volume of an object expressed in spherical co-ordinates is given by

$$V = \int_0^{2\pi}\int_0^{\pi/3}\int_0^1 r^2 \sin\phi\, dr\, d\phi\, d\theta$$

The value of the integral is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$
(C) $\frac{2\pi}{3}$ (D) $\frac{\pi}{4}$

MCQ 1.100 For which value of x will the matrix given below become singular ?

$$= \begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

(A) 4 (B) 6
(C) 8 (D) 12

YEAR 2003 ONE MARK

MCQ 1.101 $\lim_{x \to 0} \frac{\sin^2 x}{x}$ is equal to

(A) 0 (B) ∞
(C) 1 (D) -1

MCQ 1.102 The accuracy of Simpson's rule quadrature for a step size h is

(A) $O(h^2)$ (B) $O(h^3)$
(C) $O(h^4)$ (D) $O(h^5)$

MCQ 1.103 For the matrix $\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$ the eigen values are

(A) 3 and -3 (B) -3 and -5
(C) 3 and 5 (D) 5 and 0

YEAR 2003 TWO MARKS

MCQ 1.104 Consider the system of simultaneous equations

$$\begin{aligned} x + 2y + z &= 6 \\ 2x + y + 2z &= 6 \\ x + y + z &= 5 \end{aligned}$$

This system has
(A) unique solution
(B) infinite number of solutions
(C) no solution
(D) exactly two solutions

MCQ 1.105 The area enclosed between the parabola $y = x^2$ and the straight line $y = x$ is
(A) 1/8
(B) 1/6
(C) 1/3
(D) 1/2

MCQ 1.106 The solution of the differential equation $\frac{dy}{dx} + y^2 = 0$ is
(A) $y = \frac{1}{x+c}$
(B) $y = \frac{-x^3}{3} + c$
(C) ce^x
(D) unsolvable as equation is non-linear

MCQ 1.107 The vector field is $F = x\boldsymbol{i} - y\boldsymbol{j}$ (where i and j are unit vector) is
(A) divergence free, but not irrotational
(B) irrotational, but not divergence free
(C) divergence free and irrotational
(D) neither divergence free nor irrational

MCQ 1.108 Laplace transform of the function $\sin \omega t$ is
(A) $\frac{s}{s^2 + \omega^2}$
(B) $\frac{\omega}{s^2 + \omega^2}$
(C) $\frac{s}{s^2 - \omega^2}$
(D) $\frac{\omega}{s^2 - \omega^2}$

MCQ 1.109 A box contains 5 black and 5 red balls. Two balls are randomly picked one after another form the box, without replacement. The probability for balls being red is
(A) 1/90
(B) 1/2
(C) 19/90
(D) 2/9

YEAR 2002 ONE MARK

MCQ 1.110 Two dice are thrown. What is the probability that the sum of the numbers on the two dice is eight?
(A) $\frac{5}{36}$
(B) $\frac{5}{18}$
(C) $\frac{1}{4}$
(D) $\frac{1}{3}$

MCQ 1.111 Which of the following functions is not differentiable in the domain $[-1,1]$?

(A) $f(x) = x^2$
(B) $f(x) = x - 1$
(C) $f(x) = 2$
(D) $f(x) =$ maximum $(x, -x)$

MCQ 1.112 A regression model is used to express a variable Y as a function of another variable X. This implies that

(A) there is a causal relationship between Y and X
(B) a value of X may be used to estimate a value of Y
(C) values of X exactly determine values of Y
(D) there is no causal relationship between Y and X

YEAR 2002 TWO MARKS

MCQ 1.113 The following set of equations has

$$\begin{aligned} 3x + 2y + z &= 4 \\ x - y + z &= 2 \\ -2x + 2z &= 5 \end{aligned}$$

(A) no solution
(B) a unique solution
(C) multiple solutions
(D) an inconsistency

MCQ 1.114 The function $f(x,y) = 2x^2 + 2xy - y^3$ has

(A) only one stationary point at $(0,0)$
(B) two stationary points at $(0,0)$ and $\left(\frac{1}{6}, \frac{-1}{3}\right)$
(C) two stationary points at $(0,0)$ and $(1,-1)$
(D) no stationary point

MCQ 1.115 Manish has to travel from A to D changing buses at stops B and C enroute. The maximum waiting time at either stop can be 8 min each but any time of waiting up to 8 min is equally, likely at both places. He can afford up to 13 min of total waiting time if he is to arrive at D on time. What is the probability that Manish will arrive late at D ?

(A) $\frac{8}{13}$
(B) $\frac{13}{64}$
(C) $\frac{119}{128}$
(D) $\frac{9}{128}$

YEAR 2001 ONE MARK

MCQ 1.116 The divergence of vector $\boldsymbol{i} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ is

(A) $\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$
(B) 3
(C) 0
(D) 1

MCQ 1.117 Consider the system of equations given below

$$x + y = 2$$
$$2x + 2y = 5$$

This system has

(A) one solution
(B) no solution
(C) infinite solutions
(D) four solutions

MCQ 1.118 What is the derivative of $f(x) = |x|$ at $x = 0$?

(A) 1
(B) -1
(C) 0
(D) Does not exist

MCQ 1.119 The Gauss divergence theorem relates certain

(A) surface integrals to volume integrals
(B) surface integrals to line integrals
(C) vector quantities to other vector quantities
(D) line integrals to volume integrals

YEAR 2001 TWO MARKS

MCQ 1.120 The minimum point of the function $f(x) = \left(\frac{x^3}{3}\right) - x$ is at

(A) $x = 1$
(B) $x = -1$
(C) $x = 0$
(D) $x = \frac{1}{\sqrt{3}}$

MCQ 1.121 The rank of a 3×3 matrix $C(= AB)$, found by multiplying a non-zero column matrix A of size 3×1 and a non-zero row matrix B of size 1×3, is

(A) 0
(B) 1
(C) 2
(D) 3

MCQ 1.122 An unbiased coin is tossed three times. The probability that the head turns up in exactly two cases is

(A) $\frac{1}{9}$
(B) $\frac{1}{8}$
(C) $\frac{2}{3}$
(D) $\frac{3}{8}$

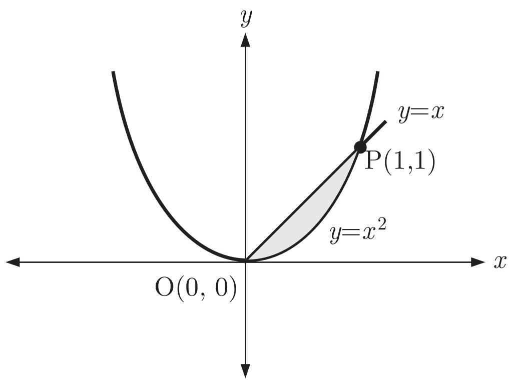
****

SOLUTION

SOL 1.1 Option (A) is correct.

For $y = x$ straight line and

$y = x^2$ parabola, curve is as given. The shaded region is the area, which is bounded by the both curves (common area).



We solve given equation as follows to gett the intersection points :

In $y = x^2$ putting $y = x$ we have $x = x^2$ or

$$x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0, 1$$

Then from $y = x$, for $x = 0 \Rightarrow y = 0$ and $x = 1 \Rightarrow y = 1$

Curve $y = x^2$ and $y = x$ intersects at point $(0,0)$ and $(1,1)$

So, the area bounded by both the curves is

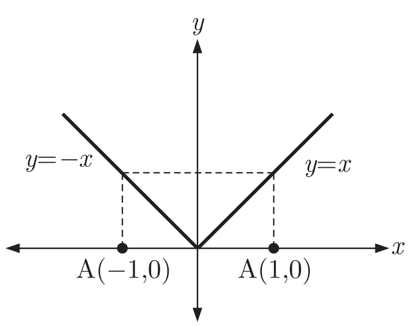
$$A = \int_{x=0}^{x=1}\int_{y=x}^{y=x^2} dy dx = \int_{x=0}^{x=1} dx \int_{y=x}^{y=x^2} dy = \int_{x=0}^{x=1} dx [y]_x^{x^2} = \int_{x=0}^{x=1} (x^2 - x) dx$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1 = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6} = \frac{1}{6} \text{unit}^2$$ Area is never negative

SOL 1.2 Option (C) is correct.

Given $f(x) = |x|$ (in $-1 \le x \le 1$)

For this function the plot is as given below.



At $x = 0$, function is continuous but not differentiable because.

For $x > 0$ and $x < 0$

$$f'(x) = 1 \text{ and } f'(x) = -1$$

$$\lim_{x \to 0^+} f'(x) = 1 \text{ and } \lim_{x \to 0^-} f'(x) = -1$$

R.H.S lim $= 1$ and L.H.S lim $= -1$

Therefore it is not differentiable.

SOL 1.3 Option (B) is correct.

Let $$y = \lim_{x \to 0} \frac{(1 - \cos x)}{x^2}$$

It forms $\left[\frac{0}{0}\right]$ condition. Hence by L-Hospital rule

$$y = \lim_{x \to 0} \frac{\frac{d}{dx}(1 - \cos x)}{\frac{d}{dx}(x^2)} = \lim_{x \to 0} \frac{\sin x}{2x}$$

Still these gives $\left[\frac{0}{0}\right]$ condition, so again applying L-Hospital rule

$$y = \lim_{x \to 0} \frac{\frac{d}{dx}(\sin x)}{2 \times \frac{d}{dx}(x)} = \lim_{x \to 0} \frac{\cos x}{2} = \frac{\cos 0}{2} = \frac{1}{2}$$

SOL 1.4 Option (D) is correct.

We have $$f(x) = x^3 + 1$$

$$f'(x) = 3x^2 + 0$$

Putting $f'(x)$ equal to zero

$$f'(x) = 0$$

$$3x^2 + 0 = 0 \Rightarrow x = 0$$

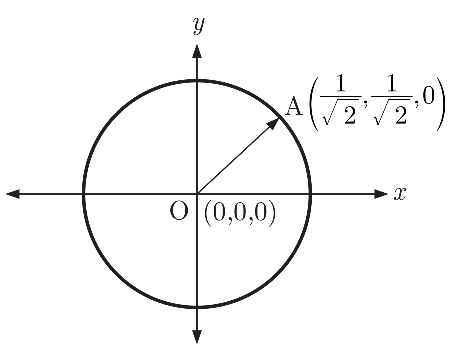
Now $$f''(x) = 6x$$

At $x = 0$, $f''(0) = 6 \times 0 = 0$ Hence $x = 0$ is the point of inflection.

SOL 1.5 Option (A) is correct.

Given : $$x^2 + y^2 + z^2 = 1$$

This is a equation of sphere with radius $r = 1$



The unit normal vector at point $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right)$ is OA

Hence $OA = \left(\frac{1}{\sqrt{2}}-0\right)i+\left(\frac{1}{\sqrt{2}}-0\right)j+(0-0)k = \frac{1}{\sqrt{2}}i+\frac{1}{\sqrt{2}}j$

SOL 1.6 Option (D) is correct.

First using the partial fraction :

$$F(s) = \frac{1}{s(s+1)} = \frac{A}{s}+\frac{B}{s+1} = \frac{A(s+1)+Bs}{s(s+1)}$$

$$\frac{1}{s(s+1)} = \frac{(A+B)s}{s(s+1)}+\frac{A}{s(s+1)}$$

Comparing the coefficients both the sides,

$$(A+B) = 0 \text{ and } A = 1,\ B = -1$$

So $\frac{1}{s(s+1)} = \frac{1}{s}-\frac{1}{s+1}$

$$F(t) = L^{-1}[F(s)]$$

$$= L^{-1}\left[\frac{1}{s(s+1)}\right] = L^{-1}\left[\frac{1}{s}-\frac{1}{s+1}\right] = L^{-1}\left[\frac{1}{s}\right]-L^{-1}\left[\frac{1}{s+1}\right]$$

$$= 1-e^{-t}$$

SOL 1.7 Option (B) is correct.

Given $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$

For finding eigen values, we write the characteristic equation as

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 5-\lambda & 3 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$\Rightarrow$ $(5-\lambda)(3-\lambda)-3 = 0$

$$\lambda^2-8\lambda+12 = 0 \Rightarrow \lambda = 2,6$$

Now from characteristic equation for eigen vector.

$$[A-\lambda I]\{x\} = [0]$$

For $\lambda = 2$

$$\begin{bmatrix} 5-2 & 3 \\ 1 & 3-2 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ $\begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$X_1+X_2 = 0 \qquad \Rightarrow X_1 = -X_2$$

So eigen vector $= \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$

Magnitude of eigen vector $= \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$$\text{Normalized eigen vector} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$$

SOL 1.8 Option (D) is correct.

Given : No. of Red balls $= 4$

No. of Black ball $= 6$

3 balls are selected randomly one after another, without replacement.

1 red and 2 black balls are will be selected as following

Manners	Probability for these sequence
$R\ B\ B$	$\frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{1}{6}$
$B\ R\ B$	$\frac{6}{10} \times \frac{4}{9} \times \frac{5}{8} = \frac{1}{6}$
$B\ B\ R$	$\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{1}{6}$

Hence Total probability of selecting 1 red and 2 black ball is

$$P = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

SOL 1.9 Option (A) is correct.

We have $$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0 \qquad ...(1)$$

Let $x = e^z$ then $$z = \log x$$

$$\frac{dz}{dx} = \frac{1}{x}$$

So, we get $$\frac{dy}{dx} = \left(\frac{dy}{dz}\right)\left(\frac{dz}{dx}\right) = \frac{1}{x}\frac{dy}{dz}$$

$$x\frac{dy}{dx} = Dy \qquad \text{where } \frac{d}{dz} = D$$

Again $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{x}\frac{dy}{dz}\right) = \frac{-1}{x^2}\frac{dy}{dz} + \frac{1}{x}\frac{d}{dz}\left(\frac{dy}{dz}\right)\frac{dz}{dx}$$

$$= \frac{-1}{x^2}\frac{dy}{dz} + \frac{1}{x}\frac{d^2y}{dz^2}\frac{dz}{dx} = \frac{1}{x^2}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right)$$

$$\frac{x^2 d^2y}{dx^2} = (D^2 - D)y = D(D-1)y$$

Now substitute in equation (i)

$$[D(D-1) + D - 4]y = 0$$

$$(D^2 - 4)y = 0 \Rightarrow D = \pm 2$$

So the required solution is $y = C_1x^2 + C_2x^{-2}$ ...(ii)

From the given limits $y(0) = 0$, equation (ii) gives

$$0 = C_1 \times 0 + C_2$$

$$C_2 = 0$$

And from $y(1) = 1$, equation (ii) gives

$$1 = C_1 + C_2$$

$$C_1 = 1$$

Substitute $C_1 \& C_2$ in equation (ii), the required solution be

$$y = x^2$$

SOL 1.10 Option (C) is correct.

For given equation matrix form is as follows

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}$$

The augmented matrix is

$$[A : B] = \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 2 & 1 & 2 & : & 5 \\ 1 & -1 & 1 & : & 1 \end{bmatrix} \qquad R_2 \to R_2 - 2R_1,\; R_3 \to R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 0 & -3 & 0 & : & -3 \\ 0 & -3 & 0 & : & -3 \end{bmatrix} \qquad R_3 \to R_3 - R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 0 & -3 & 0 & : & -3 \\ 0 & 0 & 0 & : & 0 \end{bmatrix} \qquad R_2 \to R_2 / -3$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

This gives rank of A, $\rho(A) = 2$ and Rank of $[A : B] = \rho[A : B] = 2$

Which is less than the number of unknowns (3)

$$\rho[A] = \rho[A : B] = 2 < 3$$

Hence, this gives infinite No. of solutions.

SOL 1.11 Option (B) is correct.

$$\sin\theta = \theta - \frac{\theta^3}{\underline{|3}} + \frac{\theta^5}{\underline{|5}} - \frac{\theta^7}{\underline{|7}} + \dots\dots$$

SOL 1.12 Option (D) is correct.

Let $$y = \lim_{\theta \to 0} \frac{\sin\theta}{\theta}$$

$$= \lim_{\theta \to 0} \frac{\frac{d}{d\theta}(\sin\theta)}{\frac{d}{d\theta}(\theta)} = \lim_{\theta \to 0} \frac{\cos\theta}{1} \qquad \text{Applying L-Hospital rule}$$

$$= \frac{\cos 0}{1} = 1$$

SOL 1.13 Option (C) is correct

Let a square matrix

$$A = \begin{bmatrix} x & y \\ y & x \end{bmatrix}$$

We know that the characteristic equation for the eigen values is given by

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} x-\lambda & y \\ y & x-\lambda \end{vmatrix} = 0$$

$$(x-\lambda)^2 - y^2 = 0$$

$$(x-\lambda)^2 = y^2$$

$$x - \lambda = \pm y \Rightarrow \lambda = x \pm y$$

So, eigen values are real if matrix is real and symmetric.

SOL 1.14 Option (A) is correct.

Let, $z_1 = (1+i)$, $z_2 = (2-5i)$

$$z = z_1 \times z_2 = (1+i)(2-5i)$$

$$= 2 - 5i + 2i - 5i^2 = 2 - 3i + 5 = 7 - 3i \qquad i^2 = -1$$

SOL 1.15 Option (D) is correct.

For a function, whose limits bounded between $-a$ to a and a is a positive real number. The solution is given by

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx\,; & f(x) \text{ is even} \\ 0 \qquad ; & f(x) \text{ is odd} \end{cases}$$

SOL 1.16 Option (C) is correct.

Let, $$f(x) = \int_1^3 \frac{1}{x}dx$$

From this function we get $a = 1$, $b = 3$ and $n = 3 - 1 = 2$

So, $$h = \frac{b-a}{n} = \frac{3-1}{2} = 1$$

We make the table from the given function $y = f(x) = \frac{1}{x}$ as follows :

x	$f(x) = y = \frac{1}{x}$
$x = 1$	$y_1 = \frac{1}{1} = 1$
$x = 2$	$y_2 = \frac{1}{2} = 0.5$
$x = 3$	$y_3 = \frac{1}{3} = 0.333$

Applying the Simpson's 1/3rd formula

$$\int_1^3 \frac{1}{x}dx = \frac{h}{3}[(y_1 + y_3) + 4y_2] = \frac{1}{3}[(1 + 0.333) + 4 \times 0.5]$$

$$= \frac{1}{3}[1.333 + 2] = \frac{3.333}{3} = 1.111$$

SOL 1.17 Option (D) is correct.

Given : $$\frac{dy}{dx} = (1 + y^2)x$$

$$\frac{dy}{(1 + y^2)} = xdx$$

Integrating both the sides, we get

$$\int \frac{dy}{1 + y^2} = \int xdx$$

$$\tan^{-1}y = \frac{x^2}{2} + c \quad \Rightarrow \quad y = \tan\left(\frac{x^2}{2} + c\right)$$

SOL 1.18 Option (D) is correct.

The probability of getting head $p = \frac{1}{2}$

And the probability of getting tail $q = 1 - \frac{1}{2} = \frac{1}{2}$

The probability of getting at least one head is

$$P(x \geq 1) = 1 - {}^5C_0(p)^5(q)^0 = 1 - 1 \times \left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0$$

$$= 1 - \frac{1}{2^5} = \frac{31}{32}$$

SOL 1.19 Option (C) is correct.

Given system of equations are,

$$2x_1 + x_2 + x_3 = 0 \qquad \text{...(i)}$$

$$x_2 - x_3 = 0 \qquad \text{...(ii)}$$

$$x_1 + x_2 = 0 \qquad \text{...(iii)}$$

Adding the equation (i) and (ii) we have

$$2x_1 + 2x_2 = 0$$

$$x_1 + x_2 = 0 \qquad \text{...(iv)}$$

We see that the equation (iii) and (iv) is same and they will meet at infinite points. Hence this system of equations have infinite number of solutions.

SOL 1.20 Option (D) is correct.

The volume of a solid generated by revolution about x-axis bounded by the function $f(x)$ and limits between a to b is given by

$$V = \int_a^b \pi y^2 dx$$

Given $y = \sqrt{x}$ and $a = 1$, $b = 2$

Therefore, $$V = \int_1^2 \pi(\sqrt{x})^2 dx = \pi \int_1^2 x dx = \pi\left[\frac{x^2}{2}\right]_1^2 = \pi\left[\frac{4}{2} - \frac{1}{2}\right] = \frac{3\pi}{2}$$

SOL 1.21 Option (B) is correct.

Given: $$\frac{d^3 f}{d\eta^3} + \frac{f}{2}\frac{d^2 f}{d\eta^2} = 0$$

Order is determined by the order of the highest derivation present in it. So, It is third order equation but it is a nonlinear equation because in linear equation, the product of f with $d^2 f/d\eta^2$ is not allow.

Therefore, it is a third order non-linear ordinary differential equation.

SOL 1.22 Option (D) is correct.

Let $$I = \int_{-\infty}^{\infty} \frac{dx}{1 + x^2}$$

$$= [\tan^{-1} x]_{-\infty}^{\infty} = [\tan^{-1}(+\infty) - \tan^{-1}(-\infty)]$$

$$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi \qquad \tan^{-1}(-\theta) = -\tan^{-1}(\theta)$$

SOL 1.23 Option (B) is correct.

Let, $$z = \frac{3 + 4i}{1 - 2i}$$

Divide and multiply z by the conjugate of $(1 - 2i)$ to convert it in the form of $a + bi$ we have

$$z = \frac{3 + 4i}{1 - 2i} \times \frac{1 + 2i}{1 + 2i} = \frac{(3 + 4i)(1 + 2i)}{(1)^2 - (2i)^2}$$

$$= \frac{3 + 10i + 8i^2}{1 - 4i^2} = \frac{3 + 10i - 8}{1 - (-4)}$$

$$= \frac{-5 + 10i}{5} = -1 + 2i$$

$$|z| = \sqrt{(-1)^2 + (2)^2} = \sqrt{5} \qquad |a + ib| = \sqrt{a^2 + b^2}$$

SOL 1.24 Option (C) is correct.

$$y = f(x) = \begin{cases} 2-3x & \text{if } x < \frac{2}{3} \\ 0 & \text{if } x = \frac{2}{3} \\ -(2-3x) & \text{if } x > \frac{2}{3} \end{cases}$$

Checking the continuity of the function.

At $x = \frac{2}{3}$, $Lf(x) = \lim_{h \to 0} f\left(\frac{2}{3} - h\right) = \lim_{h \to 0} 2 - 3\left(\frac{2}{3} - h\right)$

$$= \lim_{h \to 0} 2 - 2 + 3h = 0$$

and $Rf(x) = \lim_{h \to 0} f\left(\frac{2}{3} + h\right) = \lim_{h \to 0} 3\left(\frac{2}{3} + h\right) - 2$

$$= \lim_{h \to 0} 2 + 3h - 2 = 0$$

Since $L\lim_{h \to 0} f(x) = R\lim_{h \to 0} f(x)$

So, function is continuous $\forall\, x \in R$

Now checking the differentiability :

$$Lf'(x) = \lim_{h \to 0} \frac{f(\frac{2}{3} - h) - f(\frac{2}{3})}{-h} = \lim_{h \to 0} \frac{2 - 3(\frac{2}{3} - h) - 0}{-h}$$

$$= \lim_{h \to 0} \frac{2 - 2 + 3h}{-h} = \lim_{h \to 0} \frac{3h}{-h} = -3$$

and $Rf'(x) = \lim_{h \to 0} \frac{f(\frac{2}{3} + h) - f(\frac{2}{3})}{h}$

$$= \lim_{h \to 0} \frac{3(\frac{2}{3} + h) - 2 - 0}{h} = \lim_{h \to 0} \frac{2 + 3h - 2}{h} = 3$$

Since $Lf'\left(\frac{2}{3}\right) \neq Rf'\left(\frac{2}{3}\right)$, $f(x)$ is not differentiable at $x = \frac{2}{3}$.

SOL 1.25 Option (A) is correct.

Let, $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$

And λ_1 and λ_2 are the eigen values of the matrix A.

The characteristic equation is written as

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = 0 \qquad ...(i)$$

$$(2-\lambda)(3-\lambda) - 2 = 0$$

$$\lambda^2 - 5\lambda + 4 = 0 \Rightarrow \lambda = 1 \ \& \ 4$$

Putting $\lambda = 1$ in equation (i),

$$\begin{bmatrix} 2-1 & 2 \\ 1 & 3-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \text{where } \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ is eigen vector}$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 + 2x_2 = 0 \text{ or } x_1 + 2x_2 = 0$$

Let $x_2 = K$

Then $x_1 + 2K = 0 \Rightarrow x_1 = -2K$

So, the eigen vector is

$$\begin{bmatrix} -2K \\ K \end{bmatrix} \text{ or } \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Since option A$\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is in the same ratio of x_1 and x_2. Therefore option (A) is an eigen vector.

SOL 1.26 Option (A) is correct.

$f(t)$ is the inverse Laplace

So, $$f(t) = \mathcal{L}^{-1}\left[\frac{1}{s^2(s+1)}\right]$$

$$\frac{1}{s^2(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1}$$

$$= \frac{As(1+s) + B(s+1) + Cs^2}{s^2(s+1)}$$

$$= \frac{s^2(A+C) + s(A+B) + B}{s^2(s+1)}$$

Compare the coefficients of s^2, s and constant terms and we get

$$A + C = 0;\ A + B = 0 \text{ and } B = 1$$

Solving above equation, we get $A = -1$, $B = 1$ and $C = 1$

Thus $$f(t) = \mathcal{L}^{-1}\left[-\frac{1}{s} + \frac{1}{s^2} + \frac{1}{s+1}\right]$$

$$= -1 + t + e^{-t} = t - 1 + e^{-t} \qquad \mathcal{L}^{-1}\left[\frac{1}{s+a}\right] = e^{-at}$$

SOL 1.27 Option (C) is correct.

The box contains :

Number of washers = 2

Number of nuts = 3

Number of bolts = 4

Total objects = 2 + 3 + 4 = 9

First two washers are drawn from the box which contain 9 items. So the probability of drawing 2 washers is,

$$P_1 = \frac{{}^2C_2}{{}^9C_2} == \frac{1}{\frac{9!}{7!2!}} = \frac{7!2!}{9 \times 8 \times 7!} = \frac{2}{9 \times 8} = \frac{1}{36} \qquad {}^nC_n = 1$$

After this box contains only 7 objects and then 3 nuts drawn from it. So the probability of drawing 3 nuts from the remaining objects is,

$$P_2 = \frac{{}^3C_3}{{}^7C_3} = \frac{1}{\frac{7!}{4!3!}} = \frac{4!3!}{7 \times 6 \times 5 \times 4!} = \frac{1}{35}$$

After this box contain only 4 objects, probability of drawing 4 bolts from the box,

$$P_3 = \frac{{}^4C_4}{{}^4C_4} = \frac{1}{1} = 1$$

Therefore the required probability is,

$$P = P_1P_2P_3 = \frac{1}{36} \times \frac{1}{35} \times 1 = \frac{1}{1260}$$

SOL 1.28 Option (B) is correct.

Given : $h = 60^\circ - 0 = 60^\circ$

$$h = 60 \times \frac{\pi}{180} = \frac{\pi}{3} = 1.047 \text{ radians}$$

From the table, we have

$y_0 = 0,\ y_1 = 1066,\ y_2 = -323,\ y_3 = 0,\ y_4 = 323,\ y_5 = -355$ and $y_6 = 0$

From the Simpson's 1/3rd rule the flywheel Energy is,

$$E = \frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

Substitute the values, we get

$$E = \frac{1.047}{3}[(0 + 0) + 4(1066 + 0 - 355) + 2(-323 + 323)]$$

$$= \frac{1.047}{3}[4 \times 711 + 2(0)] = 993 \text{ Nm rad (Joules/cycle)}$$

SOL 1.29 Option (A) is correct.

Given : $M = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$

And $[M]^T = [M]^{-1}$

We know that when $[A]^T = [A]^{-1}$ then it is called orthogonal matrix.

$$[M]^T = \frac{I}{[M]}$$

$$[M]^T[M] = I$$

Substitute the values of M and M^T, we get

$$\begin{bmatrix} \frac{3}{5} & x \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \left(\frac{3}{5}\times\frac{3}{5}\right)+x^2 & \left(\frac{3}{5}\times\frac{4}{5}\right)+\frac{3}{5}x \\ \left(\frac{4}{5}\times\frac{3}{5}\right)+\frac{3}{5}x & \left(\frac{4}{5}\times\frac{4}{5}\right)+\left(\frac{3}{5}\times\frac{3}{5}\right) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{9}{25}+x^2 & \frac{12}{25}+\frac{3}{5}x \\ \frac{12}{25}+\frac{3}{5}x & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Comparing both sides a_{12} element,

$$\frac{12}{25}+\frac{3}{5}x = 0 \rightarrow x = -\frac{12}{25}\times\frac{5}{3} = -\frac{4}{5}$$

SOL 1.30 Option (C) is correct.

Let, $V = 3xz\boldsymbol{i} + 2xy\boldsymbol{j} - yz^2\boldsymbol{k}$

We know divergence vector field of V is given by $(\nabla \cdot V)$

So, $$\nabla \cdot V = \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right)\cdot(3xz\boldsymbol{i} + 2xy\boldsymbol{j} - yz^2\boldsymbol{k})$$

$$\nabla \cdot V = 3z + 2x - 2yz$$

At point $P(1,1,1)$

$$(\nabla \cdot V)_{P(1,1,1)} = 3\times 1 + 2\times 1 - 2\times 1\times 1 = 3$$

SOL 1.31 Option (C) is correct.

Let $$f(s) = \mathcal{L}^{-1}\left[\frac{1}{s^2+s}\right]$$

First, take the function $\frac{1}{s^2+s}$ and break it by the partial fraction,

$$\frac{1}{s^2+s} = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{(s+1)} \qquad \left\{\begin{matrix}\text{Solve by} \\ \frac{1}{(s+1)} = \frac{A}{s}+\frac{B}{s+1}\end{matrix}\right\}$$

So, $$\mathcal{L}^{-1}\left(\frac{1}{s^2+s}\right) = \mathcal{L}^{-1}\left[\frac{1}{s} - \frac{1}{(s+1)}\right] = \mathcal{L}^{-1}\left[\frac{1}{s}\right] - \mathcal{L}^{-1}\left[\frac{1}{s+1}\right] = 1 - e^{-t}$$

SOL 1.32 Option (D) is correct.

Total number of cases $= 2^3 = 8$

& Possible cases when coins are tossed simultaneously.

H	H	H
H	H	T
H	T	H
T	H	H
H	T	T
T	H	T
T	T	H
T	T	T

From these cases we can see that out of total 8 cases 7 cases contain at least one head. So, the probability of come at least one head is $= \frac{7}{8}$

SOL 1.33 Option (C) is correct.

Given : $z = x + iy$ is a analytic function

$$f(z) = u(x,y) + iv(x,y)$$

$$u = xy \quad \text{..(i)}$$

Analytic function satisfies the Cauchy-Riemann equation.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

So from equation (i),

$$\frac{\partial u}{\partial x} = y \quad \Rightarrow \quad \frac{\partial v}{\partial y} = y$$

$$\frac{\partial u}{\partial y} = x \quad \Rightarrow \quad \frac{\partial v}{\partial x} = -x$$

Let $v(x,y)$ be the conjugate function of $u(x,y)$

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy = (-x)dx + (y)dy$$

Integrating both the sides,

$$\int dv = -\int xdx + \int ydy$$

$$v = -\frac{x^2}{2} + \frac{y^2}{2} + k = \frac{1}{2}(y^2 - x^2) + k$$

SOL 1.34 Option (A) is correct.

Given $x\frac{dy}{dx} + y = x^4$

$$\frac{dy}{dx} + \left(\frac{1}{x}\right)y = x^3 \quad \text{...(i)}$$

It is a single order differential equation. Compare this with $\frac{dy}{dx} + Py = Q$ and we get

$$P = \frac{1}{x} \qquad Q = x^3$$

Its solution will be

$$y(I.F.) = \int Q(I.F.)\,dx + C$$

$$I.F. = e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\log_e x} = x$$

Complete solution is given by,

$$yx = \int x^3 \times x dx + C = \int x^4 dx + C = \frac{x^5}{5} + C \qquad \text{...(ii)}$$

and $y(1) = \frac{6}{5}$ at $x = 1 \Rightarrow y = \frac{6}{5}$ From equation (ii),

$$\frac{6}{5} \times 1 = \frac{1}{5} + C \quad \Rightarrow \quad C = \frac{6}{5} - \frac{1}{5} = 1$$

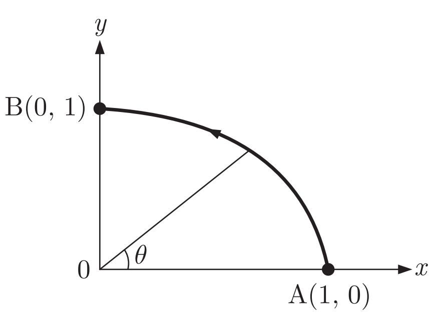
Then, from equation (ii), we get

$$yx = \frac{x^5}{5} + 1 \Rightarrow y = \frac{x^4}{5} + \frac{1}{x}$$

SOL 1.35 Option (B) is correct.

The equation of circle with unit radius and centre at origin is given by,

$$x^2 + y^2 = 1$$



Finding the integration of $(x+y)^2$ on path AB traversed in counter-clockwise sense So using the polar form

Let: $x = \cos\theta$, $y = \sin\theta$, and $r = 1$

So put the value of x and y and limits in first quadrant between 0 to $\pi/2$.

Hence,
$$I = \int_0^{\pi/2} (\cos\theta + \sin\theta)^2 d\theta$$

$$= \int_0^{\pi/2} (\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta)\,d\theta$$

$$= \int_0^{\pi/2} (1 + \sin 2\theta)\,d\theta$$

Integrating above equation, we get

$$= \left[\theta - \frac{\cos 2\theta}{2}\right]_0^{\pi/2} = \left[\left(\frac{\pi}{2} - \frac{\cos\pi}{2}\right) - \left(0 - \frac{\cos 0}{2}\right)\right]$$

$$= \left(\frac{\pi}{2} + \frac{1}{2}\right) - \left(-\frac{1}{2}\right) = \frac{\pi}{2} + 1$$

SOL 1.36 Option (A) is correct.

The given equation of surface is

$$z^2 = 1 + xy \quad \text{...(i)}$$

Let $P(x, y, z)$ be the nearest point on the surface (i), then distance from the origin is

$$d = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$$

$$d^2 = x^2 + y^2 + z^2$$

$$z^2 = d^2 - x^2 - y^2 \quad \text{...(ii)}$$

From equation (i) and (ii), we get

$$d^2 - x^2 - y^2 = 1 + xy$$

$$d^2 = x^2 + y^2 + xy + 1$$

Let $f(x, y) = d^2 = x^2 + y^2 + xy + 1$ ...(iii)

The $f(x, y)$ be the maximum or minimum according to d^2 maximum or minimum.

Differentiating equation (iii) w.r.t x and y respectively, we get

$$\frac{\partial f}{\partial x} = 2x + y \text{ or } \frac{\partial f}{\partial y} = 2y + x$$

Applying maxima minima principle and putting $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ equal to zero,

$$\frac{\partial f}{\partial x} = 2x + y = 0 \text{ or } \frac{\partial f}{\partial y} = 2y + x = 0$$

Solving these equations, we get $x = 0$, $y = 0$

So, $x = y = 0$ is only one stationary point.

Now
$$p = \frac{\partial^2 f}{\partial x^2} = 2$$

$$q = \frac{\partial^2 f}{\partial x \, \partial y} = 1$$

$$r = \frac{\partial^2 f}{\partial y^2} = 2$$

or $pr - q^2 = 4 - 1 = 3 > 0$ and r is positive.

So, $f(x, y) = d^2$ is minimum at $(0, 0)$.

Hence minimum value of d^2 at $(0, 0)$.

$$d^2 = x^2 + y^2 + xy + 1 = 1$$

$$d = 1 \text{ or } f(x, y) = 1$$

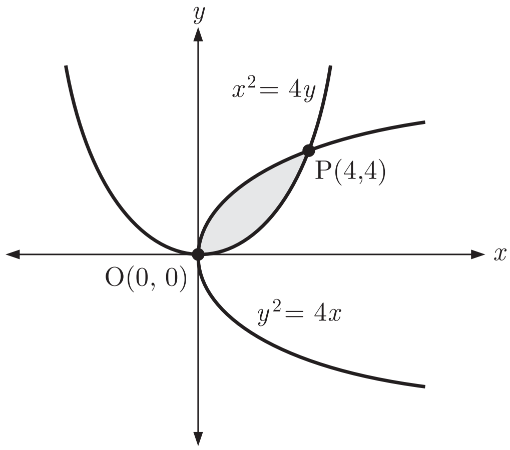
So, the nearest point is

$$z^2 = 1 + xy = 1 + 0$$

$$\Rightarrow \quad z = \pm 1$$

SOL 1.37 Option (A) is correct.

Given : $y^2 = 4x$ and $x^2 = 4y$ draw the curves from the given equations,



The shaded area shows the common area. Now finding the intersection points of the curves.

$$y^2 = 4x = 4\sqrt{4y} = 8\sqrt{y} \qquad x = \sqrt{4y} \text{ From second curve}$$

Squaring both sides

$$y^4 = 8 \times 8 \times y \Rightarrow y(y^3 - 64) = 0$$

$$y = 4 \ \& \ 0$$

Similarly put $y = 0$ in curve $x^2 = 4y$

$$x^2 = 4 \times 0 = 0 \Rightarrow x = 0$$

And Put $y = 4$

$$x^2 = 4 \times 4 = 16 \quad x = 4$$

So, $x = 4, 0$

Therefore the intersection points of the curves are $(0,0)$ and $(4,4)$.

So the enclosed area is given by

$$A = \int_{x_1}^{x_2} (y_1 - y_2)\, dx$$

Put y_1 and y_2 from the equation of curves $y^2 = 4x$ and $x^2 = 4y$

$$A = \int_0^4 \left(\sqrt{4x} - \frac{x^2}{4}\right) dx$$

$$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = 2\int_0^4 \sqrt{x}\, dx - \frac{1}{4}\int_0^4 x^2 dx$$

Integrating the equation, we get

$$A = 2\left[\frac{2}{3}x^{3/2}\right]_0^4 - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$$

$$= \frac{4}{3} \times 4^{3/2} - \frac{1}{4} \times \frac{4^3}{3} = \frac{4}{3} \times 8 - \frac{16}{3} = \frac{16}{3}$$

SOL 1.38 Option (A) is correct.

The cumulative distribution function

$$f(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a < x < b \\ 0, & x \ge b \end{cases}$$

and density function

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & a \le x \le b \\ 0, & a > x, x > b \end{cases}$$

Mean $$E(x) = \sum_{x=a}^{b} x f(x) = \frac{a+b}{2}$$

$$\text{Variance} = x^2 f(x) - \overline{x}^2 = x^2 f(x) - [x f(x)]^2$$

Substitute the value of $f(x)$

$$\text{Variance} = \sum_{x=a}^{b} x^2 \frac{1}{b-a} dx - \left\{\sum_{x=a}^{b} x \frac{1}{b-a} dx\right\}^2$$

$$= \left[\frac{x^3}{3(b-a)}\right]_a^b - \left[\left\{\frac{x^2}{2(b-a)}\right\}_a^b\right]^2$$

$$= \frac{b^3 - a^3}{3(b-a)} - \frac{(b^2-a^2)^2}{4(b-a)^2}$$

$$= \frac{(b-a)(b^2+ab+a^2)}{3(b-a)} - \frac{(b+a)^2(b-a)^2}{4(b-a)^2}$$

$$= \frac{4(b^2+ab+a^2)+3(a+b)^2}{12} = \frac{(b-a)^2}{12}$$

$$\text{Standard deviation} = \sqrt{\text{Variance}} = \sqrt{\frac{(b-a)^2}{12}} = \frac{(b-a)}{\sqrt{12}}$$

Given : $b = 1$, $a = 0$

So, standard deviation $= \dfrac{1-0}{\sqrt{12}} = \dfrac{1}{\sqrt{12}}$

SOL 1.39 Option (C) is correct.

Taylor's series expansion of $f(x)$ is given by,

$$f(x) = f(a) + \frac{(x-a)}{\underline{|1}} f'(a) + \frac{(x-a)^2}{\underline{|2}} f''(a) + \frac{(x-a)^3}{\underline{|3}} f'''(a) + ...$$

Then from this expansion the coefficient of $(x-a)^4$ is $\dfrac{f''''(a)}{\underline{|4}}$

Given $$a = 2$$

$$f(x) = e^x$$

$$f'(x) = e^x$$

$$f''(x) = e^x$$

$$f'''(x) = e^x$$
$$f''''(x) = e^x$$

Hence, for $a = 2$ the coefficient of $(x-a)^4$ is $\frac{e^2}{\lfloor 4}$

SOL 1.40 Option (D) is correct.

Given : $\ddot{x} + 3x = 0$ and $x(0) = 1$

$$(D^2 + 3)x = 0 \qquad D = \frac{d}{dt}$$

The auxiliary Equation is written as

$$m^2 + 3 = 0$$
$$m = \pm\sqrt{3}\,i = 0 \pm \sqrt{3}\,i$$

Here the roots are imaginary

$$m_1 = 0 \text{ and } m_2 = \sqrt{3}$$

Solution is given by

$$x = e^{m_1 t}(A\cos m_2 t + B\sin m_2 t)$$
$$= e^0[A\cos\sqrt{3}\,t + B\sin\sqrt{3}\,t]$$
$$= [A\cos\sqrt{3}\,t + B\sin\sqrt{3}\,t] \qquad \text{...(i)}$$

Given : $x(0) = 1$ at $t = 0$, $x = 1$

Substituting in equation (i),

$$1 = [A\cos\sqrt{3}(0) + B\sin\sqrt{3}(0)] = A + 0$$
$$A = 1$$

Differentiateing equation (i) w.r.t. t,

$$\dot{x} = \sqrt{3}[-A\sin\sqrt{3}\,t + B\cos\sqrt{3}\,t] \qquad \text{...(ii)}$$

Given $\dot{x}(0) = 0$ at $t = 0$, $\dot{x} = 0$

Substituting in equation (ii), we get

$$0 = \sqrt{3}[-A\sin 0 + B\cos 0]$$
$$B = 0$$

Substituting $A \,\&\, B$ in equation (i)

$$x = \cos\sqrt{3}\,t$$

$$x(1) = \cos\sqrt{3} = 0.99$$

SOL 1.41 Option (B) is correct.

Let $f(x) = \lim_{x \to 8} \frac{x^{1/3} - 2}{(x-8)}$ $\qquad \frac{0}{0}$ form

$$= \lim_{x \to 8} \frac{\frac{1}{3}x^{-2/3}}{1} \qquad \text{Applying L-Hospital rule}$$

Substitute the limits, we get

$$f(x) = \frac{1}{3}(8)^{-2/3} = \frac{1}{3}(2^3)^{-2/3} = \frac{1}{4 \times 3} = \frac{1}{12}$$

SOL 1.42 Option (A) is correct.

In a coin probability of getting Head

$$p = \frac{1}{2} = \frac{\text{No. of Possible cases}}{\text{No. of Total cases}}$$

Probability of getting tail

$$q = 1 - \frac{1}{2} = \frac{1}{2}$$

So the probability of getting Heads exactly three times, when coin is tossed 4 times is

$$P = {}^4C_3(p)^3(q)^1 = {}^4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1$$

$$= 4 \times \frac{1}{8} \times \frac{1}{2} = \frac{1}{4}$$

SOL 1.43 Option (C) is correct.

Let, $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$

Let the eigen values of this matrix are $\lambda_1, \lambda_2 \,\&\, \lambda_3$

Here one values is given so let $\lambda_1 = 3$

We know that

Sum of eigen values of matrix = Sum of the diagonal element of matrix A

$$\lambda_1 + \lambda_2 + \lambda_3 = 1 + 0 + p$$

$$\lambda_2 + \lambda_3 = 1 + p - \lambda_1 = 1 + p - 3 = p - 2$$

SOL 1.44 Option (D) is correct.

We know that the divergence is defined as $\nabla \cdot V$

Let $V = (x-y)\,i + (y-x)\,j + (x+y+z)\,k$

And $\nabla = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)$

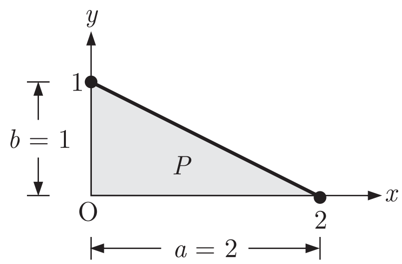
So, $\nabla \cdot V = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right) \cdot [(x-y)\,i + (y-x)\,j + (x+y+z)\,k]$

$$= \frac{\partial}{\partial x}(x-y) + \frac{\partial}{\partial y}(y-x) + \frac{\partial}{\partial z}(x+y+z)$$

$$= 1 + 1 + 1 = 3$$

SOL 1.45 Option (A) is correct.

Given :



The equation of line in intercept form is given by

$$\frac{x}{2}+\frac{y}{1}=1 \qquad \frac{x}{a}+\frac{y}{b}=1$$

$$x+2y=2 \Rightarrow x=2(1-y)$$

The limit of x is between 0 to $x=2(1-y)$ and y is 0 to 1,

Now $$\iint_p xydxdy = \int_{y=0}^{y=1}\int_{x=0}^{2(1-y)} xydxdy = \int_{y=0}^{y=1}\left[\frac{x^2}{2}\right]_0^{2(1-y)} ydy$$

$$= \int_{y=0}^{y=1} y\left[\frac{4(1-y)^2}{2}-0\right]dy$$

$$= \int_{y=0}^{y=1} 2y(1+y^2-2y)\,dy = \int_{y=0}^{y=1} 2(y+y^3-2y^2)\,dy$$

Again Integrating and substituting the limits, we get

$$\iint_p xydxdy = 2\left[\frac{y^2}{2}+\frac{y^4}{4}-\frac{2y^3}{3}\right]_0^1 = 2\left[\frac{1}{2}+\frac{1}{4}-\frac{2}{3}-0\right]$$

$$= 2\left[\frac{6+3-8}{12}\right] = \frac{2}{12} = \frac{1}{6}$$

SOL 1.46 Option (B) is correct.

Direction derivative of a function f along a vector P is given by

$$\boldsymbol{a} = \text{grad } f \cdot \frac{\boldsymbol{a}}{|\boldsymbol{a}|}$$

where $$\text{grad } f = \left(\frac{\partial f}{\partial x}i+\frac{\partial f}{\partial y}j+\frac{\partial f}{\partial z}k\right)$$

$$f(x,y,z) = x^2+2y^2+z, \quad \boldsymbol{a} = 3i-4j$$

$$\boldsymbol{a} = \text{grad}(x^2+2y^2+z)\cdot\frac{3i-4j}{\sqrt{(3)^2+(-4)^2}}$$

$$= (2xi+4yj+k)\cdot\frac{(3i-4j)}{\sqrt{25}} = \frac{6x-16y}{5}$$

At point $P(1, 1, 2)$ the direction derivative is

$$\boldsymbol{a} = \frac{6\times1-16\times1}{5} = -\frac{10}{5} = -2$$

SOL 1.47 Option (B) is correct.

Given : $2x+3y=4$

$$x + y + z = 4$$
$$x + 2y - z = a$$

It is a set of non-homogenous equation, so the augmented matrix of this system is

$$[A:B] = \begin{bmatrix} 2 & 3 & 0 & : & 4 \\ 1 & 1 & 1 & : & 4 \\ 1 & 2 & -1 & : & a \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 3 & 0 & : & 4 \\ 0 & -1 & 2 & : & 4 \\ 2 & 3 & 0 & : & 4+a \end{bmatrix} \qquad R_3 \to R_3 + R_2,\ R_2 \to 2R_2 - R_1$$

$$\sim \begin{bmatrix} 2 & 3 & 0 & : & 4 \\ 0 & -1 & 2 & : & 4 \\ 0 & 0 & 0 & : & a \end{bmatrix} \qquad R_3 \to R_3 - R_1$$

So, for a unique solution of the system of equations, it must have the condition

$$\rho[A:B] = \rho[A]$$

So, when putting $a = 0$

We get $\rho[A:B] = \rho[A]$

SOL 1.48 Option (D) is correct.

Here we check all the four options for unbounded condition.

(A) $\int_0^{\pi/4} \tan x dx = [\log|\sec x|]_0^{\pi/4} = \left[\log\left|\sec\frac{\pi}{4}\right| - \log|\sec 0|\right]$

$$= \log\sqrt{2} - \log 1 = \log\sqrt{2}$$

(B) $\int_0^{\infty} \frac{1}{x^2+1} dx = [\tan^{-1} x]_0^{\infty} = \tan^{-1}\infty - \tan^{-1}(0) = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

(C) $\int_0^{\infty} xe^{-x} dx$

Let $I = \int_0^{\infty} xe^{-x} dx = x\int_0^{\infty} e^{-x} dx - \int_0^{\infty} \left[\frac{d}{dx}(x)\int e^{-x} dx\right] dx$

$$= [-xe^{-x}]_0^{\infty} + \int_0^{\infty} e^{-x} dx = [-xe^{-x} - e^{-x}]_0^{\infty} = [-e^{-x}(x+1)]_0^{\infty}$$

$$= -[0 - 1] = 1$$

(D) $\int_0^1 \frac{1}{1-x} dx = -\int_0^1 \frac{1}{x-1} dx = -[\log(x-1)]_0^1 - [\log 0 - \log(-1)]$

Both log 0 and log (–1) undefined so it is unbounded.

SOL 1.49 Option (A) is correct.

Let $I = \oint f(z) dz$ and $f(z) = \frac{\cos z}{z}$

Then $$I = \oint \frac{\cos z}{z} dz = \oint \frac{\cos z}{|z-0|} dz \quad ...(i)$$

Given that $|z| = 1$ for unit circle. From the Cauchy Integral formula

$$\oint \frac{f(z)}{z-a} dz = 2\pi i f(a) \quad ...(ii)$$

Compare equation (i) and (ii), we can say that,

$$a = 0 \text{ and } f(z) = \cos z$$

Or, $$f(a) = f(0) = \cos 0 = 1$$

Now from equation (ii) we get

$$\oint \frac{f(z)}{z-0} dz = 2\pi i \times 1 = 2\pi i \qquad a = 0$$

SOL 1.50 Option (D) is correct.

Given $$y = \frac{2}{3} x^{3/2} \quad ...(i)$$

We know that the length of curve is given by $\int_{x_1}^{x_2} \left\{\sqrt{\left(\frac{dy}{dx}\right)^2 + 1}\right\} dx$ ...(ii)

Differentiate equation(i) w.r.t. x

$$\frac{dy}{dx} = \frac{2}{3} \times \frac{3}{2} x^{\frac{3}{2}-1} = x^{1/2} = \sqrt{x}$$

Substitute the limit $x_1 = 0$ to $x_2 = 1$ and $\frac{dy}{dx}$ in equation (ii), we get

$$\mathcal{L} = \int_0^1 \left(\sqrt{(\sqrt{x})^2 + 1}\right) dx = \int_0^1 \sqrt{x+1}\, dx$$

$$= \left[\frac{2}{3}(x+1)^{3/2}\right]_0^1 = 1.22$$

SOL 1.51 Option (B) is correct.

Let $$A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$$ λ_1 and λ_2 is the eigen values of the matrix.

For eigen values characteristic matrix is,

$$|A - \lambda I| = 0$$

$$\left|\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| = 0$$

$$\begin{vmatrix} (1-\lambda) & 2 \\ 0 & (2-\lambda) \end{vmatrix} = 0 \quad ...(i)$$

$$(1-\lambda)(2-\lambda) = 0 \Rightarrow \lambda = 1 \,\&\, 2$$

So, Eigen vector corresponding to the $\lambda = 1$ is,

$$\begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ a \end{bmatrix} = 0$$

$$2a + a = 0 \Rightarrow a = 0$$

Again for $\lambda = 2$

$$\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ b \end{bmatrix} = 0$$

$$-1 + 2b = 0 \qquad b = \frac{1}{2}$$

Then sum of $a \,\&\, b \Rightarrow a + b = 0 + \frac{1}{2} = \frac{1}{2}$

SOL 1.52 Option (C) is correct.

Given $f(x,y) = y^x$

First partially differentiate the function w.r.t. y

$$\frac{\partial f}{\partial y} = xy^{x-1}$$

Again differentiate. it w.r.t. x

$$\frac{\partial^2 f}{\partial x \partial y} = y^{x-1}(1) + x(y^{x-1}\log y) = y^{x-1}(x\log y + 1)$$

At : $x = 2,\ y = 1$

$$\frac{\partial^2 f}{\partial x \partial y} = (1)^{2-1}(2\log 1 + 1) = 1(2 \times 0 + 1) = 1$$

SOL 1.53 Option (A) is correct.

Given : $y'' + 2y' + y = 0$

$$(D^2 + 2D + 1)y = 0 \qquad \text{where } D = d/dx$$

The auxiliary equation is

$$m^2 + 2m + 1 = 0$$

$$(m+1)^2 = 0,\ m = -1, -1$$

The roots of auxiliary equation are equal and hence the general solution of the given differential equation is,

$$y = (C_1 + C_2 x)e^{m_1 x} = (C_1 + C_2 x)e^{-x} \qquad ..(i)$$

Given $y(0) = 0$ at $x = 0$, $\Rightarrow y = 0$

Substitute in equation (i), we get

$$0 = (C_1 + C_2 \times 0)e^{-0}$$

$$0 = C_1 \times 1 \Rightarrow C_1 = 0$$

Again $y(1) = 0$, at $x = 1 \Rightarrow y = 0$

Substitute in equation (i), we get

$$0 = [C_1 + C_2 \times (1)]e^{-1} = [C_1 + C_2]\frac{1}{e}$$

$$C_1 + C_2 = 0 \Rightarrow C_2 = 0$$

Substitute C_1 and C_2 in equation (i), we get

$$y = (0 + 0x)e^{-x} = 0$$

And $y(0.5) = 0$

SOL 1.54 Option (B) is correct.

Given : $y = x^2$ ...(i)

and interval [1, 5]

At $x = 1 \Rightarrow y = 1$

And at $x = 5$ $y = (5)^2 = 25$

Here the interval is bounded between 1 and 5

So, the minimum value at this interval is 1.

SOL 1.55 Option (A) is correct

Let square matrix

$$A = \begin{bmatrix} x & y \\ y & x \end{bmatrix}$$

The characteristic equation for the eigen values is given by

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} x-\lambda & y \\ y & x-\lambda \end{vmatrix} = 0$$

$$(x-\lambda)^2 - y^2 = 0$$

$$(x-\lambda)^2 = y^2$$

$$x - \lambda = \pm y$$

$$\lambda = x \pm y$$

So, eigen values are real if matrix is real and symmetric.

SOL 1.56 Option (B) is correct.

The Cauchy-Reimann equation, the necessary condition for a function $f(z)$ to be analytic is

$$\frac{\partial \varphi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$\frac{\partial \varphi}{\partial y} = -\frac{\partial \psi}{\partial x} \qquad \text{when } \frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial x} \text{ exist.}$$

SOL 1.57 Option (A) is correct.

Given : $\frac{2^2\varphi}{2x^2} + \frac{2^2\varphi}{2y^2} + \frac{2\varphi}{2x} + \frac{2\varphi}{2y} = 0$

Order is determined by the order of the highest derivative present in it.

Degree is determined by the degree of the highest order derivative present in it after the differential equation is cleared of radicals and fractions.

So, degree = 1 and order = 2

SOL 1.58 Option (B) is correct.

Given $$y = x + \sqrt{x + \sqrt{x + \sqrt{x + ...\infty}}} \quad ...(i)$$

$$y - x = \sqrt{x + \sqrt{x + \sqrt{x + ...\infty}}}$$

Squaring both the sides,

$$(y - x)^2 = x + \sqrt{x + \sqrt{x + ...\infty}}$$

$$(y - x)^2 = y \quad \text{From equation (i)}$$

$$y^2 + x^2 - 2xy = y \quad ...(ii)$$

We have to find $y(2)$, put $x = 2$ in equation (ii),

$$y^2 + 4 - 4y = y$$

$$y^2 - 5y + 4 = 0$$

$$(y - 4)(y - 1) = 0$$

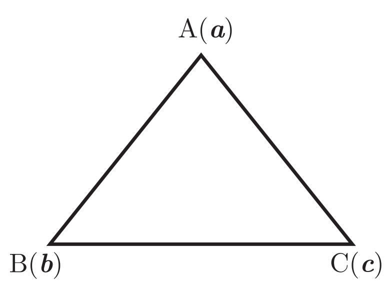
$$y = 1, 4$$

From Equation (i) we see that

For $y(2)$ $$y = 2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + ...\infty}}} > 2$$

Therefore, $$y = 4$$

SOL 1.59 Option (B) is correct.



Vector area of ΔABC,

$$A = \frac{1}{2} BC \times BA = \frac{1}{2}(c - b) \times (a - b)$$

$$= \frac{1}{2}[c \times a - c \times b - b \times a + b \times b]$$

$$= \frac{1}{2}[c \times a + b \times c + a \times b]$$

$$b \times b = 0 \text{ and } c \times b = -(b \times c)$$

$$= \frac{1}{2}[(a - b) \times (a - c)]$$

SOL 1.60 Option (C) is correct.

Given : $$\frac{dy}{dx} = y^2 \text{ or } \frac{dy}{y^2} = dx$$

Integrating both the sides

$$\int \frac{dy}{y^2} = \int dx$$

$$-\frac{1}{y} = x + C \qquad ...(i)$$

Given $y(0) = 1$ at $x = 0 \quad \Rightarrow y = 1$

Put in equation (i) for the value of C

$$-\frac{1}{1} = 0 + C \quad \Rightarrow C = -1$$

From equation (i),

$$-\frac{1}{y} = x - 1$$

$$y = -\frac{1}{x-1}$$

For this value of y, $x - 1 \neq 0$ or $x \neq 1$

And $\quad x < 1$ or $x > 1$

SOL 1.61 Option (A) is correct.

Let $\quad \phi(t) = \int_0^t f(t)\,dt$ and $\phi(0) = 0$ then $\phi'(t) = f(t)$

We know the formula of Laplace transforms of $\phi'(t)$ is

$$L[\phi'(t)] = sL[\phi(t)] - \phi(0) = sL[\phi(t)] \qquad \phi(0) = 0$$

$$L[\phi(t)] = \frac{1}{s} L[\phi'(t)]$$

Substitute the values of $\phi(t)$ and $\phi'(t)$, we get

$$L\left[\int_0^t f(t)\,dt\right] = \frac{1}{s} L[f(t)]$$

or $\quad L\left[\int_0^t f(t)\,dt\right] = \frac{1}{s} F(s)$

SOL 1.62 Option (A) is correct.

From the Trapezoidal Method

$$\int_a^b f(x)\,dx = \frac{h}{2}[f(x_0) + 2f(x_1) + 2f(x_2).....2f(x_{n-1}) + f(x_n)] \qquad ...(i)$$

$$\text{Interval } h = \frac{2\pi - 0}{8} = \frac{\pi}{4}$$

Find $\int_0^{2\pi} \sin x dx$ Here $f(x) = \sin x$

Table for the interval of $\pi/4$ is as follows

Angle θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$f(x) = \sin x$	0	0.707	1	0.707	0	-0.707	-1	-0.707	0

Now from equation(i),

$$\int_0^{2\pi} \sin x dx = \frac{\pi}{8}[0 + 2(0.707 + 1 + 0.707 + 0 - 0.707 - 1 - 0.0707 + 0)]$$

$$= \frac{\pi}{8} \times 0 = 0$$

SOL 1.63 Option (D) is correct.

The X and Y be two independent random variables.

So, $$E(XY) = E(X)E(Y) \qquad \text{(i)}$$

& covariance is defined as

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y)$$

$$= E(X)E(Y) - E(X)E(Y) \qquad \text{From eqn. (i)}$$

$$= 0$$

For two independent random variables

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$$

and $$E(X^2 Y^2) = E(X^2)E(Y^2)$$

So, option (D) is incorrect.

SOL 1.64 Option (B) is correct.

Let, $$f(x) = \lim_{x \to 0} \frac{e^x - \left(1 + x + \frac{x^2}{2}\right)}{x^3} \qquad \frac{0}{0} \text{ form}$$

$$= \lim_{x \to 0} \frac{e^x - (1 + x)}{3x^2} \qquad \frac{0}{0} \text{ form}$$

$$= \lim_{x \to 0} \frac{e^x - 1}{6x} \qquad \frac{0}{0} \text{ form}$$

$$= \lim_{x \to 0} \frac{e^x}{6} = \frac{e^0}{6} = \frac{1}{6}$$

SOL 1.65 Option (B) is correct.

Let, $$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

Let λ is the eigen value of the given matrix then characteristic matrix is

$$|A - \lambda I| = 0 \qquad \text{Here } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \text{Identity matrix}$$

$$\begin{vmatrix} 2 - \lambda & 1 \\ 0 & 2 - \lambda \end{vmatrix} = 0$$

$$(2 - \lambda)^2 = 0$$

$$\lambda = 2, 2$$

So, only one eigen vector.

SOL 1.66 Option (D) is correct.

	Column I		
P.	Gauss-Seidel method	4.	Linear algebraic equation
Q.	Forward Newton-Gauss method	1.	Interpolation
R.	Runge-Kutta method	2.	Non-linear differential equation
S.	Trapezoidal Rule	3.	Numerical integration

So, correct pairs are, P-4, Q-1, R-2, S-3

SOL 1.67 Option (B) is correct.

Given : $\frac{dy}{dx} + 2xy = e^{-x^2}$ and $y(0) = 1$

It is the first order linear differential equation so its solution is

$$y(I.F.) = \int Q(I.F.)\,dx + C$$

compare with $\frac{dy}{dx} + P(y) = Q$

So, $$I.F. = e^{\int P dx} = e^{\int 2x dx}$$

$$= e^{2\int x dx} = e^{2 \times \frac{x^2}{2}} = e^{x^2}$$

The complete solution is,

$$ye^{x^2} = \int e^{-x^2} \times e^{x^2} dx + C$$

$$= \int dx + C = x + C$$

$$y = \frac{x+c}{e^{x^2}} \qquad \text{...(i)}$$

Given $y(0) = 1$

At $x = 0 \Rightarrow y = 1$

Substitute in equation (i), we get

$$1 = \frac{C}{1} \Rightarrow C = 1$$

Then $$y = \frac{x+1}{e^{x^2}} = (x+1)e^{-x^2}$$

SOL 1.68 Option (C) is correct.

The incorrect statement is, $S = \{x : x \in A \text{ and } x \in B\}$ represents the union of set A and set B.

The above symbol ($\in$) denotes intersection of set A and set B. Therefore this statement is incorrect.

SOL 1.69 Option (D) is correct.

Total number of items = 100

Number of defective items = 20

Number of Non-defective items = 80

Then the probability that both items are defective, when 2 items are selected at random is,

$$P = \frac{{}^{20}C_2\,{}^{80}C_0}{{}^{100}C_2} = \frac{\frac{20!}{18!2!}}{\frac{100!}{98!2!}} = \frac{\frac{20\times 19}{2}}{\frac{100\times 99}{2}} = \frac{19}{495}$$

Alternate Method :

Here two items are selected without replacement.

Probability of first item being defective is

$$P_1 = \frac{20}{100} = \frac{1}{5}$$

After drawing one defective item from box, there are 19 defective items in the 99 remaining items.

Probability that second item is defective,

$$P_2 = \frac{19}{899}$$

then probability that both are defective

$$P = P_1 \times P_2 = \frac{1}{5}\times\frac{19}{99} = \frac{19}{495}$$

SOL 1.70 Option (A) is correct.

Given : $S = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$

Eigen values of this matrix is 5 and 1. We can say $\lambda_1 = 1 \quad \lambda_2 = 5$

Then the eigen value of the matrix

$$S^2 = S\,S \text{ is } \lambda_1^2,\ \lambda_2^2$$

Because. if $\lambda_1, \lambda_2, \lambda_3 ....$ are the eigen values of A, then eigen value of A^m are $\lambda_1^m, \lambda_2^m, \lambda_3^m ....$

Hence matrix S^2 has eigen values $(1)^2$ and $(5)^2 \Rightarrow 1$ and 25

SOL 1.71 Option (B) is correct.

Given $f(x) = (x-8)^{2/3} + 1$

The equation of line normal to the function is

$$(y - y_1) = m_2(x - x_1) \qquad ...(i)$$

Slope of tangent at point (0, 5) is

$$m_1 = f'(x) = \left[\frac{2}{3}(x-8)^{-1/3}\right]_{(0,5)}$$

$$m_1 = f'(x) = \frac{2}{3}(-8)^{-1/3} = -\frac{2}{3}(2^3)^{-\frac{1}{3}} = -\frac{1}{3}$$

We know the slope of two perpendicular curves is -1.

$$m_1 m_2 = -1$$

$$m_2 = -\frac{1}{m_1} = \frac{-1}{-1/3} = 3$$

The equation of line, from equation (i) is

$$(y-5) = 3(x-0)$$

$$y = 3x+5$$

SOL 1.72 Option (A) is correct.

Let $$f(x) = \int_0^{\pi/3} e^{it}\,dt = \left[\frac{e^{it}}{i}\right]_0^{\pi/3} \Rightarrow \frac{e^{i\pi/3}}{i} - \frac{e^0}{i}$$

$$= \frac{1}{i}\left[e^{\frac{\pi}{3}i} - 1\right] = \frac{1}{i}\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3} - 1\right]$$

$$= \frac{1}{i}\left[\frac{1}{2} + i\frac{\sqrt{3}}{2} - 1\right] = \frac{1}{i}\left[-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right]$$

$$= \frac{1}{i} \times \frac{i}{i}\left[-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right] = -i\left[-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right] \qquad i^2 = -1$$

$$= i\left[\frac{1}{2} - \frac{\sqrt{3}}{2}i\right] = \frac{1}{2}i - \frac{\sqrt{3}}{2}i^2 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$$

SOL 1.73 Option (B) is correct.

Given $$f(x) = \frac{2x^2 - 7x + 3}{5x^2 - 12x - 9}$$

Then $$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{2x^2 - 7x + 3}{5x^2 - 12x - 9}$$

$$= \lim_{x \to 3} \frac{4x - 7}{10x - 12} \qquad \text{Applying L – Hospital rule}$$

Substitute the limit, we get

$$\lim_{x \to 3} f(x) = \frac{4 \times 3 - 7}{10 \times 3 - 12} = \frac{12 - 7}{30 - 12} = \frac{5}{18}$$

SOL 1.74 Option (A) is correct.

(P) Singular Matrix → Determinant is zero $|A| = 0$

(Q) Non-square matrix → An $m \times n$ matrix for which $m \neq n$, is called non-square matrix. Its determinant is not defined

(R) Real Symmetric Matrix → Eigen values are always real.

(S) Orthogonal Matrix → A square matrix A is said to be orthogonal if $AA^T = I$

Its determinant is always one.

SOL 1.75 Option (B) is correct.

Given : $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 3e^{2x}$

$$[D^2 + 4D + 3]y = 3e^{2x} \qquad \frac{d}{dx} = D$$

The auxiliary Equation is,

$$m^2 + 4m + 3 = 0 \Rightarrow m = -1, -3$$

Then $C.F. = C_1e^{-x} + C_2e^{-3x}$

$$P.I. = \frac{3e^{2x}}{D^2 + 4D + 3} = \frac{3e^{2x}}{(D+1)(D+3)} \qquad \text{Put } D = 2$$

$$= \frac{3e^{2x}}{(2+1)(2+3)} = \frac{3e^{2x}}{3 \times 5} = \frac{e^{2x}}{5}$$

SOL 1.76 Option (C) is correct.

Given $EF = G$ where $G = I =$ Identity matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We know that the multiplication of a matrix and its inverse be a identity matrix

$$AA^{-1} = I$$

So, we can say that F is the inverse matrix of E

$$F = E^{-1} = \frac{[\text{adj}.E]}{|E|}$$

$$\text{adj}E = \begin{bmatrix} \cos\theta & -(\sin\theta) & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|E| = [\cos\theta \times (\cos\theta - 0)] - [(-\sin\theta) \times (\sin\theta - 0)] + 0$$

$$= \cos^2\theta + \sin^2\theta = 1$$

Hence, $F = \frac{[\text{adj}.E]}{|E|} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

SOL 1.77 Option (B) is correct.

The probability density function is,

$$f(t) = \begin{cases} 1 + t & \text{for} -1 \le t \le 0 \\ 1 - t & \text{for } 0 \le t \le 1 \end{cases}$$

For standard deviation first we have to find the mean and variance of the function.

$$\text{Mean}\,(\bar{t}) = \int_{-1}^{\infty} t f(t)\,dt = \int_{-1}^{0} t(1+t)\,dt + \int_{0}^{1} t(1-t)\,dt$$

$$= \int_{-1}^{0} (t + t^2)\,dt + \int_{0}^{1} (t - t^2)\,dt$$

$$= \left[\frac{t^2}{2} + \frac{t^3}{3}\right]_{-1}^{0} + \left[\frac{t^2}{2} - \frac{t^3}{3}\right]_{0}^{1} = \left[-\frac{1}{2} + \frac{1}{3}\right] + \left[\frac{1}{2} - \frac{1}{3}\right] = 0$$

And variance $(\sigma^2) = \int_{-\infty}^{\infty} (t - \bar{t})^2 f(t)\, dt$ $\qquad \bar{t} = 0$

$$= \int_{-1}^{0} t^2(1 + t)\, dt + \int_{0}^{1} t^2(1 - t)\, dt$$

$$= \int_{-1}^{0} (t^2 + t^3)\, dt + \int_{0}^{1} (t^2 - t^3)\, dt$$

$$= \left[\frac{t^3}{3} + \frac{t^4}{4}\right]_{-1}^{0} + \left[\frac{t^3}{3} - \frac{t^4}{4}\right]_{0}^{1}$$

$$= -\left[-\frac{1}{3} + \frac{1}{4}\right] + \left[\frac{1}{3} - \frac{1}{4} - 0\right] = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$$

Now, standard deviation

$$\sqrt{(\sigma^2)}\, s = \sqrt{\frac{1}{6}} = \frac{1}{\sqrt{6}}$$

SOL 1.78 Option (A) is correct.

The Stokes theorem is,

$$\oint_C F \cdot dr = \iint_S (\nabla \times F) \cdot n dS = \iint_S (\text{Curl}\, F) \cdot dS$$

Here we can see that the line integral $\oint_C F \cdot dr$ and surface integral $\iint_S (\text{Curl}\, F) \cdot ds$ is related to the stokes theorem.

SOL 1.79 Option (B) is correct.

Let, P = defective items

Q = non-defective items

10% items are defective, then probability of defective items

$$P = 0.1$$

Probability of non-defective item

$$Q = 1 - 0.1 = 0.9$$

The Probability that exactly 2 of the chosen items are defective is

$$= {}^{10}C_2 (P)^2 (Q)^8 = \frac{10!}{8!2!} (0.1)^2 (0.9)^8$$

$$= 45 \times (0.1)^2 \times (0.9)^8 = 0.1937$$

SOL 1.80 Option (A) is correct.

Let $f(x) = \int_{-a}^{a} (\sin^6 x + \sin^7 x)\, dx$

$$= \int_{-a}^{a} \sin^6 x dx + \int_{-a}^{a} \sin^7 x dx$$

We know that

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 0 & \text{when } f(-x) = -f(x);\text{ odd function} \\ 2\int_0^a f(x) & \text{when } f(-x) = f(x);\text{ even function} \end{cases}$$

Now, here $\sin^6 x$ is an even function and $\sin^7 x$ is an odd function. Then,

$$f(x) = 2\int_0^a \sin^6 x dx + 0 = 2\int_0^a \sin^6 x dx$$

SOL 1.81 Option (C) is correct.

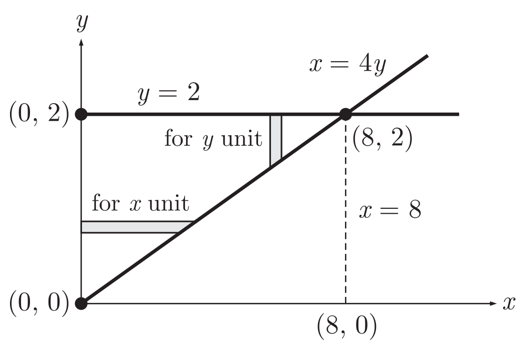
We know, from the Echelon form the rank of any matrix is equal to the Number of non zero rows.

Here order of matrix is 3×4, then, we can say that the Highest possible rank of this matrix is 3.

SOL 1.82 Option (A) is correct.

Given $$I = \int_0^8 \int_{\pi/4}^2 f(x,y)\,dydx$$

We can draw the graph from the limits of the integration, the limit of y is from $y = \frac{x}{4}$ to $y = 2$. For x the limit is $x = 0$ to $x = 8$



Here we change the order of the integration. The limit of x is 0 to 8 but we have to find the limits in the form of y then $x = 0$ to $x = 4y$ and limit of y is 0 to 2

So $$\int_0^8 \int_{x/4}^2 f(x,y)\,dydx = \int_0^2 \int_0^{4y} f(x,y)\,dxdy = \int_r^s \int_p^q f(x,y)\,dxdy$$

Comparing the limits and get

$r = 0,\ s = 2,\ p = 0,\ q = 4y$

SOL 1.83 Option (A) is correct.

Let, $$A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

The characteristic equation for eigen values is given by,

$$|A - \lambda I| = 0$$

$$A = \begin{vmatrix} 5-\lambda & 0 & 0 & 0 \\ 0 & 5-\lambda & 0 & 0 \\ 0 & 0 & 2-\lambda & 1 \\ 0 & 0 & 3 & 1-\lambda \end{vmatrix} = 0$$

Solving this, we get

$$(5-\lambda)(5-\lambda)[(2-\lambda)(1-\lambda)-3] = 0$$

$$(5-\lambda)^2[2-3\lambda+\lambda^2-3] = 0$$

$$(5-\lambda)^2(\lambda^2-3\lambda-1) = 0$$

So, $(5-\lambda)^2 = 0 \Rightarrow \lambda = 5, 5$ and $\lambda^2 - 3\lambda - 1 = 0$

$$\lambda = \frac{-(-3) \pm \sqrt{9+4}}{2} = \frac{3+\sqrt{13}}{2}, \frac{3-\sqrt{13}}{2}$$

The eigen values are $\lambda = 5, 5, \frac{3+\sqrt{13}}{2}, \frac{3-\sqrt{13}}{2}$

Let $X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

be the eigen vector for the eigen value $\lambda = 5$

Then, $(A - \lambda I)X_1 = 0$

$$(A - 5I)X_1 = 0$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 3 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

or $-3x_3 + x_4 = 0$

$$3x_3 - 4x_4 = 0$$

This implies that $x_3 = 0$, $x_4 = 0$

Let $x_1 = k_1$ and $x_2 = k_2$

So, eigen vector, $X_1 = \begin{bmatrix} k_1 \\ k_2 \\ 0 \\ 0 \end{bmatrix}$ where $k_1, k_2 \varepsilon R$

SOL 1.84 Option (C) is correct.

Given : $x + y = 2$ ...(i)

$1.01x + 0.99y = b$, $db = 1$ unit ...(ii)

We have to find the change in x in the solution of the system. So reduce y

From the equation (i) and (ii).

Multiply equation (i) by 0.99 and subtract from equation (ii)

$$1.01x + 0.99y - (0.99x + 0.99y) = b - 1.98$$

$$1.01x - 0.99x = b - 1.98$$

$$0.02x = b - 1.98$$

Differentiating both the sides, we get

$$0.02dx = db$$

$$dx = \frac{1}{0.02} = 50 \text{ unit} \qquad db = 1$$

SOL 1.85 Option (A) is correct.

Given, $x(u,v) = uv$

$$\frac{dx}{du} = v, \qquad \frac{dx}{dv} = u$$

And $y(u,v) = \frac{v}{u}$

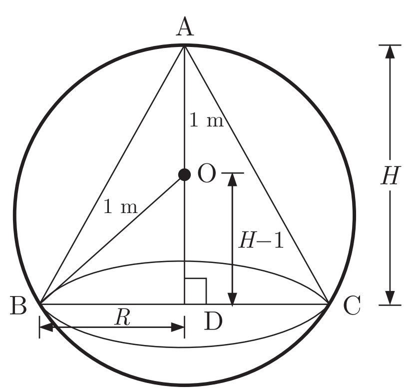
$$\frac{\partial y}{\partial u} = -\frac{v}{u^2} \qquad \frac{\partial y}{\partial v} = \frac{1}{u}$$

We know that,

$$\phi(u,v) = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix}$$

$$\phi(u,v) = \begin{bmatrix} v & u \\ \frac{-v}{u^2} & \frac{1}{u} \end{bmatrix} = v \times \frac{1}{u} - u \times \left(-\frac{v}{u^2}\right) = \frac{v}{u} + \frac{v}{u} = \frac{2v}{u}$$

SOL 1.86 Option (D) is correct.



Given : Radius of sphere $r = 1$

Let, Radius of cone $= R$

Height of the cone $= H$

Finding the relation between the volume and Height of the cone

From ΔOBD, $OB^2 = OD^2 + BD^2$

$$1 = (H-1)^2 + R^2 = H^2 + 1 - 2H + R^2$$

$$R^2 + H^2 - 2H = 0$$

$$R^2 = 2H - H^2 \qquad ...(i)$$

Volume of the cone, $V = \frac{1}{3}\pi R^2 H$

Substitute the value of R^2 from equation (i), we get

$$V = \frac{1}{3}\pi(2H - H^2)H = \frac{1}{3}\pi(2H^2 - H^3)$$

Differentiate V w.r.t to H

$$\frac{dV}{dH} = \frac{1}{3}\pi[4H - 3H^2]$$

Again differentiate $\frac{d^2V}{dH^2} = \frac{1}{3}\pi[4 - 6H]$

For minimum and maximum value, using the principal of minima and maxima.

Put $\frac{dV}{dH} = 0$

$$\frac{1}{3}\pi[4H - 3H^2] = 0$$

$$H[4 - 3H] = 0 \Rightarrow H = 0 \text{ and } H = \frac{4}{3}$$

At $H = \frac{4}{3}$, $\frac{d^2V}{dH^2} = \frac{1}{3}\pi\left[4 - 6 \times \frac{4}{3}\right] = \frac{1}{3}\pi[4-8] = -\frac{4}{3}\pi < 0$ (Maxima)

And at $H = 0$, $\frac{d^2V}{dH^2} = \frac{1}{3}\pi[4-0] = \frac{4}{3}\pi > 0$ (Minima)

So, for the largest volume of cone, the value of H should be 4/3

SOL 1.87 Option (D) is correct.

Given : $x^2\frac{dy}{dx} + 2xy = \frac{2\ln(x)}{x}$

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{2\ln(x)}{x^3}$$

Comparing this equation with the differential equation $\frac{dy}{dx} + P(y) = Q$ we have $P = \frac{2}{x}$ and $Q = \frac{2\ln(x)}{x^3}$

The integrating factor is,

$$\text{I.F.} = e^{\int P dx} = e^{\int \frac{2}{x} dx}$$

$$e^{2\ln x} = e^{\ln x^2} = x^2$$

Complete solution is written as,

$$y(I.F.) = \int Q(I.F.)\,dx + C$$

$$y(x^2) = \int \frac{2\ln x}{x^3} \times x^2 dx + C = 2\int \underset{\text{(II)}}{\ln x} \times \underset{\text{(I)}}{\frac{1}{x}} dx + C \qquad \text{...(i)}$$

Integrating the value $\int \ln x \times \frac{1}{x} dx$ Separately

Let, $$I = \int \underset{\text{(I)}}{\ln x} \times \underset{\text{(II)}}{\frac{1}{x}} dx \qquad \text{...(ii)}$$

$$= \ln x \int \frac{1}{x} dx - \int \left\{ \frac{d}{dx}(\ln x) \times \int \frac{1}{x} dx \right\} dx$$

$$= \ln x \ln x - \underbrace{\int \frac{1}{x} \times \ln x dx}_{\text{I}} \qquad \text{From equation(ii)}$$

$$2I = (\ln x)^2$$

or $$I = \frac{(\ln x)^2}{2} \qquad \text{...(iii)}$$

Substitute the value from equation (iii) in equation (i),

$$y(x^2) = \frac{2(\ln x)^2}{2} + C$$

$$x^2 y = (\ln x)^2 + C \qquad \text{...(iv)}$$

Given $y(1) = 0$, means at $x = 1 \Rightarrow y = 0$

then $$0 = (\ln 1)^2 + C \Rightarrow C = 0$$

So from equation (iv), we get

$$x^2 y = (\ln x)^2$$

Now at $x = e$, $\quad y(e) = \frac{(\ln e)^2}{e^2} = \frac{1}{e^2}$

SOL 1.88 Option (A) is correct.

Potential function of $v = x^2 yz$ at $P(1,1,1)$ is $= 1^2 \times 1 \times 1 = 1$ and at origin $O(0,0,0)$ is 0.

Thus the integral of vector function from origin to the point $(1,1,1)$ is

$$= [x^2 yz]_P - [x^2 yz]_O$$

$$= 1 - 0 = 1$$

SOL 1.89 Option (C) is correct.

Let, $$f(x) = x^3 + 3x - 7$$

From the Newton Rapson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad \text{...(i)}$$

We have to find the value of x_1, so put $n = 0$ in equation (i),

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$f(x) = x^3 + 3x - 7$$

$$f(x_0) = 1^3 + 3 \times 1 - 7 = 1 + 3 - 7 = -3 \qquad x_0 = 1$$

$$f'(x) = 3x^2 + 3$$

$$f'(x_0) = 3 \times (1)^2 + 3 = 6$$

Then, $$x_1 = 1 - \frac{(-3)}{6} = 1 + \frac{3}{6} = 1 + \frac{1}{2} = \frac{3}{2} = 1.5$$

SOL 1.90 Option (D) is correct.

We know a die has 6 faces and 6 numbers so the total number of ways

$$= 6 \times 6 = 36$$

And total ways in which sum is either 8 or 9 is 9, i.e.

$(2,6), (3,6)(3,5)(4,4)(4,5)(5,4)(5,3)(6,2)(6,3)$

Total number of tosses when both the 8 or 9 numbers are not come

$$= 36 - 9 = 27$$

Then probability of not coming sum 8 or 9 is, $= \frac{27}{36} = \frac{3}{4}$

SOL 1.91 Option (C) is correct.

Given : $$\frac{d^2y}{dx^2} + p\frac{dy}{dx} + qy = 0$$

The solution of this equation is given by,

$$y = c_1 e^{mx} + c_2 e^{nx} \qquad \text{...(i)}$$

Here m & n are the roots of ordinary differential equation

Given solution is, $$y = c_1 e^{-x} + c_2 e^{-3x} \qquad \text{...(ii)}$$

Comparing equation (i) and (ii), we get $m = -1$ and $n = -3$

Sum of roots, $$m + n = -p$$

$$-1 - 3 = -p \Rightarrow p = 4$$

and product of roots, $$mn = q$$

$$(-1)(-3) = q \Rightarrow q = 3$$

SOL 1.92 Option (C) is correct.

Given : $$\frac{d^2y}{dx^2} + p\frac{dy}{dx} + (q+1)y = 0$$

$$[D^2 + pD + (q+1)]y = 0 \qquad \frac{d}{dx} = D$$

From the previous question, put $p = 4$ and $m = 3$

$$[D^2 + 4D + 4]y = 0 \qquad \text{...(i)}$$

The auxilliary equation of equation (i) is written as

$$m^2+4m+4=0 \Rightarrow m=-2,-2$$

Here the roots of auxiliary equation are same then the solution is

$$y=(c_1+c_2x)e^{mx}=xe^{-2x} \qquad \begin{pmatrix}\text{Let } c_1=0\\ c_2=1\end{pmatrix}$$

SOL 1.93 Option (C) is correct.

Given : $x=a(\theta+\sin\theta),\ y=a(1-\cos\theta)$

First differentiate x w.r.t. θ,

$$\frac{dx}{d\theta}=a[1+\cos\theta]$$

And differentiate y w.r.t. θ

$$\frac{dy}{d\theta}=a[0-(-\sin\theta)]=a\sin\theta$$

We know, $$\frac{dy}{dx}=\frac{dy}{d\theta}\times\frac{d\theta}{dx}=\frac{dy/d\theta}{dx/d\theta}$$

Substitute the values of $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$

$$\frac{dy}{dx}=a\sin\theta\times\frac{1}{a[1+\cos\theta]}=\frac{\sin\theta}{1+\cos\theta}=\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$

$$=\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}=\tan\frac{\theta}{2} \qquad \cos\theta+1=2\cos^2\frac{\theta}{2}$$

SOL 1.94 Option (C) is correct.

Given : $P(0.866, 0.500, 0)$, so we can write

$$P=0.866i+0.5j+0k$$

$Q=(0.259, 0.966, 0)$, so we can write

$$Q=0.259i+0.966j+0k$$

For the coplanar vectors

$$P\cdot Q=|P||Q|\cos\theta$$

$$\cos\theta=\frac{P\cdot Q}{|P||Q|}$$

$$P\cdot Q=(0.866i+0.5j+0k)\cdot(0.259i+0.966j+0k)$$

$$=0.866\times0.259+0.5\times0.966$$

So, $$\cos\theta=\frac{0.866\times0.259+0.5\times0.966}{\sqrt{(0.866)^2+(0.5)^2}+\sqrt{(0.259)^2+(0.966)^2}}$$

$$=\frac{0.22429+0.483}{\sqrt{0.99}\times\sqrt{1.001}}=\frac{0.70729}{\sqrt{0.99}\times\sqrt{1.001}}=0.707$$

$$\theta=\cos^{-1}(0.707)=45^\circ$$

SOL 1.95 Option (B) is correct.

Let $$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

We know that the sum of the eigen value of a matrix is equal to the sum of the diagonal elements of the matrix

So, the sum of eigen values is,

$$1+5+1 = 7$$

SOL 1.96 Option (D) is correct.

Given : Total number of cards = 52 and two cards are drawn at random.

Number of kings in playing cards = 4

So the probability that both cards will be king is given by,

$$P = \frac{{}^4C_1}{{}^{52}C_1} \times \frac{{}^3C_1}{{}^{51}C_1} = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221} \qquad {}^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}$$

SOL 1.97 Option (B) is correct.

Given : $$U(t-a) = \begin{cases} 0, & \text{for } t < a \\ 1, & \text{for } t \geq a \end{cases}$$

From the definition of Laplace Transform

$$\mathcal{L}[F(t)] = \int_0^\infty e^{-st} f(t)\,dt$$

$$\mathcal{L}[U(t-a)] = \int_0^\infty e^{-st} U(t-a)\,dt$$

$$= \int_0^a e^{-st}(0) + \int_a^\infty e^{-st}(1)\,dt = 0 + \int_a^\infty e^{-st}\,dt$$

$$\mathcal{L}[U(t-a)] = \left[\frac{e^{-st}}{-s}\right]_a^\infty = 0 - \left[\frac{e^{-as}}{-s}\right] = \frac{e^{-as}}{s}$$

SOL 1.98 Option (D) is correct.

First we have to make the table from the given data

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	1			
		1		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

Take $x_0 = 0$ and $h = 1$

Then $$P = \frac{x - x_0}{h} = x$$

From Newton's forward Formula

$$f(x) = f(x_0) + \frac{P}{\underline{|1}}\Delta f(0) + \frac{P(P-1)}{\underline{|2}}\Delta^2 f(0) + \frac{P(P-1)(P-2)}{\underline{|3}}\Delta^3 f(0)$$

$$= f(0) + x\Delta f(0) + \frac{x(x-1)}{2}\Delta^2 f(0) + \frac{x(x-1)(x-2)}{6}\Delta^3 f(0)$$

$$= 1 + x(1) + \frac{x(x-1)}{2}(-2) + \frac{x(x-1)(x-2)}{6}(12)$$

$$= 1 + x - x(x-1) + 2x(x-1)(x-2)$$

$$f(x) = 2x^3 - 7x^2 + 6x + 1$$

SOL 1.99 Option (A) is correct.

Given : $$V = \int_0^{2\pi}\int_0^{\pi/3}\int_0^1 r^2 \sin\phi\, dr d\phi d\theta$$

First integrating the term of r, we get

$$V = \int_0^{2\pi}\int_0^{\pi/3}\left[\frac{r^3}{3}\right]_0^1 \sin\phi\, d\phi d\theta = \int_0^{2\pi}\int_0^{\pi/3}\frac{1}{3}\sin\phi\, d\phi d\theta$$

Integrating the term of ϕ, we have

$$V = \frac{1}{3}\int_0^{2\pi}[-\cos\phi]_0^{\pi/3} d\theta$$

$$= -\frac{1}{3}\int_0^{2\pi}\left[\cos\frac{\pi}{3} - \cos 0\right] d\theta = -\frac{1}{3}\int_0^{2\pi}\left[\frac{1}{2} - 1\right] d\theta$$

$$= -\frac{1}{3}\int_0^{2\pi}\left(-\frac{1}{2}\right) d\theta = -\frac{1}{3}\times\left(-\frac{1}{2}\right)\int_0^{2\pi} d\theta$$

Now, integrating the term of θ, we have

$$V = \frac{1}{6}[\theta]_0^{2\pi} = \frac{1}{6}[2\pi - 0] = \frac{\pi}{3}$$

SOL 1.100 Option (A) is correct.

Let, $$A = \begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

For singularity of the matrix $|A| = 0$

$$\begin{vmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{vmatrix} = 0$$

$$8[0 - 2\times 6] - x[0 - 24] + 0[24 - 0] = 0$$

$$8\times(-12) + 24x = 0$$

$$-96 + 24x = 0 \Rightarrow x = \frac{96}{24} = 4$$

SOL 1.101 Option (A) is correct

Let, $$f(x) = \lim_{x \to 0} \frac{\sin^2 x}{x} = \lim_{x \to 0} \frac{\sin^2 x}{x} \times \frac{x}{x}$$

$$= \lim_{x \to 0} \left(\frac{\sin x}{x}\right)^2 \times x \qquad \lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$= (1)^2 \times 0 = 0$$

Alternative :

Let $$f(x) = \lim_{x \to 0} \frac{\sin^2 x}{x} \qquad \left[\frac{0}{0}\text{form}\right]$$

$$f(x) = \lim_{x \to 0} \frac{2\sin x \cos x}{1} \qquad \text{Apply L-Hospital rule}$$

$$= \lim_{x \to 0} \frac{\sin 2x}{1} = \frac{\sin 0}{1} = 0$$

SOL 1.102 Option (D) is correct.

Accuracy of Simpson's rule quadrature is $O(h^5)$

SOL 1.103 Option (C) is correct.

Let, $$A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$

The characteristic equation for the eigen value is given by,

$$|A - \lambda I| = 0 \qquad I = \text{Identity matrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\left| \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 4-\lambda & 1 \\ 1 & 4-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(4-\lambda) - 1 = 0$$

$$(4-\lambda)^2 - 1 = 0$$

$$\lambda^2 - 8\lambda + 15 = 0$$

Solving above equation, we get

$$\lambda = 5, 3$$

SOL 1.104 Option (C) is correct.

Given : $$x + 2y + z = 6$$

$$2x + y + 2z = 6$$

$$x + y + z = 5$$

Comparing to $Ax = B$, we get

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 6 \\ 6 \\ 5 \end{bmatrix}$$

Write the system of simultaneous equations in the form of Augmented matrix,

$$[A:B] = \begin{bmatrix} 1 & 2 & 1 & : & 6 \\ 2 & 1 & 2 & : & 6 \\ 1 & 1 & 1 & : & 5 \end{bmatrix} \qquad R_2 \to R_2 - 2R_1 \text{ and } R_3 \to 2R_3 - R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 6 \\ 0 & -3 & 0 & : & -6 \\ 0 & 1 & 0 & : & 4 \end{bmatrix} \qquad R_3 \to 3R_3 + R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 6 \\ 0 & -3 & 0 & : & -6 \\ 0 & 0 & 0 & : & 6 \end{bmatrix}$$

It is a echelon form of matrix.

Since $\rho[A] = 2$ and $\rho[A:B] = 3$

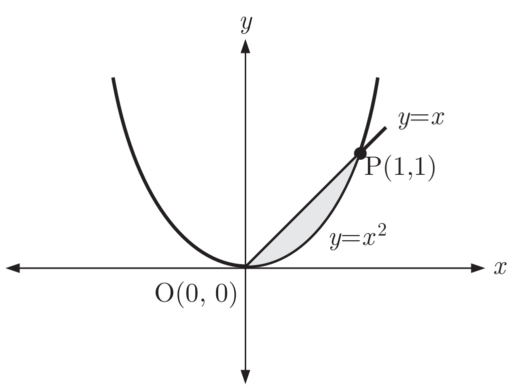
$$\rho[A] \neq \rho[A:B]$$

So, the system has no solution and system is inconsistent.

SOL 1.105 Option (B) is correct.

Given : $y = x^2$ and $y = x$.

The shaded area shows the area, which is bounded by the both curves.



Solving given equation, we get the intersection points as,

In $y = x^2$ putting $y = x$ we have $x = x^2$ or $x^2 - x = 0$ which gives $x = 0, 1$

Then from $y = x$ we can see that curve $y = x^2$ and $y = x$ intersects at point $(0,0)$ and $(1,1)$. So, the area bounded by both the curves is

$$A = \int_{x=0}^{x=1} \int_{y=x}^{y=x^2} dy dx = \int_{x=0}^{x=1} dx \int_{y=x}^{y=x^2} dy = \int_{x=0}^{x=1} dx [y]_x^{x^2}$$

$$= \int_{x=0}^{x=1} (x^2 - x) = \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1 = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6} = \frac{1}{6}\text{unit}^2$$

Area is never negative

SOL 1.106 Option (A) is correct.

$$\frac{dy}{dx} + y^2 = 0$$

$$\frac{dy}{dx} = -y^2$$

$$-\frac{dy}{y^2} = dx$$

Integrating both the sides, we have

$$-\int \frac{dy}{y^2} = \int dx$$

$$y^{-1} = x + c \quad \Rightarrow y = \frac{1}{x+c}$$

SOL 1.107 Option (C) is correct.

Given : $\boldsymbol{F} = x\boldsymbol{i} - y\boldsymbol{j}$

First Check divergency, for divergence,

$$\text{Grade } \boldsymbol{F} = \nabla \cdot \boldsymbol{F} = \left[\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right] \cdot [xi - yj] = 1 - 1 = 0$$

So we can say that $\boldsymbol{F}$ is divergence free.

Now checking the irrationalit;. For irritation the curl $\boldsymbol{F} = 0$

$$\text{Curl } \boldsymbol{F} = \nabla \times \boldsymbol{F} = \left[\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right] \times [xi - yj]$$

$$= \begin{bmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & -y & 0 \end{bmatrix} = i[0-0] - j[0-0] + k[0-0] = 0$$

So, vector field is irrotational. We can say that the vector field is divergence free and irrotational.

SOL 1.108 Option (B) is correct.

Let $f(t) = \sin \omega t$

From the definition of Laplace transformation

$$\mathcal{L}[F(t)] = \int_0^\infty e^{-st} f(t)\,dt = \int_0^\infty e^{-st} \sin \omega t dt$$

$$= \int_0^\infty e^{-st} \left(\frac{e^{i\omega t} - e^{-i\omega t}}{2i}\right) dt$$

$$\sin\omega t = \frac{e^{i\omega t} - e^{-i\omega t}}{2i} = \frac{1}{2i}\int_0^\infty (e^{-st}e^{i\omega t} - e^{-st}e^{-i\omega t})\,dt$$

$$= \frac{1}{2i}\int_0^\infty [e^{(-s+i\omega)t} - e^{-(s+i\omega)t}]dt$$

Integrating above equation, we get

$$\sin\omega t = \frac{1}{2i}\left[\frac{e^{(-s+i\omega)t}}{-s+i\omega} - \frac{e^{-(s+i\omega)t}}{-(s+i\omega)}\right]_0^\infty$$

$$= \frac{1}{2i}\left[\frac{e^{(-s+i\omega)t}}{-s+i\omega} + \frac{e^{-(s+i\omega)t}}{(s+i\omega)}\right]_0^\infty$$

Substitute the limits, we get

$$\sin\omega t = \frac{1}{2i}\left[0 + 0 - \left(\frac{e^0}{(-s+i\omega)} + \frac{e^{-0}}{s+i\omega}\right)\right]$$

$$= -\frac{1}{2i}\left[\frac{s+i\omega+i\omega-s}{(-s+i\omega)(s+i\omega)}\right]$$

$$= -\frac{1}{2i}\times\frac{2i\omega}{(i\omega)^2 - s^2} = \frac{-\omega}{-\omega^2 - s^2} = \frac{\omega}{\omega^2 + s^2}$$

Alternative :

From the definition of Laplace transformation

$$\mathcal{L}[F(t)] = \int_0^\infty e^{-st}\sin\omega t dt$$

We know $\int e^{at}\sin bt dt = \frac{e^{at}}{a^2+b^2}[a\sin bt - b\cos bt]$ $\quad\begin{pmatrix} a = -s \text{ and} \\ b = \omega \end{pmatrix}$

Then, $\mathcal{L}[\sin\omega t] = \left[\frac{e^{-st}}{s^2+\omega^2}(-s\sin\omega t - \omega\cos\omega t)\right]_0^\infty$

$$= \left[\frac{e^{-\infty}}{s^2+\omega^2}(-s\sin\infty - \omega\cos\infty)\right] - \left[\frac{e^{-0}}{s^2+\omega^2}(-s\sin 0 - \omega\cos 0)\right]$$

$$= 0 - \frac{1}{s^2+\omega^2}[0-\omega] = -\frac{1}{s^2+\omega^2}(-\omega)$$

$$\mathcal{L}[\sin\omega t] = \frac{\omega}{s^2+\omega^2}$$

SOL 1.109 Option (D) is correct.

Given : black balls = 5, Red balls = 5, Total balls = 10

Here, two balls are picked from the box randomly one after the other without replacement. So the probability of both the balls are red is

$$P = \frac{{}^5C_0 \times {}^5C_2}{{}^{10}C_2} = \frac{\frac{5!}{0!\times 5!}\times\frac{5!}{3!2!}}{\frac{10!}{3!2!}} = \frac{1\times 10}{45} = \frac{10}{45} = \frac{2}{9} \qquad {}^nC_r = \frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}}$$

Alternate Method :

Given : Black balls = 5,

$$\text{Red balls} = 5$$
$$\text{Total balls} = 10$$

The probability of drawing a red bell,

$$P_1 = \frac{5}{10} = \frac{1}{2}$$

If ball is not replaced, then box contains 9 balls.
So, probability of drawing the next red ball from the box.

$$P_2 = \frac{4}{9}$$

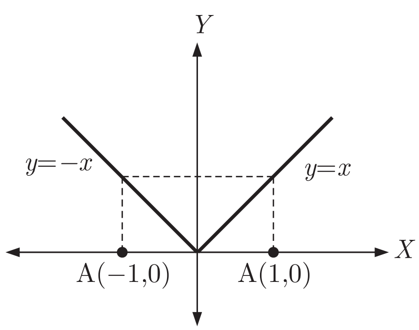
Hence, probability for both the balls being red is,

$$P = P_1 \times P_2 = \frac{1}{2} \times \frac{4}{9} = \frac{2}{9}$$

SOL 1.110 Option (A) is correct.
We know that a dice has 6 faces and 6 numbers so the total number of cases (outcomes) $= 6 \times 6 = 36$
And total ways in which sum of the numbers on the dices is eight,
(2, 6) (3, 5) (4, 4) (5, 3) (6, 2)
So, the probability that the sum of the numbers eight is,

$$p = \frac{5}{36}$$

SOL 1.111 Option (D) is correct.
We have to draw the graph on x-y axis from the given functions.



$$f(x) = \begin{cases} -x & x \le -1 \\ 0 & x = 0 \\ x & x \ge 1 \end{cases}$$

It clearly shows that $f(x)$ is differential at $x = -1$, $x = 0$ and $x = 1$, i.e. in the domain $[-1, 1]$.
So, (a), (b) and (c) are differential and $f(x)$ is maximum at $(x, -x)$.

SOL 1.112 Option (B) is correct.
If the scatter diagram indicates some relationship between two variables X

and Y, then the dots of the scatter diagram will be concentrated round a curve. This curve is called the curve of regression.
Regression analysis is used for estimating the unknown values of one variable corresponding to the known value of another variable.

SOL 1.113 Option (B) is correct.

Given :
$$3x+2y+z = 4$$
$$x-y+z = 2$$
$$-2x+2z = 5$$

The Augmented matrix of the given system of equation is

$$[A:B] = \begin{bmatrix} 3 & 2 & 1 & : & 4 \\ 1 & -1 & 1 & : & 2 \\ -2 & 0 & 2 & : & 5 \end{bmatrix} R_3 \to R_3+2R_2,\ R_2 \to R_2-R_1$$

$$\sim \begin{bmatrix} 3 & 2 & 1 & : & 4 \\ -2 & -3 & 0 & : & -2 \\ 0 & -2 & 4 & : & 9 \end{bmatrix}$$

Here $\rho[A:B] = \rho[A] = 3 = n$ (number of unknown)
Then the system of equation has a unique solution.

SOL 1.114 Option (B) is correct.

Given : $f(x,y) = 2x^2+2xy-y^3$

Partially differentiate this function w.r.t x and y,

$$\frac{\partial f}{\partial x} = 4x+2y,\quad \frac{\partial f}{\partial y} = 2x-3y^2$$

For the stationary point of the function, put $\partial f/\partial x$ and $\partial f/\partial y$ equal to zero.

$$\frac{\partial f}{\partial x} = 4x+2y = 0 \quad \Rightarrow \quad 2x+y = 0 \qquad \text{...(i)}$$

and
$$\frac{\partial f}{\partial y} = 2x-3y^2 = 0 \quad \Rightarrow \quad 2x-3y^2 = 0 \qquad \text{...(ii)}$$

From equation (i), $y = -2x$ substitute in equation (ii),

$$2x-3(-2x)^2 = 0$$
$$2x-3\times 4x^2 = 0$$
$$6x^2-x = 0 \Rightarrow x = 0,\ \frac{1}{6}$$

From equation (i),

For $x = 0$, $\quad y = -2\times(0) = 0$

and for $x = \frac{1}{6}$, $\quad y = -2\times\frac{1}{6} = -\frac{1}{3}$

So, two stationary point at $(0,0)$ and $\left(\frac{1}{6}, -\frac{1}{3}\right)$

SOL 1.115 Option (B) is correct.

$$\text{Sample space} = \begin{matrix} (1,1),(1,2)\dots(1,8) \\ (2,1), & (2,2) & \dots & (2,8) \\ (3,1), & (3,2) & \dots & (3,8) \\ \vdots & \vdots & \vdots & \vdots \\ (8,1), & (8,2) & \dots & (8,8) \end{matrix}$$

$$\text{Total number of sample space} = 8 \times 8 = 64$$

Now, the favourable cases when Manish will arrive late at D

$$= (6,8),\ (8,6)...(8,8)$$

$$\text{Total number of favourable cases} = 13$$

So,
$$\text{Probability} = \frac{\text{Total number of favourable cases}}{\text{Totol number of sample space}}$$

$$= \frac{13}{64}$$

SOL 1.116 Option (B) is correct.

Divergence is defined as $\nabla \cdot r$

where $r = xi + yj + zk$

and $\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$

So, $\nabla \cdot r = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right) \cdot (xi + yj + zk)$

$$\nabla \cdot r = 1 + 1 + 1 = 3$$

SOL 1.117 Option (B) is correct.

Given :
$$x + y = 2$$
$$2x + 2y = 5$$

The Augmented matrix of the given system of equations is

$$[A:B] = \begin{bmatrix} 1 & 1 & : & 2 \\ 2 & 2 & : & 5 \end{bmatrix}$$

Applying row operation, $R_2 \to R_2 - 2R_1$

$$[A:B] = \begin{bmatrix} 1 & 1 & : & 2 \\ 0 & 0 & : & 1 \end{bmatrix}$$

$$\rho[A] = 1 \neq \rho[A:B] = 2$$

So, the system has no solution.

SOL 1.118 Option (D) is correct.

Given : $f(x) = |x|$

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$Lf'(x) = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h \to 0} \frac{-(-h)}{-h} - 0 = -1$$

$$Rf'(x) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{h-0}{h} = 1$$

Since $Lf'(0) \neq Rf'(0)$

So, derivative of $f(x)$ at $x = 0$ does not exist.

SOL 1.119 Option (A) is correct.

The surface integral of the normal component of a vector function F taken around a closed surface S is equal to the integral of the divergence of F taken over the volume V enclosed by the surface S.

Mathematically $$\iint_S F \cdot n\, dS = \iiint_V \text{div}\, F dv$$

So, Gauss divergence theorem relates surface integrals to volume integrals.

SOL 1.120 Option (A) is correct.

Given : $$f(x) = \frac{x^3}{3} - x$$

$$f'(x) = x^2 - 1$$

$$f''(x) = 2x$$

Using the principle of maxima – minima and put $f'(x) = 0$

$$x^2 - 1 = 0 \Rightarrow x = \pm 1$$

Hence at $x = -1$, $f''(x) = -2 < 0$ (Maxima)

at $x = 1$, $f''(x) = 2 > 0$ (Minima)

So, $f(x)$ is minimum at $x = 1$

SOL 1.121 Option (B) is correct.

Let $$A = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}, B = \begin{bmatrix} a_2 & b_2 & c_2 \end{bmatrix}$$

$$C = AB$$

Let $$= \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \times \begin{bmatrix} a_2 & b_2 & c_2 \end{bmatrix} = \begin{bmatrix} a_1a_2 & a_1b_2 & a_1c_2 \\ b_1a_2 & b_1b_2 & b_1c_2 \\ c_1a_2 & c_1b_2 & c_1c_2 \end{bmatrix}$$

The 3×3 minor of this matrix is zero and all the 2×2 minors are also zero. So the rank of this matrix is 1.

$$\rho[C] = 1$$

SOL 1.122 Option (D) is correct.

In a coin probability of getting head $p = \frac{1}{2}$ and probability of getting tail,

$$q = 1 - \frac{1}{2} = \frac{1}{2}$$

When unbiased coin is tossed three times, then total possibilities are

H H H
H H T
H T H
T H H
H T T
T T H
T H T
T T T

From these cases, there are three cases, when head comes exactly two times. So, the probability of getting head exactly two times, when coin is tossed 3 times is,

$$P = {}^3C_2(p)^2(q)^1 = 3 \times \left(\frac{1}{2}\right)^2 \times \frac{1}{2} = \frac{3}{8}$$

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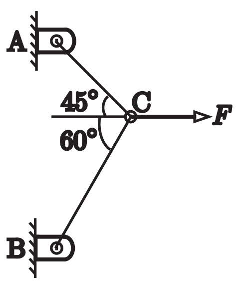
CHAPTER 2

ENGINEERING MECHANICS

YEAR 2012 TWO MARKS

• Common Data For Q.1 and 2

Two steel truss members, AC and BC, each having cross sectional area of $100\,mm^2$, are subjected to a horizontal force F as shown in figure. All the joints are hinged.



MCQ 2.1 If $F = 1\,kN$, the magnitude of the vertical reaction force developed at the point B in kN is

(A) 0.63 (B) 0.32

(C) 1.26 (D) 1.46

MCQ 2.2 The maximum force F is kN that can be applied at C such that the axial stress in any of the truss members DOES NOT exceed $100\,MPa$ is

(A) 8.17 (B) 11.15

(C) 14.14 (D) 22.30

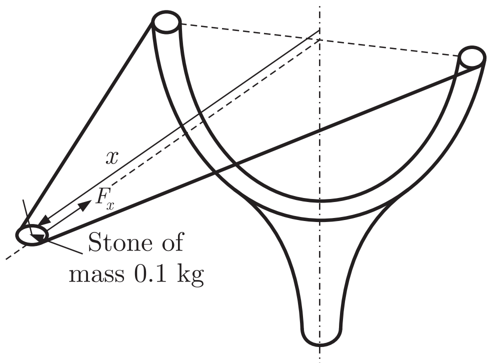
YEAR 2011 ONE MARK

MCQ 2.3 The coefficient of restitution of a perfectly plastic impact is

(A) 0 (B) 1

(C) 2 (D) ∞

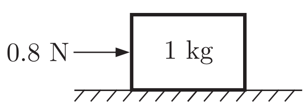
MCQ 2.4 A stone with mass of 0.1 kg is catapulted as shown in the figure. The total force F_x (in N) exerted by the rubber band as a function of distance x (in m) is given by $F_x = 300x^2$. If the stone is displaced by $0.1\,\text{m}$ from the un-stretched position ($x = 0$) of the rubber band, the energy stored in the rubber band is



(A) 0.01 J (B) 0.1 J
(C) 1 J (D) 10 J

YEAR 2011 TWO MARKS

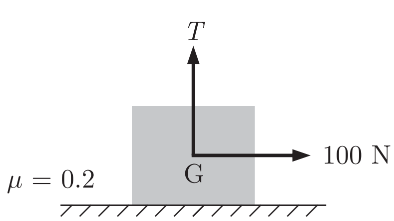
MCQ 2.5 A 1 kg block is resting on a surface with coefficient of friction $\mu = 0.1$. A force of $0.8\,\text{N}$ is applied to the block as shown in the figure. The friction force is



(A) 0 (B) 0.8 N
(C) 0.98 N (D) 1.2 N

YEAR 2009 ONE MARK

MCQ 2.6 A block weighing $981\,\text{N}$ is resting on a horizontal surface. The coefficient of friction between the block and the horizontal surface is $\mu = 0.2$. A vertical cable attached to the block provides partial support as shown. A man can pull horizontally with a force of $100\,\text{N}$. What will be the tension, T (in N) in the cable if the man is just able to move the block to the right ?

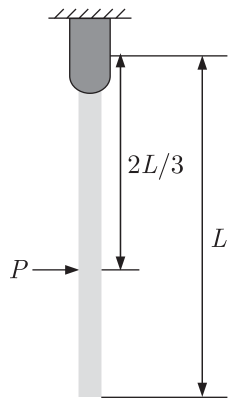


(A) 176.2 (B) 196.0

(C) 481.0 (D) 981.0

YEAR 2009 TWO MARKS

MCQ 2.7 A uniform rigid rod of mass M and length L is hinged at one end as shown in the adjacent figure. A force P is applied at a distance of $2L/3$ from the hinge so that the rod swings to the right. The reaction at the hinge is



(A) $-P$ (B) 0

(C) $P/3$ (D) $2P/3$

YEAR 2008 ONE MARK

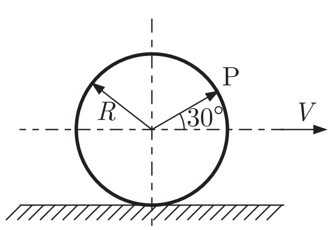
MCQ 2.8 A straight rod length $L(t)$, hinged at one end freely extensible at the other end, rotates through an angle $\theta(t)$ about the hinge. At time t, $L(t) = 1$ m, $\dot{L}(t) = 1$ m/s, $\theta(t) = \frac{\pi}{4}$ rad and $\dot{\theta}(t) = 1$ rad/s. The magnitude of the velocity at the other end of the rod is

(A) 1 m/s (B) $\sqrt{2}$ m/s

(C) $\sqrt{3}$ m/s (D) 2 m/s

YEAR 2008 TWO MARKS

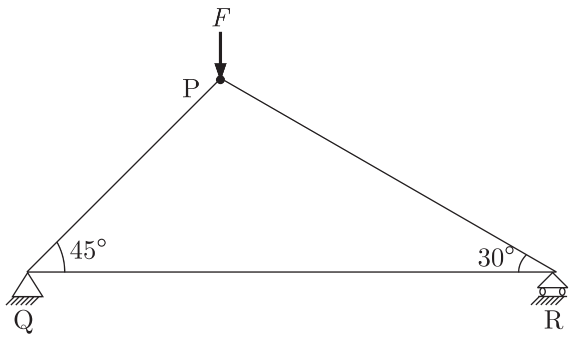
MCQ 2.9 A circular disk of radius R rolls without slipping at a velocity V. The magnitude of the velocity at point P (see figure) is



(A) $\sqrt{3}\,V$ (B) $\sqrt{3}\,V/2$

(C) $V/2$ (D) $2V/\sqrt{3}$

MCQ 2.10 Consider a truss PQR loaded at P with a force F as shown in the figure -



The tension in the member QR is

(A) 0.5 F (B) 0.63 F

(C) 0.73 F (D) 0.87 F

YEAR 2007 ONE MARK

MCQ 2.11 During inelastic collision of two particles, which one of the following is conserved ?

(A) Total linear momentum only

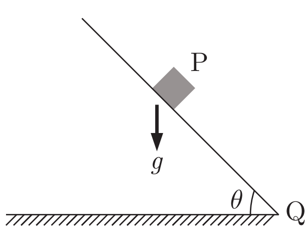
(B) Total kinetic energy only

(C) Both linear momentum and kinetic energy

(D) Neither linear momentum nor kinetic energy

YEAR 2007 TWO MARKS

MCQ 2.12 A block of mass M is released from point P on a rough inclined plane with inclination angle θ, shown in the figure below. The co-efficient of friction is μ. If $\mu < \tan\theta$, then the time taken by the block to reach another point Q on the inclined plane, where $PQ = s$, is



(A) $\sqrt{\dfrac{2s}{g\cos\theta(\tan\theta-\mu)}}$ (B) $\sqrt{\dfrac{2s}{g\cos\theta(\tan\theta+\mu)}}$

(C) $\sqrt{\frac{2s}{g\sin\theta(\tan\theta-\mu)}}$

(D) $\sqrt{\frac{2s}{g\sin\theta(\tan\theta+\mu)}}$

YEAR 2006 TWO MARKS

MCQ 2.13 If a system is in equilibrium and the position of the system depends upon many independent variables, the principles of virtual work states that the partial derivatives of its total potential energy with respect to each of the independent variable must be

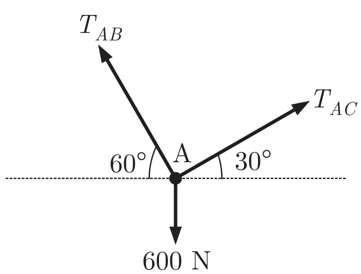
(A) -1.0

(B) 0

(C) 1.0

(D) ∞

MCQ 2.14 If point A is in equilibrium under the action of the applied forces, the values of tensions T_{AB} and T_{AC} are respectively



(A) 520 N and 300 N

(B) 300 N and 520 N

(C) 450 N and 150 N

(D) 150 N and 450 N

YEAR 2005 ONE MARK

MCQ 2.15 The time variation of the position of a particle in rectilinear motion is given by $x = 2t^3 + t^2 + 2t$. If v is the velocity and a is the acceleration of the particle in consistent units, the motion started with

(A) $v = 0, a = 0$

(B) $v = 0, a = 2$

(C) $v = 2, a = 0$

(D) $v = 2, a = 2$

MCQ 2.16 A simple pendulum of length of 5 m, with a bob of mass 1 kg, is in simple harmonic motion. As it passes through its mean position, the bob has a speed of 5 m/s. The net force on the bob at the mean position is

(A) zero

(B) 2.5 N

(C) 5 N

(D) 25 N

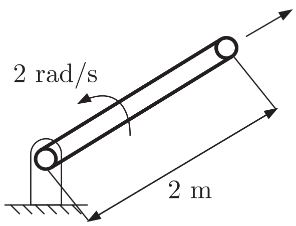
YEAR 2005 TWO MARKS

MCQ 2.17 Two books of mass 1 kg each are kept on a table, one over the other. The coefficient of friction on every pair of contacting surfaces is 0.3. The lower book is pulled with a horizontal force F. The minimum value of F for which slip occurs between the two books is

(A) zero (B) 1.06 N

(C) 5.74 N (D) 8.83 N

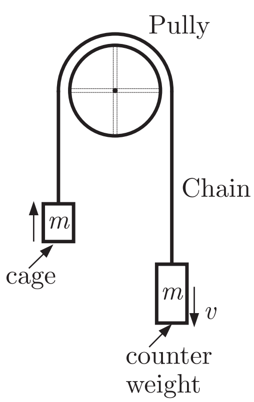
MCQ 2.18 A shell is fired from a cannon. At the instant the shell is just about to leave the barrel, its velocity relative to the barrel is 3 m/s, while the barrel is swinging upwards with a constant angular velocity of 2 rad/s. The magnitude of the absolute velocity of the shell is



(A) 3 m/s (B) 4 m/s

(C) 5 m/s (D) 7 m/s

MCQ 2.19 An elevator (lift) consists of the elevator cage and a counter weight, of mass m each. The cage and the counterweight are connected by chain that passes over a pulley. The pulley is coupled to a motor. It is desired that the elevator should have a maximum stopping time of t seconds from a peak speed v. If the inertias of the pulley and the chain are neglected, the minimum power that the motor must have is

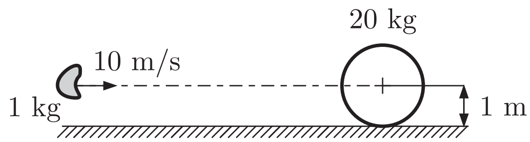


(A) $\frac{1}{2}mV^2$ (B) $\frac{mV^2}{2t}$

(C) $\frac{mV^2}{t}$ (D) $\frac{2mV^2}{t}$

MCQ 2.20 A 1 kg mass of clay, moving with a velocity of 10 m/s, strikes a stationary wheel and sticks to it. The solid wheel has a mass of 20 kg and a radius of

1 m. Assuming that the wheel is set into pure rolling motion, the angular velocity of the wheel immediately after the impact is approximately



(A) zero

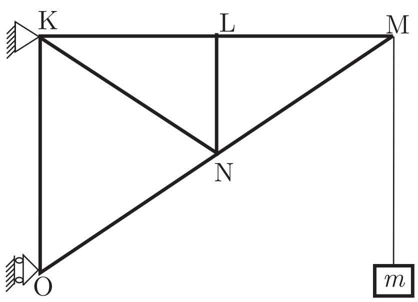
(B) $\frac{1}{3}$ rad/s

(C) $\sqrt{\frac{10}{3}}$ rad/s

(D) $\frac{10}{3}$ rad/s

YEAR 2004 ONE MARK

MCQ 2.21 The figure shows a pin-jointed plane truss loaded at the point M by hanging a mass of 100 kg. The member LN of the truss is subjected to a load of



(A) 0 Newton

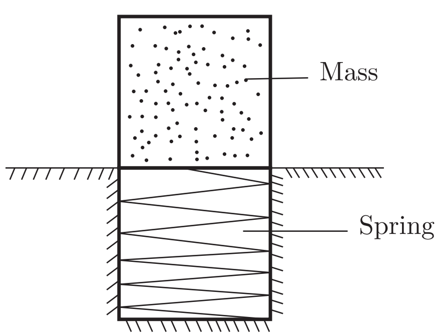
(B) 490 Newtons in compression

(C) 981 Newtons in compression

(D) 981 Newtons in tension

YEAR 2004 TWO MARKS

MCQ 2.22 An ejector mechanism consists of a helical compression spring having a spring constant of $k = 981 \times 10^3$ N/m. It is pre-compressed by 100 mm from its free state. If it is used to eject a mass of 100 kg held on it, the mass will move up through a distance of



(A) 100 mm (B) 500 mm

(C) 581 mm (D) 1000 mm

MCQ 2.23 A rigid body shown in the figure (a) has a mass of 10 kg. It rotates with a uniform angular velocity 'ω'. A balancing mass of 20 kg is attached as shown in figure (b). The percentage increase in mass moment of inertia as a result of this addition is

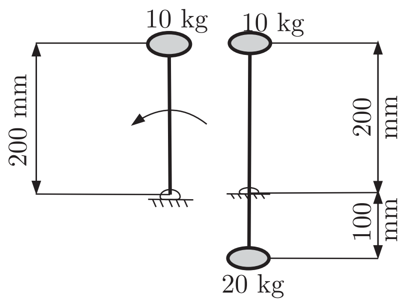
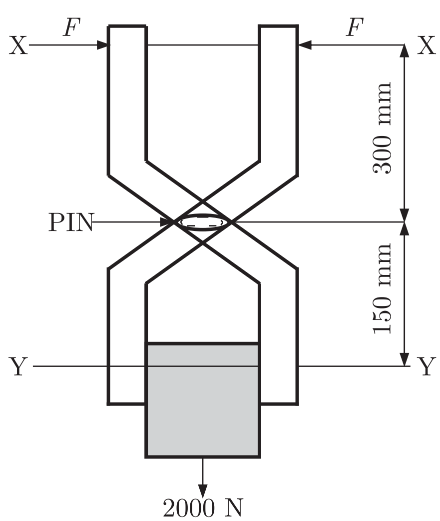


fig. (a) fig. (b)

(A) 25% (B) 50%

(C) 100% (D) 200%

MCQ 2.24 The figure shows a pair of pin-jointed gripper-tongs holding an object weighting 2000 N. The coefficient of friction (μ) at the gripping surface is 0.1 XX is the line of action of the input force and YY is the line of application of gripping force. If the pin-joint is assumed to be frictionless, the magnitude of force F required to hold the weight is

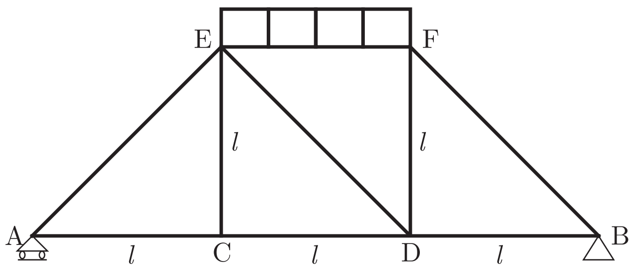


(A) 1000 N (B) 2000 N

(C) 2500 N (D) 5000 N

YEAR 2003 ONE MARK

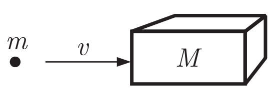
MCQ 2.25 A truss consists of horizontal members (AC,CD, DB and EF) and vertical members (CE and DF) having length l each. The members AE, DE and BF are inclined at 45° to the horizontal. For the uniformly distributed load "p" per unit length on the member EF of the truss shown in figure given below, the force in the member CD is



(A) $\frac{pl}{2}$ (B) pl

(C) 0 (D) $\frac{2pl}{3}$

MCQ 2.26 A bullet of mass "m" travels at a very high velocity v (as shown in the figure) and gets embedded inside the block of mass "M" initially at rest on a rough horizontal floor. The block with the bullet is seen to move a distance "s" along the floor. Assuming μ to be the coefficient of kinetic friction between the block and the floor and "g" the acceleration due to gravity what is the velocity v of the bullet ?



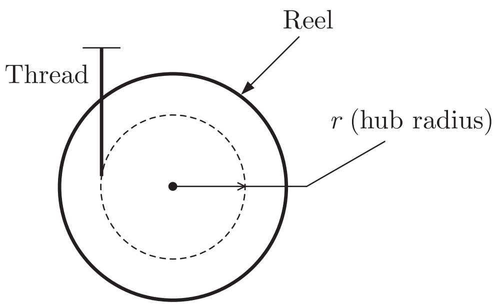
(A) $\frac{M+m}{m}\sqrt{2\mu gs}$ (B) $\frac{M-m}{m}\sqrt{2\mu gs}$

(C) $\frac{\mu(M+m)}{m}\sqrt{2\mu gs}$ (D) $\frac{M}{m}\sqrt{2\mu gs}$

YEAR 2003 TWO MARKS

• **Common Data For Q.Data for Q. 27 & 28 are given below. Solve the problems and choose correct answers.**

A reel of mass "m" and radius of gyration "k" is rolling down smoothly from rest with one end of the thread wound on it held in the ceiling as depicated in the figure. Consider the thickness of thread and its mass negligible in comparison with the radius "r" of the hub and the reel mass "m". Symbol "g" represents the acceleration due to gravity.



MCQ 2.27 The linear acceleration of the reel is

(A) $\frac{gr^2}{(r^2+k^2)}$

(B) $\frac{gk^2}{(r^2+k^2)}$

(C) $\frac{grk}{(r^2+k^2)}$

(D) $\frac{mgr^2}{(r^2+k^2)}$

MCQ 2.28 The tension in the thread is

(A) $\frac{mgr^2}{(r^2+k^2)}$

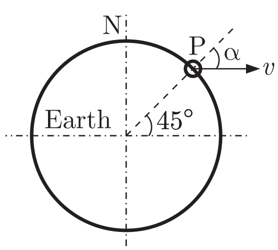
(B) $\frac{mgrk}{(r^2+k^2)}$

(C) $\frac{mgk^2}{(r^2+k^2)}$

(D) $\frac{mg}{(r^2+k^2)}$

YEAR 2001 ONE MARK

MCQ 2.29 A particle P is projected from the earth surface at latitude 45° with escape velocity $v = 11.19\,\text{km/s}$. The velocity direction makes an angle α with the local vertical. The particle will escape the earth's gravitational field



(A) only when $\alpha = 0$

(B) only when $\alpha = 45^\circ$

(C) only when $\alpha = 90^\circ$

(D) irrespective of the value of α

MCQ 2.30 The area moment of inertia of a square of size 1 unit about its diagonal is

(A) $\frac{1}{3}$

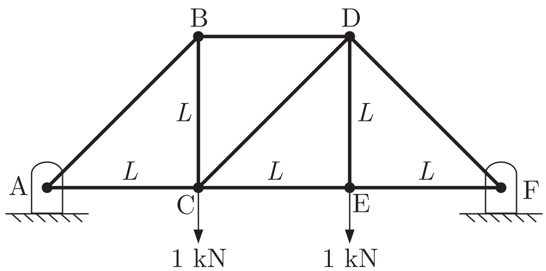
(B) $\frac{1}{4}$

(C) $\frac{1}{12}$

(D) $\frac{1}{6}$

YEAR 2001 TWO MARKS

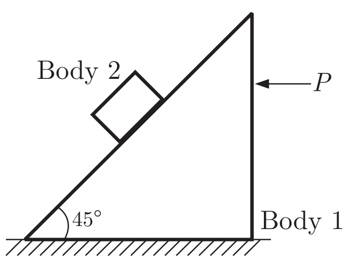
MCQ 2.31 For the loading on truss shown in the figure, the force in member CD is



(A) zero (B) 1 kN

(C) $\sqrt{2}$ kN (D) $\frac{1}{\sqrt{2}}$ kN

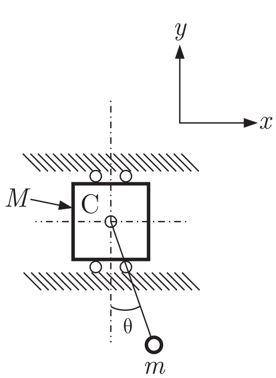
MCQ 2.32 Bodies 1 and 2 shown in the figure have equal mass m. All surfaces are smooth. The value of force P required to prevent sliding of body 2 on body 1 is



(A) $P = 2$ mg (B) $P = \sqrt{2}$ mg

(C) $P = 2\sqrt{2}$ mg (D) $P = mg$

MCQ 2.33 Mass M slides in a frictionless slot in the horizontal direction and the bob of mass m is hinged to mass M at C, through a rigid massless rod. This system is released from rest with $\theta = 30°$. At the instant when $\theta = 0°$, the velocities of m and M can be determined using the fact that, for the system (i.e., m and M together)



(A) the linear momentum in x and y directions are conserved but the energy is not conserved.

(B) the linear momentum in x and y directions are conserved and the energy is also conserved.

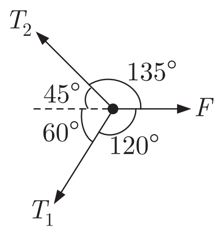
(C) the linear momentum in x direction is conserved and the energy is also conserved.

(D) the linear momentum in y direction is conserved and the energy is also conserved.

****

SOLUTION

SOL 2.1 Option (A) is correct.



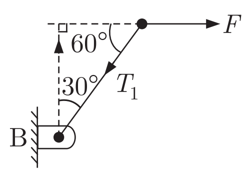
From above figure. Three forces are acting on a common point. Hence by Lami's Theorem.

$$\frac{F}{\sin(105^\circ)} = \frac{T_2}{\sin 120^\circ} = \frac{T_1}{\sin 135^\circ}$$

$\Rightarrow$
$$\frac{T_1}{\sin 135^\circ} = \frac{F}{\sin 105^\circ} = \frac{1}{\sin 105^\circ}$$

$$T_1 = 0.7320\,\text{kN}$$

Hence vertical reaction at B,



$$R_{NT_1} = T_1 \cos 30^\circ = 0.73205 \times \cos 30^\circ = 0.634\,\text{kN}$$

SOL 2.2 Option (B) is correct.

From Previous question

$$\frac{F}{\sin 105^\circ} = \frac{T_2}{\sin 120^\circ}$$

$$T_2 = \frac{\sin 120^\circ}{\sin 135} \times F = 0.8965F$$

and
$$T_1 = (0.73205)F$$

$$T_2 > T_1$$

$$\sigma = 100\,\text{MPa (given)}$$

As we know
$$F = \sigma \times A_1$$

$\Rightarrow$
$$F_{max} = \sigma_{max} \times A_1$$

$$T_2 = 100 \times 100$$

$$0.8965F = 100 \times 100$$

$$F = \frac{100 \times 100}{0.8965} = 11154.5\,\text{N} = 11.15\,\text{kN}$$

SOL 2.3 Option (A) is correct.

From the Newton's Law of collision of Elastic bodies.

$$\text{Velocity of separation} = e \times \text{Velocity of approach}$$

$$(V_2 - V_1) = e(U_1 - U_2)$$

Where e is a constant of proportionality & it is called the coefficient of restitution and its value lies between 0 to 1. The coefficient of restitution of a perfectly plastic impact is zero, because all the K.E. will be absorbed during perfectly plastic impact.

SOL 2.4 Option (B) is correct.

Given : $F_x = 300x^2$, Position of x is, $x = 0$ to $x = 0.1$

The energy stored in the rubber band is equal to work done by the stone.

Hence $dE = F_x dx$

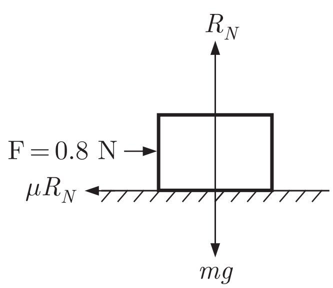
Integrating both the sides & put the value of F & limits

$$\int_0^E dE = \int_0^{0.1} 300x^2 dx$$

$$E = 300\left[\frac{x^3}{3}\right]_0^{0.1} = 300\left[\frac{(0.1)^3}{3}\right] = 0.1\,\text{Joule}$$

SOL 2.5 Option (B) is correct.

Given : $m = 1\,\text{kg}$, $\mu = 0.1$; From FBD : $R_N = mg$



Now static friction force,

$$f_S = \mu R_N = \mu mg = 0.1 \times 1 \times 9.8 = 0.98\,\text{N}$$

Applied force $F = 0.8\,\text{N}$ is less then, the static friction $f_S = 0.98\,\text{N}$

$$F < f_S$$

So, we can say that the friction developed will equal to the applied force

$$F = 0.8\,\text{N}$$

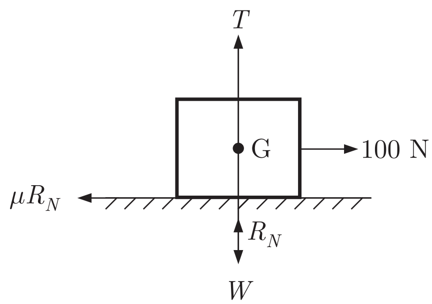
SOL 2.6 Option (C) is correct.

Given : $W = 981\,\text{N}$, $\mu = 0.2$

First of all we have to make a FBD of the block

Here, R_N = Normal reaction force

T = Tension in string



Using the balancing of forces, we have

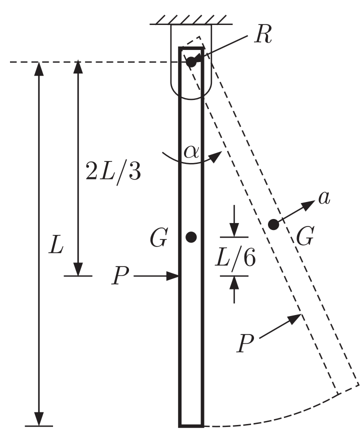
$\Sigma F_x = 0$: $\mu R_N = 100\text{ N}$

$$R_N = \frac{100}{\mu} = \frac{100}{0.2} = 500\text{ N}$$

and $\Sigma F_y = 0$ or downward forces = upward forces

$$W = T + R_N \Rightarrow T = W - R_N = 981 - 500 = 481\text{ N}$$

SOL 2.7 Option (B) is correct.



When rod swings to the right, linear acceleration a and angular acceleration α comes in action. Centre of gravity (G) acting at the mid-point of the rod. Let R be the reaction at the hinge.

Linear acceleration $a = r.\alpha = \frac{L}{2} \times \alpha = \frac{2a}{L}$ ...(i)

and about point G, for rotational motion

$$\sum M_G = I_G \times \alpha$$

$$R\left(\frac{L}{2}\right) + P\left(\frac{L}{6}\right) = \frac{ML^2}{12}\left(\frac{2a}{L}\right)$$ From equation (i)

$$R + \frac{P}{3} = \frac{Ma}{3}$$

$$a = \frac{3R}{M} + \frac{P}{M}$$ ...(ii)

By equilibrium of forces in normal direction to the rod

$$\sum F_m = 0: \qquad P - R = Ma = M\left(\frac{3R}{M} + \frac{P}{M}\right) \qquad \text{From equation (ii)}$$

$$P - R = 3R + P$$

$\Rightarrow$ $R = 0$ So, reaction at the hinge is zero.

SOL 2.8 Option (D) is correct.

Let : V_t = Tangential Velocity

V_r = Relative Velocity

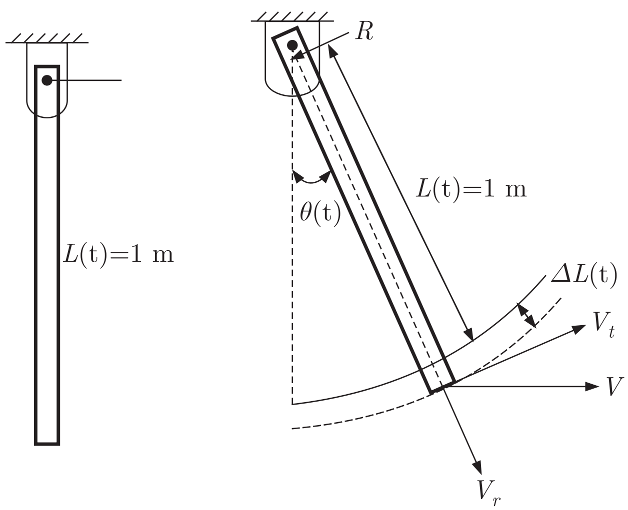
V = Resultant Velocity

Let rod of length $L(t)$ increases by an amount $\Delta L(t)$.

Given $L(t) = 1$ m, $\dot{L}(t) = 1$ m/sec, $\theta(t) = \frac{\pi}{4}$ rad, $\dot{\theta}(t) = 1$ rad/sec

Time taken by the rod to turn $\frac{\pi}{4}$ rad is,

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{\theta(t)}{\dot{\theta}(t)} = \frac{\pi/4}{1} = \frac{\pi}{4} \text{ sec}$$



So, increase in length of the rod during this time will be

$$\Delta L(t) = L(t) \times t = \frac{\pi}{4} \times 1 = \frac{\pi}{4} \text{ meter}$$

Rod turn $\frac{\pi}{4}$ radian. So, increased length after $\frac{\pi}{4}$ sec, (New length)

$$= \left(1 + \frac{\pi}{4}\right) = 1.785 \text{ m}$$

Now, tangential velocity,

$$V_t = R.\omega = 1.785 \times 1 = 1.785 \text{ m/sec} \qquad \omega = \dot{\theta}(t)$$

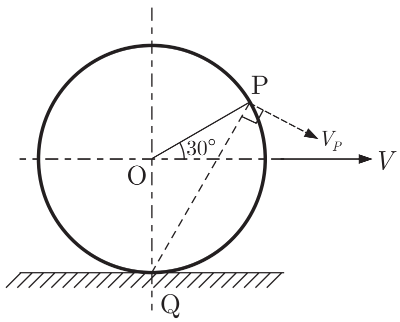
Radial velocity, $V_r = \dot{L}(t) = 1$ m/sec

Therefore, the resultant velocity will be

$$V_R = \sqrt{V_t^2 + V_r^2} = \sqrt{(1.785)^2 + (1)^2} = 2.04 \simeq 2 \text{ m/sec}$$

SOL 2.9 Option (A) is correct.

When disc rolling along a straight path, without slipping. The centre of the wheel O moves with some linear velocity and each particle on the wheel rotates with some angular velocity.



Thus, the motion of any particular on the periphery of the wheel is a combination of linear and angular velocity.

Let wheel rotates with angular velocity $= \omega$ rad/sec.

So, $$\omega = \frac{V}{R} \quad \text{....(i)}$$

Velocity at point P is, $$V_P = \omega \times PQ \quad \text{...(ii)}$$

From triangle OPQ $$PQ \sqrt{(OQ)^2 + (OP)^2 - 2OQ \times OP \times \cos(\angle POQ)}$$
$$= \sqrt{(R)^2 + (R)^2 - 2RR\cos 120^\circ}$$
$$= \sqrt{(R)^2 + (R)^2 + (R)^2} = \sqrt{3}\,R \quad \text{...(iii)}$$

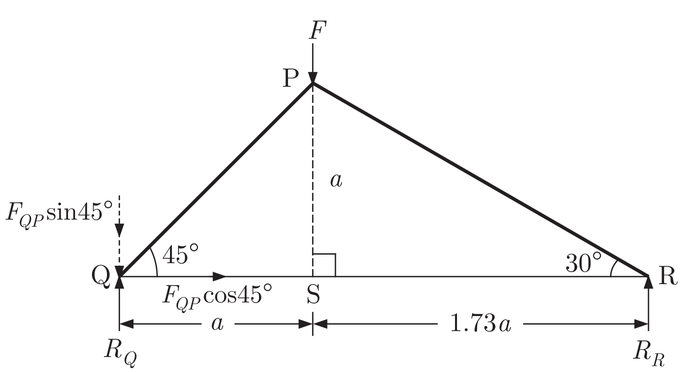
From equation (i), (ii) and (iii)

$$V_P = \frac{V}{R} \times \sqrt{3}\,R = \sqrt{3}\,V$$

SOL 2.10 Option (B) is correct.

The forces which are acting on the truss PQR is shown in figure.

We draw a perpendicular from the point P, that intersects QR at point S.



Let $PS = QS = a$

R_Q & R_R are the reactions acting at point Q & R respectively.

Now from the triangle PRS

$$\tan 30^\circ = \frac{PS}{SR} \Rightarrow SR = \frac{PS}{\tan 30^\circ} = \frac{a}{\frac{1}{\sqrt{3}}} = \sqrt{3}\,a = 1.73a$$

Taking the moment about point R,

$$R_Q \times (a + 1.73a) = F \times 1.73a$$

$$R_Q = \frac{1.73a\text{ F}}{2.73a} = \frac{1.73\text{ F}}{2.73} = 0.634\text{ F}$$

From equilibrium of the forces, we have

$$R_R + R_Q = F$$

$$R_R = F - R_Q = F - 0.634\text{ F} = 0.366\text{ F}$$

To find tension in QR we have to use the method of joint at point Q, and $\Sigma F_y = 0$

$$F_{QP}\sin 45^\circ = R_Q$$

$$F_{QP} = \frac{0.634\text{ F}}{\frac{1}{\sqrt{2}}} = 0.8966\text{ F}$$

and, $\Sigma F_x = 0$

$$F_{QP}\cos 45^\circ = F_{QR} \Rightarrow F_{QR} = 0.8966\text{ F} \times \frac{1}{\sqrt{2}} = 0.634\text{ F} \simeq 0.63\text{ F}$$

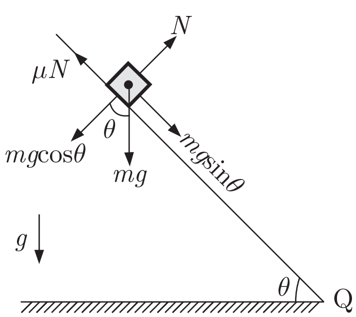
SOL 2.11 Option (A) is correct.

In both elastic & in inelastic collision total linear momentum remains conserved. In the inelastic collision loss in kinetic energy occurs because the coefficient of restitution is less than one and loss in kinetic energy is given by the relation,

$$\Delta K.E. = \frac{m_1 m_2}{2(m_1 + m_2)}(u_1 - u_2)^2(1 - e^2)$$

SOL 2.12 Option (A) is correct.

First of all we resolve all the force which are acting on the block.



Given : $PQ = s$ where $N =$ Normal fraction force

$\mu < \tan\theta$

Now from Newton's second law,

$$F = ma$$

$$mg\sin\theta - \mu N = ma \qquad a = \text{Acceleration of block}$$

$$mg\sin\theta - \mu mg\cos\theta = ma \qquad N = mg\cos\theta$$

$$g\sin\theta - \mu g\cos\theta = a$$

$$a = g\cos\theta\left[\frac{\sin\theta}{\cos\theta} - \mu\right] = g\cos\theta(\tan\theta - \mu) \qquad ...(i)$$

From the Newton;s second law of Motion,

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}g\cos\theta(\tan\theta - \mu)t^2 \qquad u = 0$$

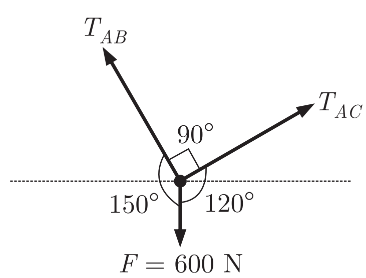
$$t = \sqrt{\frac{2s}{g\cos\theta(\tan\theta - \mu)}}$$

SOL 2.13 Option (B) is correct.

If a system of forces acting on a body or system of bodies be in equilibrium and the system has to undergo a small displacement consistent with the geometrical conditions, then the algebraic sum of the virtual works done by all the forces of the system is zero and total potential energy with respect to each of the independent variable must be equal to zero.

SOL 2.14 Option (A) is correct.

First we solve this problem from Lami's theorem. Here three forces are given. Now we have to find the angle between these forces



Applying Lami's theorem, we have

$$\frac{F}{\sin 90^\circ} = \frac{T_{AB}}{\sin 120^\circ} = \frac{T_{AC}}{\sin 150^\circ}$$

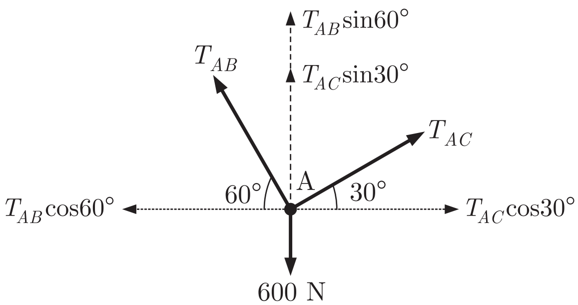
$$\frac{600}{1} = \frac{T_{AB}}{\sqrt{3}/2} = \frac{T_{AC}}{1/2}$$

$$T_{AB} = 600 \times \frac{\sqrt{3}}{2} = 300\sqrt{3} \approx 520 \text{ N}$$

$$T_{AC} = \frac{600}{2} = 300 \text{ N}$$

Alternative :

Now we using the Resolution of forces.



Resolve the T_{AB} & T_{AC} in x & y direction (horizontal & vertical components)

We use the Resolution of forces in x & y direction

$\Sigma F_x = 0,$ $$T_{AB}\cos 60° = T_{AC}\cos 30°$$

$$\frac{T_{AB}}{T_{AC}} = \frac{\sqrt{3}}{2}\times\frac{2}{1} = \sqrt{3} \quad \text{...(i)}$$

$\Sigma F_y = 0,$ $$T_{AB}\sin 60° + T_{AC}\sin 30° = 600\text{ N}$$

$$\frac{\sqrt{3}}{2}T_{AB} + \frac{1}{2}T_{AC} = 600\text{ N}$$

$$\sqrt{3}\,T_{AB} + T_{AC} = 1200\text{ N} \quad T_{AC} = \frac{T_{AB}}{\sqrt{3}} \text{ From equation (i)}$$

Now, $$\sqrt{3}\,T_{AB} + \frac{T_{AB}}{\sqrt{3}} = 1200\text{ N}$$

$$4T_{AB} = 1200\sqrt{3}$$

$$T_{AB} = \frac{1200\sqrt{3}}{4} = 520\text{ N}$$

and $$T_{AC} = \frac{T_{AB}}{\sqrt{3}} = \frac{520}{\sqrt{3}} = 300\text{ N}$$

SOL 2.15 Option (D) is correct.

Given ; $$x = 2t^3 + t^2 + 2t$$

We know that,

$$v = \frac{dx}{dt} = \frac{d}{dt}(2t^3 + t^2 + 2t) = 6t^2 + 2t + 2 \quad \text{...(i)}$$

We have to find the velocity & acceleration of particle, when motion stared,

So at $t = 0,$ $v = 2$

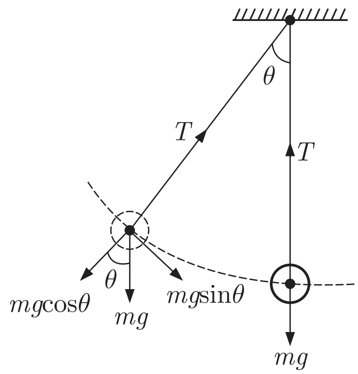
Again differentiate equation (i) w.r.t. t

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 12t + 2$$

At $t = 0,$ $a = 2$

SOL 2.16 Option (A) is correct.

We have to make the diagram of simple pendulum

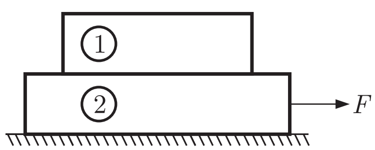


Here, We can see easily from the figure that tension in the string is balanced by the weight of the bob and net force at the mean position is always zero.

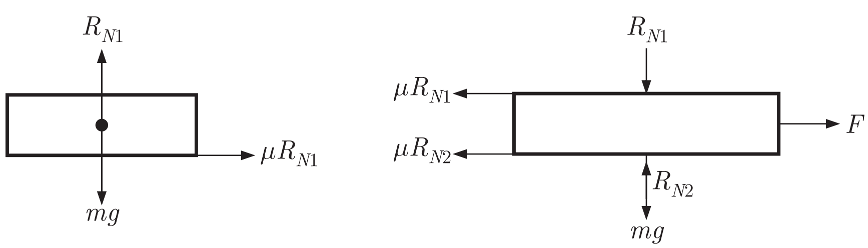
SOL 2.17 Option (D) is correct.

Given : $m_1 = m_2 = 1\,\text{kg},\ \mu = 0.3$

The FBD of the system is shown below :



For first book FBD of book second



For Book (1) $\Sigma F_y = 0$ $\qquad R_{N1} = mg$ ...(i)

Then, Friction Force $F_{N1} = \mu R_{N1} = \mu mg$

From FBD of book second,

$\Sigma F_x = 0,$ $\qquad F = \mu R_{N1} + \mu R_{N2}$

$\Sigma F_y = 0,$ $\qquad R_{N_2} = R_{N1} + mg = mg + mg = 2\,\text{mg}$ ...(ii)

For slip occurs between the books when

$$F \geq \mu R_{N1} + \mu R_{N2} \geq \mu mg + \mu \times 2\,\text{mg}$$

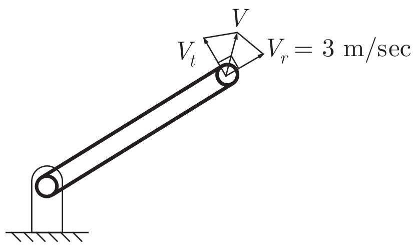
$$F \geq \mu(3\,\text{mg}) \geq 0.3(3 \times 1 \times 9.8) \geq 8.82$$

It means the value of F is always greater or equal to the 8.82, for which slip occurs between two books.

So, $F = 8.83\text{ N}$

SOL 2.18 Option (C) is correct.

Given : $\omega = 2\text{ rad/sec}$, $r = 2\text{ m}$



Tangential velocity of barrel, $V_t = r\omega = 2 \times 2 = 4\text{ m/sec}$

$$V = V_r i + V_t j = 3i + 4j$$

Resultant velocity of shell, $|V| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5\text{ m/sec}$

SOL 2.19 Option (C) is correct.

Given : Mass of cage & counter weight = m kg each

Peak speed = V

Initial velocity of both the cage and counter weight.

$$V_1 = V\text{ m/sec}$$

Final velocity of both objects

$$V_2 = 0$$

Initial kinetic Energy, $E_1 = \frac{1}{2}mV^2 + \frac{1}{2}mV^2 = mV^2$

Final kinetic Energy $E_2 = \frac{1}{2}m(0)^2 + \frac{1}{2}m(0)^2 = 0$

Now, Power = Rate of change of K.E.

$$= \frac{E_1 - E_2}{t} = \frac{mV^2}{t}$$

SOL 2.20 Option (B) is correct.

Given : $m_1 = 1\text{ kg}$, $V_1 = 10\text{ m/sec}$, $m_2 = 20\text{ kg}$, $V_2 =$ Velocity after striking the wheel $r = 1$ meter

Applying the principal of linear momentum on the system

$$\frac{dP}{dt} = 0 \Rightarrow P = \text{constant}$$

Initial Momentum = Final Momentum

$$m_1 \times V_1 = (m_1 + m_2)V_2$$

$$V_2 = \frac{m_1 V_1}{(m_1 + m_2)} = \frac{1 \times 10}{1 + 20} = \frac{10}{21}$$

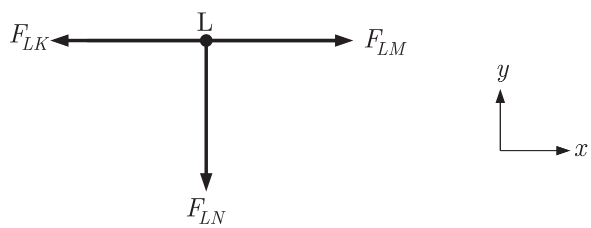
Now after the collision the wheel rolling with angular velocity ω.

So, $$V_2 = r\omega \Rightarrow \omega = \frac{V_2}{r} = \frac{10}{21 \times 1} = 0.476$$

It is nearly equal to 1/3.

SOL 2.21 Option (A) is correct.

First of all we consider all the forces, which are acting at point L.



Now sum all the forces which are acting along x direction,

$$F_{LK} = F_{LM} \qquad \text{Both are acting in opposite direction}$$

Also summation of all the forces, which are acting along y-direction.

$$F_{LN} = 0 \qquad \text{Only one forces acting in } y\text{-direction}$$

So the member LN is subjected to zero load.

SOL 2.22 Option (A) is correct.

Given : $k = 981 \times 10^3$ N/m, $x_i = 100\text{mm} = 0.1$ m, $m = 100$ kg

Let, when mass $m = 100$ kg is put on the spring then spring compressed by x mm. From the conservation of energy :

Energy stored in free state = Energy stored after the mass is attach.

$$(\text{K.E.})_i = (\text{K.E.})_f + (\text{P.E.})_f$$

$$\frac{1}{2}kx_i^2 = \frac{1}{2}kx^2 + mg(x + 0.1)$$

$$kx_i^2 = kx^2 + 2mg(x + 0.1)$$

Substitute the values, we get

$$981 \times 10^3 \times (0.1)^2 = (981 \times 10^3 \times x^2) + [2 \times 100 \times 9.81 \times (x + 0.1)]$$

$$10^3 \times 10^{-2} = 10^3 x^2 + 2(x + 0.1)$$

$$10 = 1000x^2 + 2x + 0.2$$

$$1000x^2 + 2x - 9.8 = 0$$

Solving above equation, we get

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4 \times 1000(-9.8)}}{2 \times 1000} = \frac{-2 \pm \sqrt{4 + 39200}}{2000} = \frac{-2 \pm 198}{2000}$$

On taking -ve sign, we get

$$x = \frac{-2-198}{2000} = -\frac{1}{10},\ m = -100 \text{ mm}$$

(-ve sign shows the compression of the spring)

SOL 2.23 Option (B) is correct.

Given : First Mass, $m_1 = 10$ kg

Balancing Mass, $m_2 = 20$ kg

We know the mass moment of inertia, $I = mk^2$

Where, k = Radius of gyration

Case (I) : When mass of 10 kg is rotates with uniform angular velocity 'ω'

$$I_1 = m_1 k_1^2 = 10 \times (0.2)^2 = 10 \times 0.04 = 0.4 \text{ kg m}^2 \qquad k_1 = 0.2 \text{ m}$$

Case (II) : When balancing mass of 20 kg is attached then moment of inertia

$$I_2 = 10 \times (0.2)^2 + 20 \times (0.1)^2 = 0.4 + 0.2 = 0.6 \quad \text{Here } k_1 = 0.2 \text{ m}$$

and $k_2 = 0.1$ m

Percent increase in mass moment of inertia,

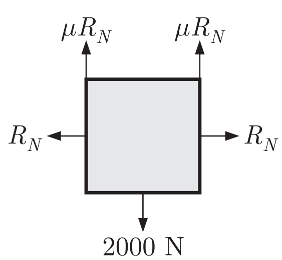
$$I = \frac{I_2 - I_1}{I_1} \times 100 = \frac{0.6-0.4}{0.4} \times 100 = \frac{1}{2} \times 100 = 50\%$$

SOL 2.24 Option (D) is correct.

Given : Weight of object $W = 2000$ N

Coefficient of Friction $\mu = 0.1$

First of all we have to make the FBD of the system.



Here, R_N = Normal reaction force acting by the pin joint.

$F = \mu R_N$ = Friction force

In equilibrium condition of all the forces which are acting in y direction.

$$\mu R_N + \mu R_N = 2000 \text{ N}$$

$$\mu R_N = 1000 \text{ N}$$

$$R_N = \frac{1000}{0.1} = 10000 \text{ N} \qquad \mu = 0.1$$

Taking the moment about the pin, we get

$$10000 \times 150 = F \times 300$$

$$F = 5000 \text{ N}$$

SOL 2.25 Option (A) is correct.

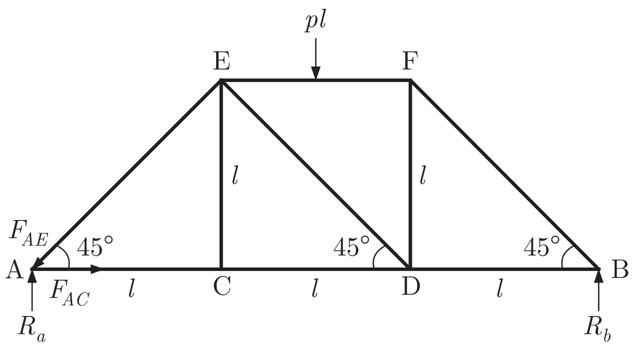
Given : $AC = CD = DB = EF = CE = DF = l$

At the member EF uniform distributed load is acting, the U.D.L. is given as "p" per unit length.

So, the total load acting on the element EF of length l

= Lord per unit length $\times$ Total length of element

$= p \times l = pl$



This force acting at the mid point of EF. From the FBD we get that at A and B reactions are acting because of the roller supports, in the upward direction. In equilibrium condition,

Upward force = Downward forces

$$R_a + R_b = pl \qquad ...(i)$$

And take the moment about point A,

$$pl \times \left(l + \frac{l}{2}\right) = R_b(l + l + l)$$

$$pl \times \frac{3}{2}l = R_b \times 3l \quad \Rightarrow R_b = \frac{pl}{2}$$

Substitute the value of R_b in equation (i), we get

$$R_a + \frac{pl}{2} = pl$$

$$R_a = pl - \frac{pl}{2} = \frac{pl}{2} = R_b = \frac{pl}{2}$$

At point A we use the principal of resolution of forces in the y-direction,

$$\sum F_y = 0 : F_{AE} \sin 45^\circ = R_a = \frac{pl}{2}$$

$$F_{AE} = \frac{pl}{2} \times \frac{1}{\sin 45^\circ} = \frac{pl}{2} \times \sqrt{2} = \frac{pl}{\sqrt{2}}$$

And $$F_{AC} = F_{AE} \cos 45^\circ = \frac{pl}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{pl}{2}$$

At C, No external force is acting. So,

$$F_{AC} = \frac{pl}{2} = F_{CD}$$

SOL 2.26 Option (A) is correct.

Given : Mass of bullet $= m$

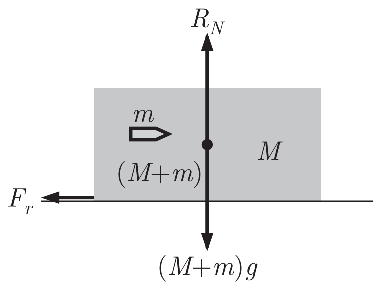
Mass of block $= M$

Velocity of bullet $= v$

Coefficient of Kinematic friction = μ

Let, Velocity of system (Block + bullet) after striking the bullet $= u$

We have to make the FBD of the box after the bullet strikes,



Friction Force (Retardation)= F_r

Applying principal of conservation of linear momentum,

$$\frac{dP}{dt} = 0 \text{ or } P = mV = \text{cons tan t.}$$

So,
$$mv = (M+m)u$$
$$u = \frac{mv}{M+m} \qquad \text{...(i)}$$

And, from the FBD the vertical force (reaction force),

$$R_N = (M+m)g$$
$$F_r = \mu R_N = \mu(M+m)g$$

Frictional retardation
$$a = \frac{-F_r}{(m+M)} = \frac{-\mu(M+m)g}{M+m} = -\mu g \qquad \text{...(ii)}$$

Negative sign show the retardation of the system (acceleration in opposite direction). From the Newton's third law of motion,

$$V_f^2 = u^2 + 2as$$

V_f = Final velocity of system (block + bullet) = 0

$$u^2 + 2as = 0$$
$$u^2 = -2as = -2\times(-\mu g)\times s = 2\mu gs \qquad \text{From equation (ii)}$$

Substitute the value of u from equation (i), we get

$$\left(\frac{mv}{M+m}\right)^2 = 2\mu gs$$
$$\frac{m^2v^2}{(M+m)^2} = 2\mu gs$$
$$v^2 = \frac{2\mu gs(M+m)^2}{m^2}$$

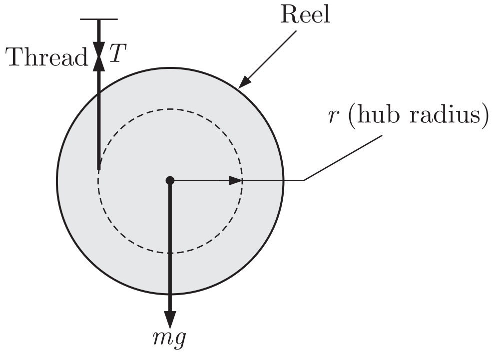
$$v = \sqrt{2\mu gs} \times \left(\frac{M+m}{m}\right) = \frac{M+m}{m}\sqrt{2\mu gs}$$

SOL 2.27 Option (A) is correct.

Given : Mass of real $= m$

Radius of gyration $= k$

We have to make FBD of the system,



Where, T = Tension in the thread

mg = Weight of the system

Real is rolling down. So Angular acceleration (α) comes in the action

From FBD, For vertical translation motion,

$$mg - T = ma \quad ...(i)$$

and for rotational motion,

$$\Sigma M_G = I_G \alpha$$

$$T \times r = mk^2 \times \frac{a}{r} \qquad I_G = mk^2,\ \alpha = a/r$$

$$T = \frac{mk^2}{r^2} \times a \quad ...(ii)$$

From equation (i) & (ii) Substitute the value of T in equation (i), we get

$$mg - \frac{mk^2}{r^2} \times a = ma$$

$$mg = a\left[\frac{mk^2}{r^2} + m\right]$$

$$a = \frac{gr^2}{k^2 + r^2} \quad ...(iii)$$

SOL 2.28 Option (C) is correct.

From previous question, $T = mg - ma$

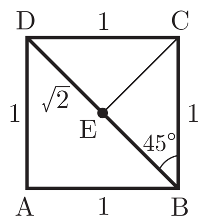
Substitute the value of a from equation (iii), we get

$$T = mg - m \times \frac{gr^2}{(k^2 + r^2)} = \frac{mg(k^2 + r^2) - mgr^2}{(k^2 + r^2)} = \frac{mgk^2}{k^2 + r^2}$$

SOL 2.29 Option (D) is correct.

We know that a particle requires the velocity of 11.2 km/s for escape it from the earth's gravitational field. The angle α does not effect on it.

SOL 2.30 Option (C) is correct.



The BD is the diagonal of the square $ABCD$ and $\angle CBD = 45^\circ$.

From the ΔBCE $\quad \sin 45^\circ = \frac{CE}{BC} \Rightarrow CE = 1 \times \sin 45^\circ = \frac{1}{\sqrt{2}}$ unit

where CE is the height of the triangle ΔBCD.

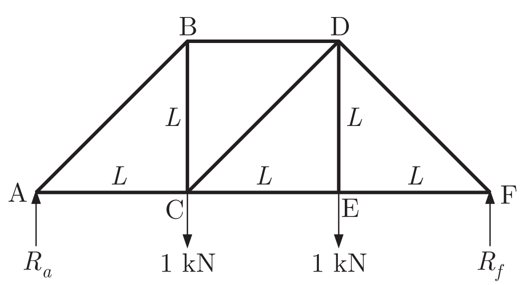
Now, the area moment of inertia of a triangle about its base BD is $\frac{bh^3}{12}$

where $b =$ base of triangle & $h =$ height of triangle

So, the triangle ΔABD are same and required moment of inertia of the square $ABCD$ about its diagonal is,

$$I = 2 \times \frac{1}{12} \times (BD) \times (CE)^3 = \frac{1}{6} \times \sqrt{2} \times \left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{12} \text{ unit}$$

SOL 2.31 Option (A) is correct.



The reactions at the hinged support will be in only vertical direction as external loads are vertical.

Now, consider the FBD of entire truss. In equilibrium of forces.

$$R_a + R_f = 1 + 1 = 2 \text{ kN} \qquad \text{...(i)}$$

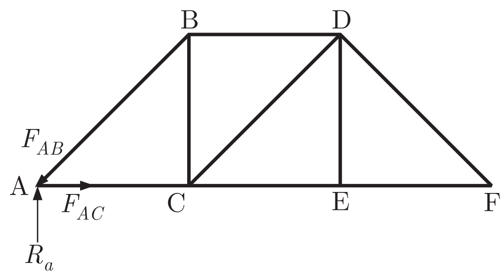
Taking moment about point A, we get

$$R_f \times 3L = 1 \times L + 1 \times 2L = 3L$$

$$R_f = 1 \text{ kN}$$

From equation (i), $\quad R_a = 2 - 1 = 1 \text{ kN}$

First consider the FBD of joint A with the direction of forces assumed in the figure.



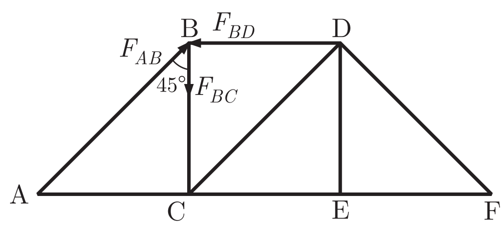
Resolving force vertically, we get

$$R_a = F_{AB}\sin 45^\circ$$

$$F_{AB} = \frac{1}{\sin 45^\circ} = \sqrt{2}\text{ kN (Compression)}$$

Resolving forces horizontally

$$F_{AC}\ F_{AB}\cos 45^\circ = \sqrt{2}\times\frac{1}{\sqrt{2}} = 1\text{ kN (Tension)}$$



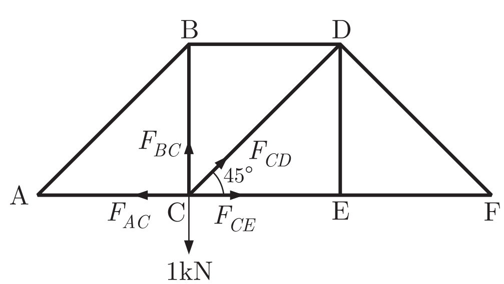
Consider the FBD of joint B with known value of force F_{AB} in member AB
Resolving forces vertically,

$$F_{BC} = F_{AB}\cos 45^\circ = \sqrt{2}\times\frac{1}{\sqrt{2}} = 1\text{ kN}\quad\text{(Tension)}$$

Resolving forces horizontally,

$$F_{BD} = F_{AB}\sin 45^\circ = \sqrt{2}\times\frac{1}{\sqrt{2}} = 1\text{ kN (Compression)}$$

Consider the FBD of joint C with known value of force F_{BC} and F_{AC}

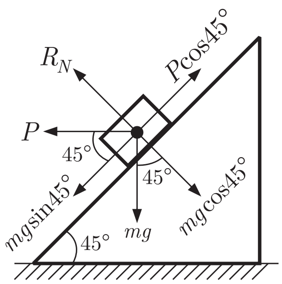


Resolving forces vertically,

$$1 = F_{BC} + F_{CD}\sin 45^\circ$$

$$1 = 1 + F_{CD}\sin 45^\circ \Rightarrow F_{CD} = 0$$

SOL 2.32 Option (D) is correct.



From the *FBD* of the system.

$$R_N = mg\cos 45^\circ$$

All surfaces are smooth, so there is no frictional force at the surfaces.

The downward force $mg\sin 45^\circ$ is balanced by $P\cos 45^\circ$.

$$mg\sin 45^\circ = P\cos 45^\circ$$

$$mg \times \frac{1}{\sqrt{2}} = P \times \frac{1}{\sqrt{2}} \Rightarrow P = mg$$

SOL 2.33 Option (C) is correct.

In this case the motion of mass m is only in x-direction. So, the linear momentum is only in x-direction & it remains conserved.

Also from Energy conservation law the energy remains constant i.e. energy is also conserved.

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CHAPTER 3

STRENGTH OF MATERIALS

YEAR 2012 ONE MARK

MCQ 3.1 A thin walled spherical shell is subjected to an internal pressure. If the radius of the shell is increased by 1% and the thickness is reduced by 1%, with the internal pressure remaining the same, the percentage change in the circumferential (hoop) stress is

(A) 0 (B) 1

(C) 1.08 (D) 2.02

MCQ 3.2 A cantilever beam of length L is subjected to a moment M at the free end. The moment of inertia of the beam cross section about the neutral axis is I and the Young's modulus is E. The magnitude of the maximum deflection is

(A) $\frac{ML^2}{2EI}$ (B) $\frac{ML^2}{EI}$

(C) $\frac{2ML^2}{EI}$ (D) $\frac{4ML^2}{EI}$

MCQ 3.3 For a long slender column of uniform cross section, the ratio of critical buckling load for the case with both ends clamped to the case with both the ends hinged is

(A) 1 (B) 2

(C) 4 (D) 8

YEAR 2012 TWO MARKS

MCQ 3.4 The homogeneous state of stress for a metal part undergoing plastic deformation is

$$T = \begin{pmatrix} 10 & 5 & 0 \\ 5 & 20 & 0 \\ 0 & 0 & -10 \end{pmatrix}$$

where the stress component values are in MPa. Using Von Mises Yield criterion, the value of estimated shear yield stress, in MPa is

(A) 9.50 (B) 16.07

(C) 28.52 (D) 49.41

MCQ 3.5 The state of stress at a point under plane stress condition is

$$\sigma_{xx} = 40\,\text{MPa},\ \sigma_{yy} = 100\,\text{MPa} \text{ and } \tau_{xy} = 40\,\text{MPa}$$

The radius of the Mohr's circle representing the given state of stress in MPa is

(A) 40 (B) 50

(C) 60 (D) 100

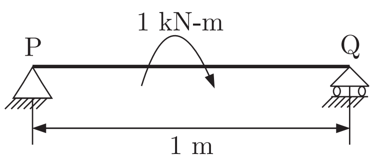
MCQ 3.6 A solid steel cube constrained on all six faces is heated so that the temperature rises uniformly by ΔT. If the thermal coefficient of the material is α, Young's modulus is E and the Poisson's ratio is υ, the thermal stress developed in the cube due to heating is

(A) $-\frac{\alpha(\Delta T)E}{(1-2\upsilon)}$ (B) $-\frac{2\alpha(\Delta T)E}{(1-2\upsilon)}$

(C) $-\frac{3\alpha(\Delta T)E}{(1-2\upsilon)}$ (D) $-\frac{\alpha(\Delta T)E}{3(1-2\upsilon)}$

YEAR 2011 ONE MARK

MCQ 3.7 A simply supported beam PQ is loaded by a moment of 1 kNm at the mid-span of the beam as shown in the figure The reaction forces R_P and R_Q at supports P and Q respectively are



(A) 1 kN downward, 1 kN upward

(B) 0.5 kN upward, 0.5 kN downward

(C) 0.5 kN downward, 0.5 kN upward

(D) 1 kN upward, 1 kN upward

MCQ 3.8 A column has a rectangular cross-section of 10×20 mm and a length of 1 m. The slenderness ratio of the column is close to

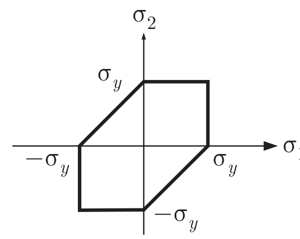
(A) 200 (B) 346

(C) 477 (D) 1000

MCQ 3.9 Match the following criteria of material failure, under biaxial stresses σ_1 and σ_2 and yield stress σ_y, with their corresponding graphic representations.

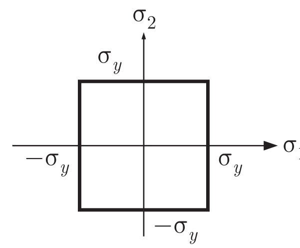
P. Maximum-normal-stress criterion

L.



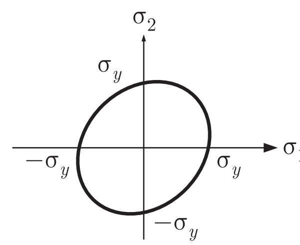
Q. Maximum-distortion-energy criterion

M.



R. Maximum-shear-stress criterion

N.



(A) P-M, Q-L, R-N (B) P-N, Q-M, R-L

(C) P-M, Q-N, R-L (D) P-N, Q-L, R-M

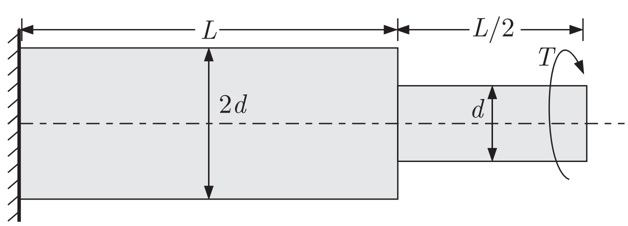
MCQ 3.10 A thin cylinder of inner radius 500 mm and thickness 10 mm is subjected to an internal pressure of 5 MPa. The average circumferential (hoop) stress in MPa is

(A) 100 (B) 250

(C) 500 (D) 1000

YEAR 2011 TWO MARKS

MCQ 3.11 A torque T is applied at the free end of a stepped rod of circular cross-section as shown in the figure. The shear modulus of material of the rod is G. The expression for d to produce an angular twist θ at the free end is



(A) $\left(\frac{32TL}{\pi\theta G}\right)^{\frac{1}{4}}$

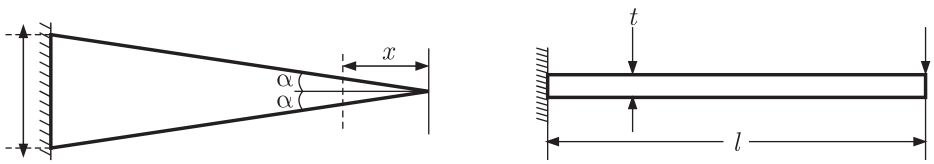
(B) $\left(\frac{18TL}{\pi\theta G}\right)^{\frac{1}{4}}$

(C) $\left(\frac{16TL}{\pi\theta G}\right)^{\frac{1}{4}}$

(D) $\left(\frac{2TL}{\pi\theta G}\right)^{\frac{1}{4}}$

• Common Data For Q. 12 and 13 :

A triangular-shaped cantilever beam of uniform-thickness is shown in the figure The Young's modulus of the material of the beam is E. A concentrated load P is applied at the free end of the beam.



MCQ 3.12 The area moment of inertia about the neutral axis of a cross-section at a distance x measured from the free end is

(A) $\frac{bxt^3}{6l}$

(B) $\frac{bxt^3}{12l}$

(C) $\frac{bxt^3}{24l}$

(D) $\frac{xt^3}{12l}$

MCQ 3.13 The maximum deflection of the beam is

(A) $\frac{24Pl^3}{Ebt^3}$

(B) $\frac{12Pl^3}{Ebt^3}$

(C) $\frac{3Pl^3}{Ebt^3}$

(D) $\frac{6Pl^3}{Ebt^3}$

YEAR 2010 ONE MARK

MCQ 3.14 The state of plane-stress at a point is given by $\sigma_x = -200$ MPa, $\sigma_y = 100$ MPa $\tau_{xy} = 100$ MPa. The maximum shear stress (in MPa) is

(A) 111.8

(B) 150.1

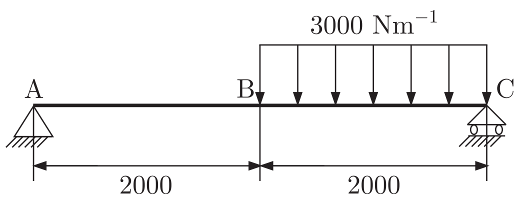
(C) 180.3

(D) 223.6

YEAR 2010 TWO MARKS

• Common Data For Q.15 and Q.16

A massless beam has a loading pattern as shown in the figure. The beam is of rectangular cross-section with a width of 30 mm and height of 100 mm



MCQ 3.15 The maximum bending moment occurs at

(A) Location B

(B) 2675 mm to the right of A

(C) 2500 mm to the right of A

(D) 3225 mm to the right of A

MCQ 3.16 The maximum magnitude of bending stress (in MPa) is given by

(A) 60.0

(B) 67.5

(C) 200.0

(D) 225.0

YEAR 2009 ONE MARK

MCQ 3.17 If the principal stresses in a plane stress problem are $\sigma_1 = 100$ MPa, $\sigma_2 = 40$ MPa, the magnitude of the maximum shear stress (in MPa) will be

(A) 60

(B) 50

(C) 30

(D) 20

MCQ 3.18 A solid circular shaft of diameter d is subjected to a combined bending moment M and torque, T. The material property to be used for designing the shaft using the relation $\frac{16}{\pi d^3}\sqrt{M^2 + T^2}$ is

(A) ultimate tensile strength (S_u)

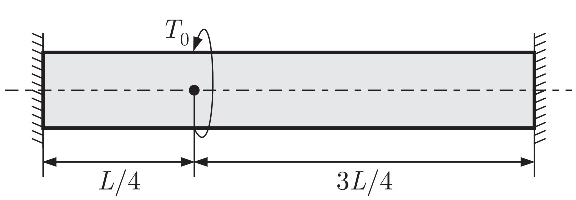
(B) tensile yield strength (S_y)

(C) torsional yield strength (S_{sy})

(D) endurance strength (S_e)

YEAR 2009 TWO MARKS

MCQ 3.19 A solid shaft of diameter d and length L is fixed at both the ends. A torque, T_0 is applied at a distance $\frac{L}{4}$ from the left end as shown in the figure given below.



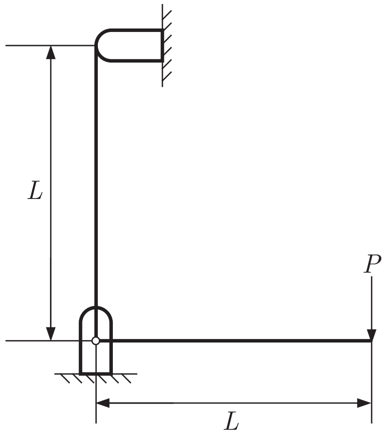
The maximum shear stress in the shaft is

(A) $\frac{16T_0}{\pi d^3}$

(B) $\frac{12T_0}{\pi d^3}$

(C) $\frac{8T_0}{\pi d^3}$ (D) $\frac{4T_0}{\pi d^3}$

MCQ 3.20 A frame of two arms of equal length L is shown in the adjacent figure. The flexural rigidity of each arm of the frame is EI. The vertical deflection at the point of application of load P is



(A) $\frac{PL^3}{3EI}$ (B) $\frac{2PL^3}{3EI}$

(C) $\frac{PL^3}{EI}$ (D) $\frac{4PL^3}{3EI}$

YEAR 2008 ONE MARK

MCQ 3.21 The transverse shear stress acting in a beam of rectangular cross-section, subjected to a transverse shear load, is

(A) variable with maximum at the bottom of the beam

(B) variable with maximum at the top of the beam

(C) uniform

(D) variable with maximum on the neutral axis

MCQ 3.22 A rod of length L and diameter D is subjected to a tensile load P. Which of the following is sufficient to calculate the resulting change in diameter ?

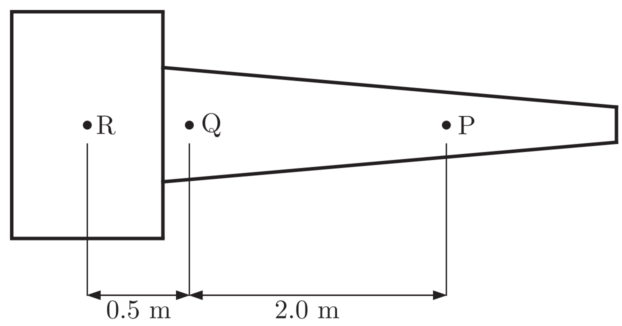
(A) Young's modulus

(B) Shear modulus

(C) Poisson's ratio

(D) Both Young's modulus and shear modulus

MCQ 3.23 A cantilever type gate hinged at Q is shown in the figure. P and R are the centers of gravity of the cantilever part and the counterweight respectively. The mass of the cantilever part is 75 kg. The mass of the counter weight, for static balance, is



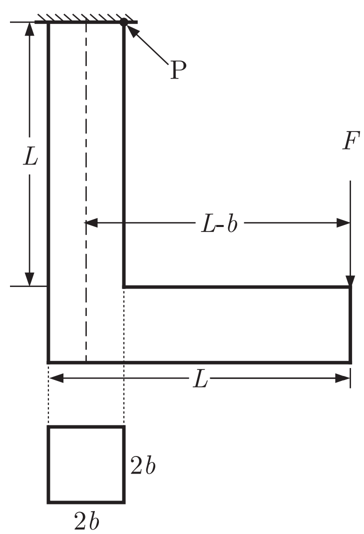
(A) 75 kg
(B) 150 kg
(C) 225 kg
(D) 300 kg

MCQ 3.24 An axial residual compressive stress due to a manufacturing process is present on the outer surface of a rotating shaft subjected to bending. Under a given bending load, the fatigue life of the shaft in the presence of the residual compressive stress is

(A) decreased
(B) increased or decreased, depending on the external bending load
(C) neither decreased nor increased
(D) increased

YEAR 2008 TWO MARKS

MCQ 3.25 For the component loaded with a force F as shown in the figure, the axial stress at the corner point P is



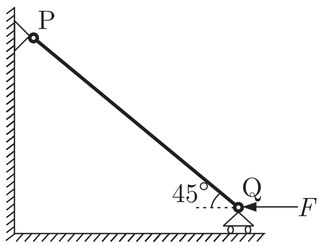
(A) $\frac{F(3L-b)}{4b^3}$
(B) $\frac{3(3L+b)}{4b^3}$
(C) $\frac{F(3L-4b)}{4b^3}$
(D) $\frac{F(3L-2b)}{4b^3}$

MCQ 3.26 A solid circular shaft of diameter 100 mm is subjected to an axial stress of 50 MPa. It is further subjected to a torque of 10 kNm. The maximum principal stress experienced on the shaft is closest to

(A) 41 MPa (B) 82 MPa

(C) 164 MPa (D) 204 MPa

MCQ 3.27 The rod PQ of length L and with flexural rigidity EI is hinged at both ends. For what minimum force F is it expected to buckle ?



(A) $\frac{\pi^2 EI}{L^2}$ (B) $\frac{\sqrt{2}\,\pi^2 EI}{L^2}$

(C) $\frac{\pi^2 EI}{\sqrt{2}\,L^2}$ (D) $\frac{\pi^2 EI}{2L^2}$

MCQ 3.28 A compression spring is made of music wire of 2 mm diameter having a shear strength and shear modulus of 800 MPa and 80 GPa respectively. The mean coil diameter is 20 mm, free length is 40 mm and the number of active coils is 10. If the mean coil diameter is reduced to 10 mm, the stiffness of the spring is approximately

(A) decreased by 8 times (B) decreased by 2 times

(C) increased by 2 times (D) increased by 8 times

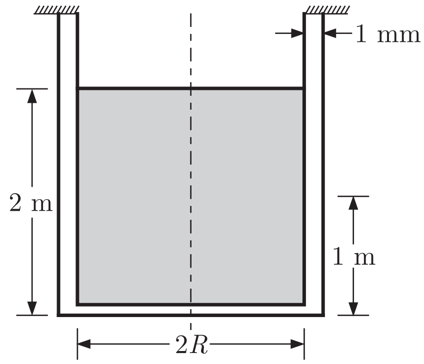
MCQ 3.29 A two dimensional fluid element rotates like a rigid body. At a point within the element, the pressure is 1 unit. Radius of the Mohr's circle, characterizing the state of stress at that point, is

(A) 0.5 unit (B) 0 unit

(C) 1 unit (D) 2 unit

• Common Data For Q. 30 and 31 :

A cylindrical container of radius $R = 1$ m, wall thickness 1 mm is filled with water up to a depth of 2 m and suspended along its upper rim. The density of water is $1000\ \text{kg/m}^3$ and acceleration due to gravity is $10\ \text{m/s}^2$. The self-weight of the cylinder is negligible. The formula for hoop stress in a thin-walled cylinder can be used at all points along the height of the cylindrical container.



MCQ 3.30 The axial and circumference stress (σ_d, σ_c) experienced by the cylinder wall at mid-depth (1 m as shown) are

(A) (10, 10)MPa (B) (5, 10)MPa

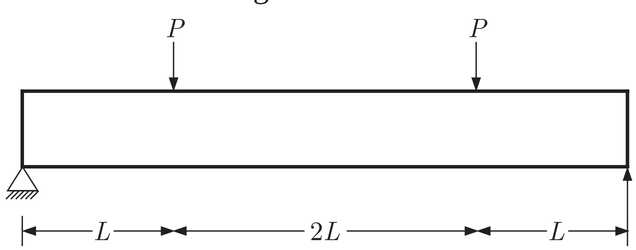
(C) (10, 5)MPa (D) (5, 5)MPa

MCQ 3.31 If the Young's modulus and Poisson's ratio of the container material are 100 GPa and 0.3, respectively, the axial strain in the cylinder wall at mid-depth is

(A) 2×10^{-5} (B) 6×10^{-5}

(C) 7×10^{-5} (D) 1.2×10^{-4}

MCQ 3.32 The strain energy stored in the beam with flexural rigidity EI and loaded as shown in the figure is

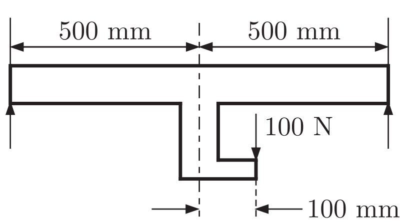


(A) $\frac{P^2L^3}{3EI}$ (B) $\frac{2P^2L^3}{3EI}$

(C) $\frac{4P^2L^3}{3EI}$ (D) $\frac{8P^2L^3}{3EI}$

YEAR 2007 ONE MARK

MCQ 3.33 In a simply-supported beam loaded as shown below, the maximum bending moment in Nm is



(A) 25 (B) 30

(C) 35 (D) 60

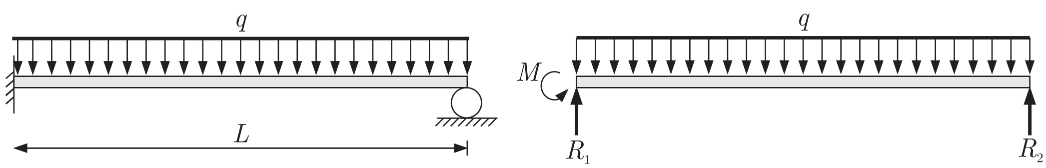
MCQ 3.34 A steel rod of length L and diameter D, fixed at both ends, is uniformly heated to a temperature rise of ΔT. The Young's modulus is E and the co-efficient of linear expansion is α. The thermal stress in the rod is

(A) 0 (B) $\alpha\Delta T$

(C) $E\alpha\Delta T$ (D) $E\alpha\Delta TL$

YEAR 2007 TWO MARKS

MCQ 3.35 A uniformly loaded propped cantilever beam and its free body diagram are shown below. The reactions are



(A) $R_1 = \frac{5qL}{8}, R_2 = \frac{3qL}{8}, M = \frac{qL^2}{8}$ (B) $R_1 = \frac{3qL}{8}, R_2 = \frac{5qL}{8}, M = \frac{qL^2}{8}$

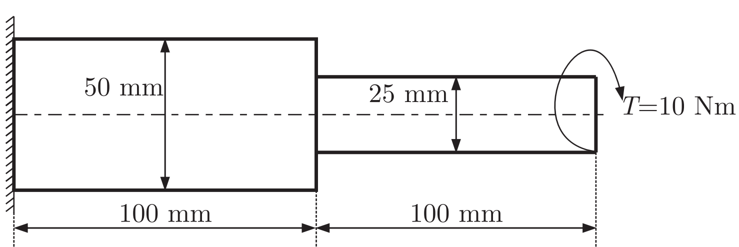
(C) $R_1 = \frac{5qL}{8}, R_2 = \frac{3qL}{8}, M = 0$ (D) $R_1 = \frac{3qL}{8}, R_2 = \frac{5qL}{8}, M = 0$

MCQ 3.36 A $200 \times 100 \times 50$ mm steel block is subjected to a hydrostatic pressure of 15 MPa. The Young's modulus and Poisson's ratio of the material are 200 GPa and 0.3 respectively. The change in the volume of the block in mm^3 is

(A) 85 (B) 90

(C) 100 (D) 110

MCQ 3.37 A stepped steel shaft shown below is subjected to 10 Nm torque. If the modulus of rigidity is 80 GPa, the strain energy in the shaft in N-mm is

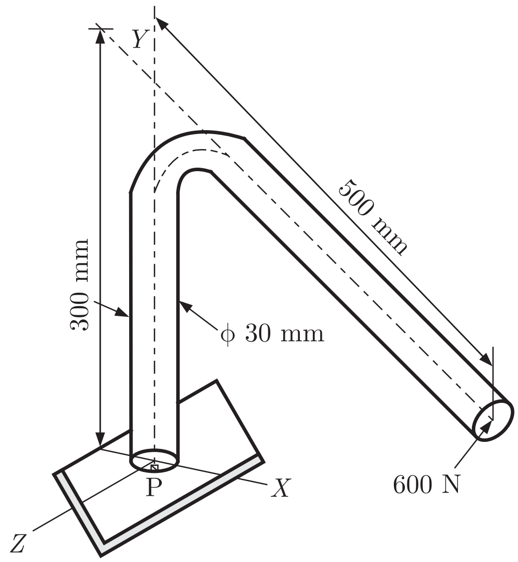


(A) 4.12 (B) 3.46

(C) 1.73 (D) 0.86

• Common Data For Q. 38 and 39 :

A machine frame shown in the figure below is subjected to a horizontal force of 600 N parallel to Z-direction.



MCQ 3.38 The normal and shear stresses in MPa at point P are respectively

(A) 67.9 and 56.6 (B) 56.6 and 67.9

(C) 67.9 and 0.0 (D) 0.0 and 56.6

MCQ 3.39 The maximum principal stress in MPa and the orientation of the corresponding principal plane in degrees are respectively

(A) -32.0 and -29.52 (B) 100.0 and 60.48

(C) -32.0 and 60.48 (D) 100.0 and -29.52

YEAR 2006 ONE MARK

MCQ 3.40 For a circular shaft of diameter d subjected to torque T, the maximum value of the shear stress is

(A) $\frac{64T}{\pi d^3}$ (B) $\frac{32T}{\pi d^3}$

(C) $\frac{16T}{\pi d^3}$ (D) $\frac{8T}{\pi d^3}$

MCQ 3.41 A pin-ended column of length L, modulus of elasticity E and second moment of the cross-sectional area is I loaded eccentrically by a compressive load P. The critical buckling load (P_{cr}) is given by

(A) $P_{cr} = \frac{EI}{\pi^2 L^2}$ (B) $P_{cr} = \frac{\pi^2 EI}{3L^2}$

(C) $P_{cr} = \frac{\pi EI}{L^2}$ (D) $P_{cr} = \frac{\pi^2 EI}{L^2}$

YEAR 2006 TWO MARKS

MCQ 3.42 According to Von-Mises' distortion energy theory, the distortion energy under three dimensional stress state is represented by

(A) $\frac{1}{2E}[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\upsilon(\sigma_1\sigma_2+\sigma_3\sigma_2+\sigma_1\sigma_3)]$

(B) $\frac{1-2\upsilon}{6E}[\sigma_1^2+\sigma_2^2+\sigma_3^2+2(\sigma_1\sigma_2+\sigma_3\sigma_2+\sigma_1\sigma_3)]$

(C) $\frac{1+\upsilon}{3E}[\sigma_1^2+\sigma_2^2+\sigma_3^2-(\sigma_1\sigma_2+\sigma_3\sigma_2+\sigma_1\sigma_3)]$

(D) $\frac{1}{3E}[\sigma_1^2+\sigma_2^2+\sigma_3^2-\upsilon(\sigma_1\sigma_2+\sigma_3\sigma_2+\sigma_1\sigma_3)]$

MCQ 3.43 A steel bar of 40 mm $\times$ 40 mm square cross-section is subjected to an axial compressive load of 200 kN. If the length of the bar is 2 m and $E = 200$ GPa, the elongation of the bar will be

(A) 1.25 mm (B) 2.70 mm

(C) 4.05 mm (D) 5.40 mm

MCQ 3.44 A bar having a cross-sectional area of 700 mm^2 is subjected to axial loads at the positions indicated. The value of stress in the segment QR is

(A) 40 MPa (B) 50 MPa

(C) 70 MPa (D) 120 MPa

• Common Data For Q. 45 and Q. 46 :

A simply supported beam of span length 6 m and 75 mm diameter carries a uniformly distributed load of 1.5 kN/m

MCQ 3.45 What is the maximum value of bending moment ?

(A) 9 kN-m (B) 13.5 kN-m

(C) 81 kN-m (D) 125 kN-m

MCQ 3.46 What is the maximum value of bending stress ?

(A) 162.98 MPa (B) 325.95 MPa

(C) 625.95 MPa (D) 651.90 MPa

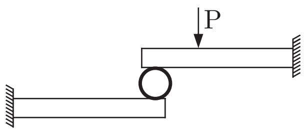
YEAR 2005 ONE MARK

MCQ 3.47 A uniform, slender cylindrical rod is made of a homogeneous and isotropic material. The rod rests on a frictionless surface. The rod is heated uniformly. If the radial and longitudinal thermal stresses are represented by σ_r and σ_z, respectively, then

(A) $\sigma_r = 0, \sigma_z = 0$ (B) $\sigma_r \neq 0, \sigma_z = 0$

(C) $\sigma_r = 0, \sigma_z \neq 0$ (D) $\sigma_r \neq 0, \sigma_z \neq 0$

MCQ 3.48 Two identical cantilever beams are supported as shown , with their free ends in contact through a rigid roller. After the load P is applied, the free ends will have



(A) equal deflections but not equal slopes

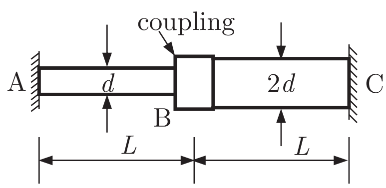
(B) equal slopes but not equal deflections

(C) equal slopes as well as equal deflections

(D) neither equal slopes nor equal deflections

YEAR 2005 TWO MARKS

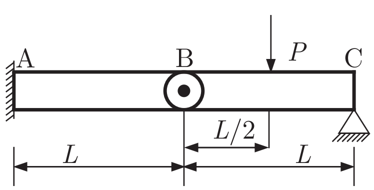
MCQ 3.49 The two shafts AB and BC, of equal length and diameters d and $2d$, are made of the same material. They are joined at B through a shaft coupling, while the ends A and C are built-in (cantilevered). A twisting moment T is applied to the coupling. If T_A and T_C represent the twisting moments at the ends A and C, respectively, then



(A) $T_C = T_A$ (B) $T_C = 8T_A$

(C) $T_C = 16T_A$ (D) $T_A = 16T_C$

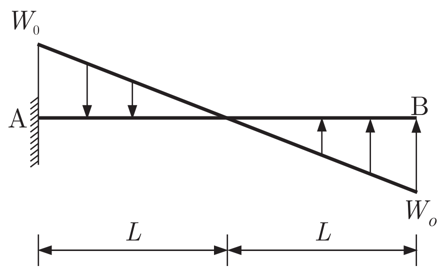
MCQ 3.50 A beam is made up of two identical bars AB and BC, by hinging them together at B. The end A is built-in (cantilevered) and the end C is simply-supported. With the load P acting as shown, the bending moment at A is

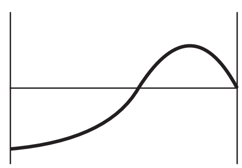


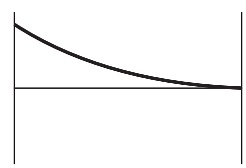
(A) zero (B) $\frac{PL}{2}$

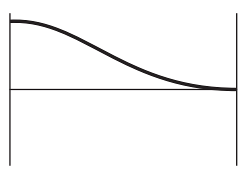
(C) $\frac{3PL}{2}$ (D) indeterminate

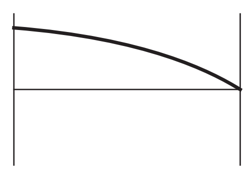
MCQ 3.51 A cantilever beam carries the anti-symmetric load shown, where W_0 is the peak intensity of the distributed load. Qualitatively, the correct bending moment diagram for this beam is



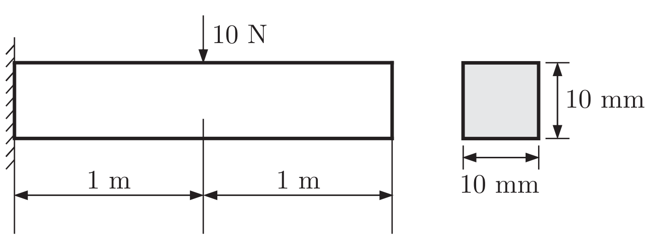
(A) 

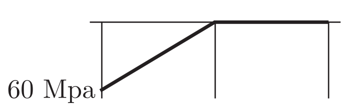
(B) 

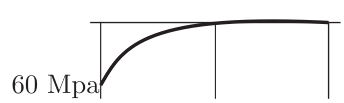
(C) 

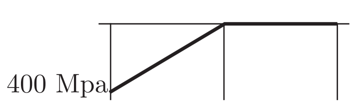
(D) 

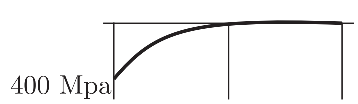
MCQ 3.52 A cantilever beam has the square cross section of 10 mm × 10 mm. It carries a transverse load of 10 N. Consider only the bottom fibres of the beam, the correct representation of the longitudinal variation of the bending stress is



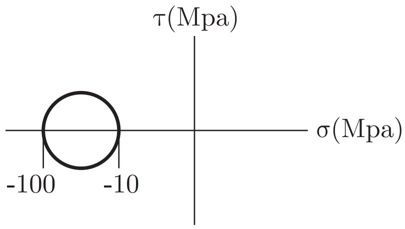
(A) 

(B) 

(C) 

(D) 

MCQ 3.53 The Mohr's circle of plane stress for a point in a body is shown. The design is to be done on the basis of the maximum shear stress theory for yielding. Then, yielding will just begin if the designer chooses a ductile material whose yield strength is



(A) 45 MPa (B) 50 MPa

(C) 90 MPa (D) 100 MPa

MCQ 3.54 A weighing machine consist of a 2 kg pan resting on a spring. In this condition, with the pan resting on the spring, the length of the spring is 200 mm. When a mass of 20 kg is placed on the pan, the length of the spring becomes 100 mm. For the spring, the un-deformed length L and the spring constant k (stiffness) are

(A) $L = 220$ mm, $k = 1862$ N/m (B) $L = 210$ mm, $k = 1960$ N/m

(C) $L = 200$ mm, $k = 1960$ N/m (D) $L = 200$ mm, $k = 2156$ N/m

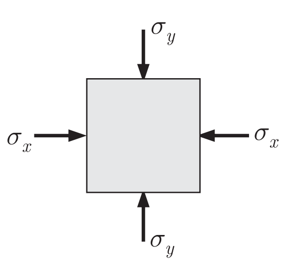
YEAR 2004 ONE MARK

MCQ 3.55 In terms of Poisson's ratio (υ) the ratio of Young's Modulus (E) to Shear Modulus (G) of elastic materials is

(A) $2(1+\upsilon)$ (B) $2(1-\upsilon)$

(C) $\frac{1}{2}(1+\upsilon)$ (D) $\frac{1}{2}(1-\upsilon)$

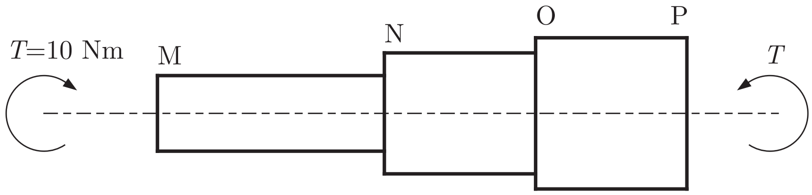
MCQ 3.56 The figure shows the state of stress at a certain point in a stressed body. The magnitudes of normal stresses in x and y directions are 100 MPa and 20 MPa respectively. The radius of Mohr's stress circle representing this state of stress is



(A) 120 (B) 80

(C) 60 (D) 40

MCQ 3.57 A torque of 10 Nm is transmitted through a stepped shaft as shown in figure. The torsional stiffness of individual sections of length MN, NO and OP are 20 Nm/rad, 30 Nm/rad and 60 Nm/rad respectively. The angular deflection between the ends M and P of the shaft is

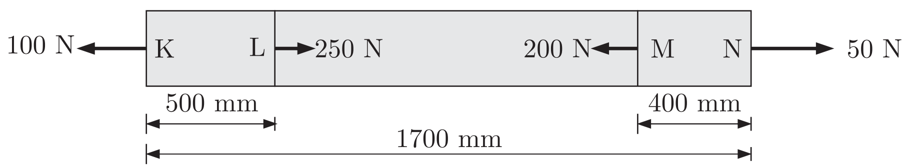


(A) 0.5 rad (B) 1.0 rad

(C) 5.0 rad (D) 10.0 rad

YEAR 2004 TWO MARKS

MCQ 3.58 The figure below shows a steel rod of 25 mm^2 cross sectional area. It is loaded at four points, K, L, M and N. Assume $E_{steel} = 200$ GPa. The total change in length of the rod due to loading is



(A) 1 μm (B) -10 μm

(C) 16 μm (D) -20 μm

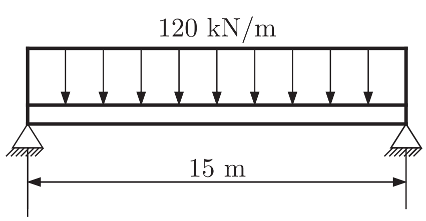
MCQ 3.59 A solid circular shaft of 60 mm diameter transmits a torque of 1600 N.m. The value of maximum shear stress developed is

(A) 37.72 MPa (B) 47.72 MPa

(C) 57.72 MPa (D) 67.72 MPa

- **Common Data For Q. 60 and 61 are given below.**

A steel beam of breadth 120 mm and height 750 mm is loaded as shown in the figure. Assume $E_{steel} = 200$ GPa.



MCQ 3.60 The beam is subjected to a maximum bending moment of

(A) 3375 kN-m (B) 4750 kN-m

(C) 6750 kN-m (D) 8750 kN-m

MCQ 3.61 The value of maximum deflection of the beam is

(A) 93.75 mm (B) 83.75 mm

(C) 73.75 mm (D) 63.75 mm

YEAR 2003 ONE MARK

MCQ 3.62 The second moment of a circular area about the diameter is given by (D is the diameter).

(A) $\frac{\pi D^4}{4}$ (B) $\frac{\pi D^4}{16}$

(C) $\frac{\pi D^4}{32}$ (D) $\frac{\pi D^4}{64}$

MCQ 3.63 A concentrated load of P acts on a simply supported beam of span L at a distance $L/3$ from the left support. The bending moment at the point of application of the load is given by

(A) $\frac{PL}{3}$ (B) $\frac{2PL}{3}$

(C) $\frac{PL}{9}$ (D) $\frac{2PL}{9}$

MCQ 3.64 Two identical circular rods of same diameter and same length are subjected to same magnitude of axial tensile force. One of the rod is made out of mild steel having the modulus of elasticity of 206 GPa. The other rod is made out of cast iron having the modulus of elasticity of 100 GPa. Assume both the materials to be homogeneous and isotropic and the axial force causes the same amount of uniform stress in both the rods. The stresses developed are within the proportional limit of the respective materials. Which of the following observations is correct ?

(A) Both rods elongate by the same amount

(B) Mild steel rod elongates more than the cast iron rod

(C) Cast iron rod elongates more than the mild steel rods

(D) As the stresses are equal strains are also equal in both the rods

MCQ 3.65 The beams, one having square cross section and another circular cross-section, are subjected to the same amount of bending moment. If the cross sectional area as well as the material of both the beams are same then

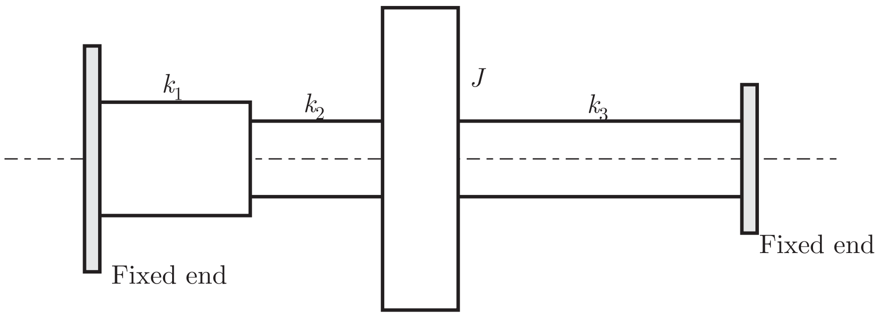
(A) maximum bending stress developed in both the beams is same

(B) the circular beam experience more bending stress than the square one

(C) the square beam experience more bending stress than the circular one

(D) as the material is same, both the beams will experience same deformation.

MCQ 3.66 Consider the arrangement shown in the figure below where J is the combined polar mass moment of inertia of the disc and the shafts. k_1, k_2, k_3 are the torsional stiffness of the respective shafts. The natural frequency of torsional oscillation of the disc is given by



(A) $\sqrt{\dfrac{k_1 + k_2 + k_3}{J}}$ (B) $\sqrt{\dfrac{k_1k_2 + k_2k_3 + k_3k_1}{J(k_1 + k_2)}}$

(C) $\sqrt{\dfrac{k_1 + k_2 + k_3}{J(k_1k_2 + k_2k_3 + k_3k_1)}}$ (D) $\sqrt{\dfrac{k_1k_2 + k_2k_3 + k_3k_1}{J(k_2 + k_3)}}$

MCQ 3.67 Maximum shear stress developed on the surface of a solid circular shaft under pure torsion is 240 MPa. If the shaft diameter is doubled then the maximum shear stress developed corresponding to the same torque will be

(A) 120 MPa (B) 60 MPa

(C) 30 MPa (D) 15 MPa

YEAR 2003 TWO MARKS

MCQ 3.68 A simply supported laterally loaded beam was found to deflect more than a specified value. Which of the following measures will reduce the deflection ?

(A) Increase the area moment of inertia

(B) Increase the span of the beam

(C) Select a different material having lesser modulus of elasticity

(D) Magnitude of the load to be increased

MCQ 3.69 A shaft subjected to torsion experiences a pure shear stress τ on the surface. The maximum principal stress on the surface which is at 45° to the axis will have a value

(A) $\tau \cos 45^\circ$ (B) $2\tau \cos 45^\circ$

(C) $\tau \cos^2 45^\circ$ (D) $2\tau \sin 45^\circ \cos 45^\circ$

• **Common Data For Q. 70 and 71 are given below.**

The state of stress at a point "P" in a two dimensional loading is such that the Mohr's circle is a point located at 175 MPa on the positive normal stress axis.

MCQ 3.70 The maximum and minimum principal stresses respectively from the Mohr's circle are

(A) $+175$ MPa, -175 MPa (B) $+175$ MPa, $+175$ MPa

(C) 0, -175 MPa (D) 0, 0

MCQ 3.71 The directions of maximum and minimum principal stresses at the point "P" from the Mohr's circle are

(A) 0, 90° (B) 90°, 0

(C) 45°, 135° (D) all directions

YEAR 2002 ONE MARK

MCQ 3.72 The total area under the stress-strain curve of mild steel specimen tested upto failure under tension is a measure of

(A) ductility

(B) ultimate strength

(C) stiffness

(D) toughness

MCQ 3.73 The number of components in a stress tensor defining stress at a point in three dimensions is

(A) 3 (B) 4

(C) 6 (D) 9

YEAR 2002 TWO MARKS

MCQ 3.74 The relationship between Young's modulus (E), Bulk modulus (K) and Poisson's ratio (υ) is given by

(A) $E = 3K(1-2\upsilon)$ (B) $K = 3E(1-2\upsilon)$

(C) $E = 3K(1-\upsilon)$ (D) $K = 3E(1-\upsilon)$

MCQ 3.75 The ratio of Euler's bucking loads of columns with the same parameters having (i) both ends fixed, and (ii) both ends hinged is

(A) 2 (B) 4

(C) 6 (D) 8

YEAR 2001 ONE MARK

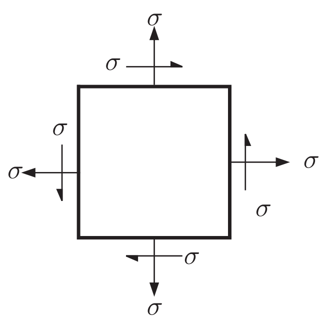
MCQ 3.76 The shape of the bending moment diagram for a uniform cantilever beam carrying a uniformly distributed load over its length is

(A) a straight line (B) a hyperbola

(C) an ellipse (D) a parabola

YEAR 2001 TWO MARKS

MCQ 3.77 The maximum principal stress for the stress state shown in the figure is



(A) σ (B) 2σ

(C) 3σ (D) 1.5σ

****

SOLUTION

SOL 3.1 Option (D) is correct.

For thin walled spherical shell circumferential (hoop) stress is

$$\sigma = \frac{pd}{4t} = \frac{pr}{2t}$$

For initial condition let radius r_1 and thickness t_1, then

$$\sigma_1 = \frac{pr_1}{2t_1} \qquad ...(i)$$

For final condition radius r_2 increased by 1%, then

$$r_2 = r_1 + \frac{r_1}{100} = 1.01 r_1$$

Thickness t_2 decreased by 1% then

$$t_2 = t_1 - \frac{t_1}{100} = 0.99 t_1$$

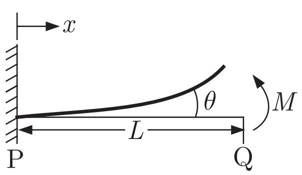
and $$\sigma_2 = \frac{pr_2}{2t_2} = \frac{p \times 1.01 r_1}{1 \times 9.99 t_1} = 1.0202 \frac{pr_1}{2t_1}$$

From Eq. (i) $$\sigma_2 = 1.0202 \times \sigma_1$$

Change in hoop stress (%)

$$\sigma_c = \frac{\sigma_2 - \sigma_1}{\sigma_1} \times 100 = \frac{1.0202\sigma_1 - \sigma_1}{\sigma_1} \times 100 = 2.02\%$$

SOL 3.2 Option (A) is correct.



Since $$EI \frac{d^2 y}{dx^2} = M$$

Integrating $$EI \frac{dy}{dx} = mx + C_1 \qquad ...(i)$$

At $x = 0$, $$\frac{dy}{dx} = 0$$

So $$EI(0) = M(0) + C_1 \Rightarrow C_1 = 0$$

Hence Eq.(i) becomes

$$EI \frac{dy}{dx} = mx$$

Again integrating $$EIy = \frac{mx^2}{2} + C_2 \qquad ...(ii)$$

At $x = 0,\ y = 0,$ $\qquad EI(0) = \frac{m(0)^2}{2} + C_2$

$$C_2 = 0$$

Then Eq. (ii) becomes

$$EIy = \frac{Mx^2}{2}$$

$$y = \frac{Mx^2}{2EI} \qquad \Rightarrow y_{max} = \frac{ML^2}{2EI}\ (\text{At } x = L, y = y_{max})$$

SOL 3.3 Option (C) is correct.

Critical buckling load, $= \frac{\pi^2 EI}{L^2}$ ...(i)

For both ends clamped $L = \frac{L}{2}$

For both ends hinged $L = L$

Ratio for both ends clamped to both ends hinged is $= \dfrac{\frac{\pi^2 EI}{\left(\frac{L}{2}\right)^2}}{\frac{\pi^2 EI}{L^2}} = \frac{4}{L^2} \times \frac{L^2}{1} = 4$

SOL 3.4 Option (B) is correct.

According to Von Mises Yield criterion

$$\sigma_Y^2 = \frac{1}{2}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]$$

Given, $\quad T = \begin{bmatrix} 10 & 5 & 0 \\ 5 & 20 & 0 \\ 0 & 0 & -10 \end{bmatrix}$

From given Matrix

$$\sigma_x = 10 \qquad \tau_{xy} = 5$$

$$\sigma_y = 20 \qquad \tau_{yz} = 0$$

$$\sigma_z = -10 \qquad \tau_{zx} = 0$$

So, $\quad \sigma_Y^2 = \frac{1}{2}[(10-20)^2 + (20+10)^2 + (-10-10)^2 + 6(5^2 + 0^2 + 0^2)]$

$$= \frac{1}{2} \times [100 + 900 + 400 + (6 \times 25)] = 27.83\text{ MPa}$$

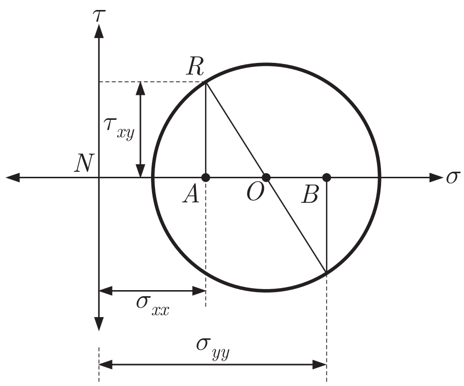
Shear yield stress

$$\tau_Y = \frac{\sigma_Y}{\sqrt{3}} = \frac{27.83}{\sqrt{3}} = 16.06\text{ MPa}$$

SOL 3.5 Option (B) is correct.

Given, $\sigma_{xx} = 40\text{ MPa} = AN$, $\sigma_{yy} = 100\text{ MPa} = BN$, $\tau_{xy} = 40\text{ MPa} = AR$

Diagram for Mohr's circle



Radius of Mohr's circle $OR = \sqrt{(AR)^2 + (AO)^2}$

$$AO = \frac{AB}{2} = \frac{BN - AN}{2} = \frac{100 - 40}{2} = 30$$

Therefore, $OR = \sqrt{(40)^2 + (30)^2} = 50\text{ MPa}$

SOL 3.6 Option (A) is correct.

For a solid cube strain in x, y and z axis are

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\upsilon(\sigma_y + \sigma_z)}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\upsilon(\sigma_x + \sigma_z)}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\upsilon(\sigma_x + \sigma_y)}{E}$$

From symmetry of cube, $\varepsilon_x = \varepsilon_y = \varepsilon_z = \varepsilon$

and $\sigma_x = \sigma_y = \sigma_z = \sigma$

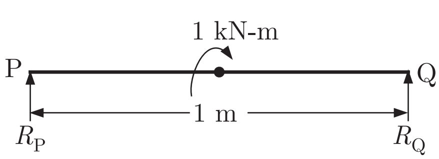
So $\varepsilon = \frac{(1 - 2\upsilon)}{E} \times \sigma$

Where $\varepsilon = -\alpha\Delta T$ (Thermal compression stress)

Therefore, $\sigma = \frac{\varepsilon \times E}{(1 - 2\upsilon)} = -\frac{-\alpha\Delta TE}{(1 - 2\upsilon)} = -\frac{\alpha\Delta TE}{(1 - 2\upsilon)}$

SOL 3.7 Option (A) is correct.

First of all we have to make a free body diagram of the given beam.



Here R_P and R_Q are the reaction forces acting at P and Q.

For equilibrium of forces on the beam,

$$R_P + R_Q = 0 \qquad \text{...(i)}$$

Taking the moment about the point P,

$$R_Q \times 1 = 1\,\text{kN-m} \qquad \Rightarrow \quad R_Q = 1\,\text{kN-m}$$

From equation (i), $R_P = -R_Q = -1\,\text{kN-m}$

Since, our assumption that R_P acting in the upward direction, is wrong,
So, R_P acting in downward direction and R_Q acting in upward direction.

SOL 3.8 Option (B) is correct.

Given : $l = 1$ meter, $b = 20$ mm, $h = 10$ mm

We know that, Slenderness ratio $= \frac{l}{k}$

Where, $k = \sqrt{\frac{I}{A}} = \sqrt{\frac{bh^3/12}{b \times h}}$

Substitute the values, we get

$$k = \sqrt{\frac{\frac{1}{12} \times 20 \times (10)^3 \times 10^{-12}}{10 \times 20 \times 10^{-6}}} = \sqrt{\frac{20 \times 10^{-3}}{12 \times 10 \times 20}}$$

$$= \sqrt{8.33 \times 10^{-6}} = 2.88 \times 10^{-3}\,\text{m}$$

$$\text{Slenderness ratio} = \frac{1}{2.88 \times 10^{-3}} = 347.22 \simeq 346$$

SOL 3.9 Option (C) is correct.

(P) Maximum-normal stress criterion → (M)

(Q) Maximum-distortion energy criterion → (N)

(R) Maximum-shear-stress criterion → (L)

So correct pairs are, P-M, Q-N, R-L

SOL 3.10 Option (B) is correct.

Given : $r = 500$ mm, $t = 10$ mm, $p = 5$ MPa

We know that average circumferential (hoop) stress is given by,

$$\sigma_h = \frac{pd}{2t} = \frac{5 \times (2 \times 500)}{2 \times 10} = 250\,\text{MPa}$$

SOL 3.11 Option (B) is correct.

Here we see that shafts are in series combination. For series combination Total angular twist,

$$\theta = \theta_1 + \theta_2 \qquad \text{...(i)}$$

From the torsional equation,

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l} \qquad \Rightarrow \quad \theta = \frac{Tl}{GJ} \qquad J = \frac{\pi}{32}d^4$$

$$\theta = \frac{32\,Tl}{\pi d^4 G}$$

Now, from equation (i),

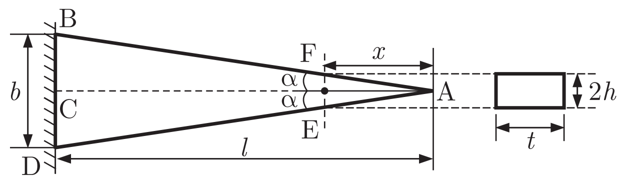
$$\theta = \frac{32T(L)}{\pi(2d)^4G} + \frac{32T\left(\frac{L}{2}\right)}{\pi d^4 G} = \frac{32TL}{\pi d^4 G}\left[\frac{1}{16}+\frac{1}{2}\right] = \frac{32TL}{\pi d^4G}\times\frac{9}{16} = \frac{18TL}{\pi d^4 G}$$

$$d = \left(\frac{18TL}{\pi\theta G}\right)^{\frac{1}{4}}$$

SOL 3.12 Option (B) is correct.

Let, b = width of the base of triangle $ABD = BD$

t = thickness of conilever beam



From the similar triangle (Figure (i)) ΔABC or ΔAFE

$$\frac{b/2}{l} = \frac{h}{x} \qquad \text{let } OE = h$$

$$h = \frac{bx}{2l} \qquad ...(i)$$

Now from figure (ii), For a rectangular cross section,

$$I = \frac{(2h)t^3}{12} = 2\times\frac{bx}{2l}\times\frac{t^3}{12} = \frac{bxt^3}{12l} \qquad \text{From equation (i)}$$

SOL 3.13 Option (D) is correct.

We know that deflection equation is

$$EI\frac{d^2t}{dx^2} = M = P\times x$$

$$\frac{d^2y}{dx^2} = \frac{1}{EI}P\times x$$

From previous part of the question

$$\frac{d^2y}{dx^2} = \frac{1}{E\times\frac{bxt^3}{12L}}\times Px = \frac{12PL}{Ebt^3}$$

On Integrating, we get

$$\frac{dy}{dx} = \frac{12PLx}{Ebt^3} + C_1 \qquad ...(i)$$

When $x = L$, $\frac{dy}{dx} = 0$

So, $$0 = \frac{12PL^2}{Ebt^3} + C_1 \quad \Rightarrow \quad C_1 = -\frac{12PL^2}{Ebt^3}$$

Again integrating equation (i),

$$y = \frac{12PL}{Ebt^3} \times \frac{x^2}{2} + C_1 x + C_2 \qquad \text{...(ii)}$$

When $x = L$, $y = 0$

So,
$$0 = \frac{12PL}{2Ebt^3} \times L^2 + C_1 L + C_2 = \frac{6PL^3}{Ebt^3} - \frac{12PL^3}{Ebt^3} + C_2$$

$$C_2 = \frac{6PL^3}{Ebt^3}$$

From equation (ii),

$$y = \frac{6PLx^2}{Ebt^3} - \frac{12PL^2 x}{Ebt^3} + \frac{6PL^3}{Ebt^3} \qquad \text{...(iii)}$$

The maximum deflection occurs at $x = 0$, from equation (iii),

$$y_{\max} = 0 + 0 + \frac{6PL^3}{Ebt^3} = \frac{6PL^3}{Ebt^3}$$

SOL 3.14 Option (C) is correct.

Given : $\sigma_x = -200\,\text{MPa}$, $\sigma_y = 100\,\text{MPa}$, $\tau_{xy} = 100\,\text{MPa}$

We know that maximum shear stress is given by,

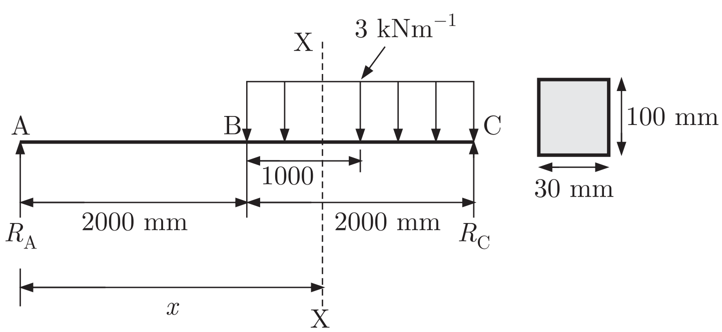
$$\tau_{\max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

Substitute the values, we get

$$\tau_{\max} = \frac{1}{2}\sqrt{(-200 - 100)^2 + 4 \times (100)^2}$$

$$= \frac{1}{2}\sqrt{90000 + 40000} = 180.27 \simeq 180.3\,\text{MPa}$$

SOL 3.15 Option (C) is correct.



First of all we have to make the FBD of the given system.

Let R_A and R_C are the reactions acting at point A and C respectively.

In the equilibrium condition of forces,

$$R_A + R_C = 6000\,\text{N} \qquad \text{...(i)}$$

Taking moment about point A,

$$R_C \times 4 = 6000 \times 3$$

$$R_C = \frac{18000}{4} = 4500\,\text{N} = 4.5\,\text{kN}$$

And from equation (i),

$$R_A = 6000 - 4500 = 1500\text{ N} = 1.5\text{ kN}$$

Taking a section $X - X$ at a distance x from A and taking the moment about this section

$$M_{XX} = R_A \times x - 3(x-2) \times \frac{(x-2)}{2} \qquad F = 3(x-2) \text{ and } d = \frac{x-2}{2}$$

$$= 1.5x - 1.5(x-2)^2 \qquad \text{...(ii)}$$

For maximum Bending moment,

$$\frac{d}{dx}(M_{XX}) = 0$$

$$1.5 - 2 \times 1.5(x-2) = 0$$

$$1.5 - 3x + 6 = 0$$

$$-3x = -7.5$$

$$x = 2.5\text{ m} = 2500\text{ mm}$$

So the maximum bending moment occurs at 2500 mm to the right of A.

SOL 3.16 Option (B) is correct.

From the equation (ii) of the previous part, we have

Maximum bending moment at $x = 2.5$ m is,

$$(BM)_{2.5\text{ m}} = 1.5 \times 2.5 - 1.5(2.5-2)^2 = 3.375\text{ kN-m}$$

From the bending equation,

$$\sigma_b = \frac{M}{I} \times y = \frac{M}{\frac{bh^3}{12}} \times \frac{h}{2} = \frac{6M}{bh^2}$$

Substitute the values, we get

$$\sigma_b = \frac{6 \times 3375}{0.030 \times (0.1)^2} = 67.5 \times 10^6\text{ N/m}^2 = 67.5\text{ MPa}$$

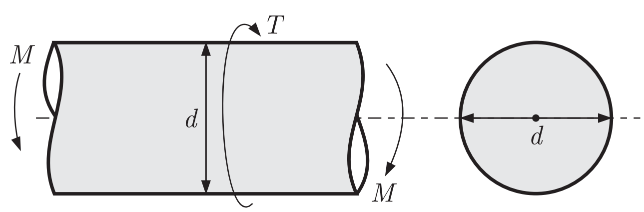
SOL 3.17 Option (C) is correct.

Given : $\sigma_1 = 100$ MPa, $\sigma_2 = 40$ MPa

We know, the maximum shear stress for the plane complex stress is given by

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{100-40}{2} = \frac{60}{2} = 30\text{ MPa}$$

SOL 3.18 Option (C) is correct.



We know that, for a shaft of diameter d is subjected to combined bending moment M and torque T, the equivalent Torque is,

$$T_e = \sqrt{M^2 + T^2}$$

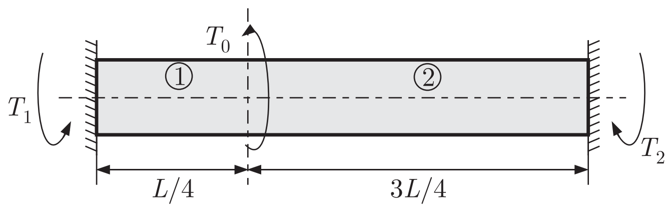
Induced shear stress is,

$$\tau = \frac{16T}{\pi d^3} = \frac{16}{\pi d^3} \times \sqrt{M^2 + T^2}$$

Now, for safe design, τ should be less than $\frac{S_{sy}}{N}$

Where, $S_{sy} =$ Torsional yield strength and $N =$ Factor of safety

SOL 3.19 Option (B) is correct.



First, the shaft is divided in two parts (1) and (2) and gives a twisting moment T_1 (in counter-clockwise direction) and T_2 (in clock wise direction) respectively.

By the nature of these twisting moments, we can say that shafts are in parallel combination.

So, $$T_0 = T_1 + T_2 \quad \text{...(i)}$$

From the torsional equation,

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l} \qquad \Rightarrow \quad T = \frac{GJ\theta}{l}$$

But, here $$G_1 = G_2$$

$$\theta_1 = \theta_2 \qquad \text{For parallel connection}$$

$$J_1 = J_2 \qquad \text{Diameter is same}$$

So, $$T_1 l_1 = T_2 l_2$$

$$T_1 \times \frac{L}{4} = T_2 \times \frac{3L}{4}$$

$$T_1 = 3T_2$$

Now, From equation (i),

$$T_0 = 3T_2 + T_2 = 4T_2$$

$$T_2 = \frac{T_0}{4}$$

And $$T_1 = \frac{3T_0}{4}$$

Here $T_1 > T_2$

So, maximum shear stress is developed due to T_1,

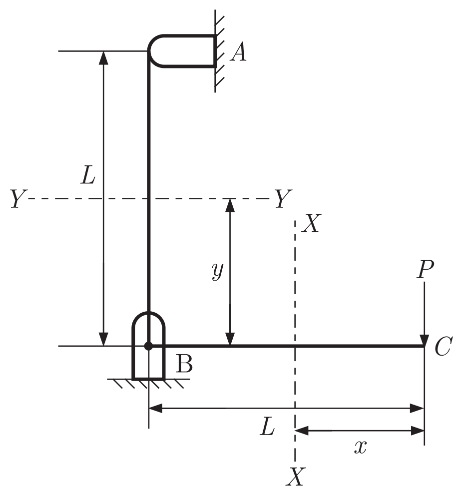
$$\frac{T_1}{J} = \frac{\tau_{max}}{r} \qquad \Rightarrow \tau_{max} = \frac{T_1}{J} \times r$$

Substitute the values, we get

$$\tau_{max} = \frac{\left(\frac{3T_0}{4}\right)}{\frac{\pi}{32}d^4} \times \frac{d}{2} = \frac{32 \times 3T_0}{8\pi \times d^3} = \frac{12T_0}{\pi d^3}$$

SOL 3.20 Option (D) is correct.

We have to solve this by Castigliano's theorem.



We have to take sections XX and YY along the arm BC and AB respectively and find the total strain energy.

So, Strain energy in arm BC is,

$$U_{BC} = \int_0^L \frac{M_x^2}{2EI}dx = \int_0^L \frac{(Px)^2}{2EI}dx \qquad M_x = P \times x$$

Integrating the equation and putting the limits, we get

$$U_{BC} = \frac{P^2}{2EI}\left[\frac{x^3}{3}\right]_0^L = \frac{P^2L^3}{6EI}$$

Similarly for arm AB, we have

$$U_{AB} = \int_0^L \frac{M_y^2}{2EI}dy = \int_0^L \frac{P^2L^2}{2EI}dy \qquad M_y = P \times L$$

$$= \frac{P^2L^3}{2EI}$$

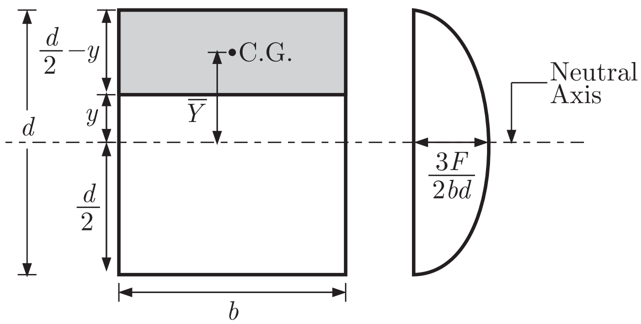
So, total strain energy stored in both the arms is,

$$U = U_{AB} + U_{BC} = \frac{P^2L^3}{2EI} + \frac{P^2L^3}{6EI} = \frac{2P^2L^3}{3EI}$$

From the Castigliano's theorem, vertical deflection at point A is,

$$\delta_A = \frac{\delta U}{\delta P} = \frac{\delta}{\delta P}\left(\frac{2P^2L^3}{3EI}\right) = \frac{4PL^3}{3EI}$$

SOL 3.21 Option (D) is correct.



For a rectangle cross-section:

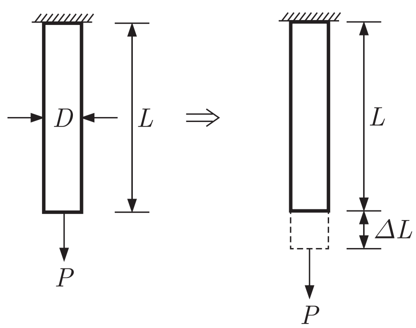
$$\tau_v = \frac{FA\overline{Y}}{Ib} = \frac{6F}{bd^3}\left(\frac{d^2}{4} - y^2\right) \qquad F = \text{Transverse shear load}$$

Maximum values of τ_v occurs at the neutral axis where, $y = 0$

$$\text{Maximum } \tau_v = \frac{6F}{bd^3} \times \frac{d^2}{4} = \frac{3F}{2bd} = \frac{3}{2}\tau_{\text{mean}} \qquad \tau_{mean} = \frac{F}{bd}$$

So, transverse shear stress is variable with maximum on the neutral axis.

SOL 3.22 Option (D) is correct.



From the application of load P, the length of the rod increases by an amount of ΔL

$$\Delta L = \frac{PL}{AE} = \frac{PL}{\frac{\pi}{4}D^2E} = \frac{4PL}{\pi D^2 E}$$

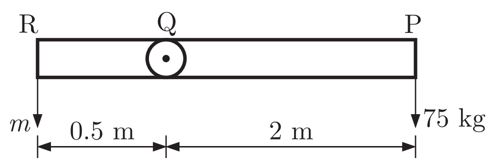
And increase in length due to applied load P in axial or longitudinal direction, the shear modulus is comes in action.

$$G = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{\tau_s}{\Delta L/L} = \frac{\tau_s L}{\Delta L}$$

So, for calculating the resulting change in diameter both young's modulus and shear modulus are used.

SOL 3.23 Option (D) is correct.

First of all we have to make the FBD of the given system.



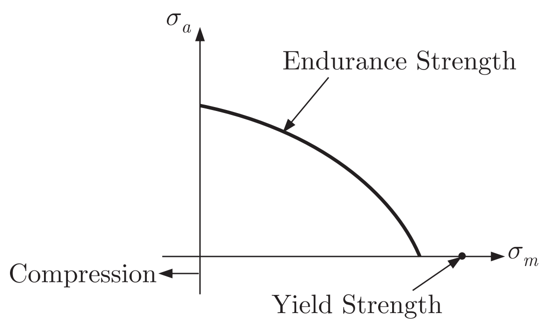
Let mass of the counter weight $= m$.

Here point Q is the point of contraflexure or point of inflection or a virtual hinge.

So, $$M_Q = 0$$

$$m \times 0.5 = 75 \times 2 \qquad \Rightarrow \quad m = 300\text{ kg}$$

SOL 3.24 Option (D) is correct.



(Gerber's Parabola)

The figure shown the Gerber's parabola. It is the characteristic curve of the fatigue life of the shaft in the presence of the residual compressive stress. The fatigue life of the material is effectively increased by the introduction of a compressive mean stress, whether applied or residual.

SOL 3.25 Option (D) is correct.

Here corner point P is fixed. At point P double stresses are acting, one is due to bending and other stress is due to the direct Load.

So, bending stress, (From the bending equation)

$$\sigma_b = \frac{M}{I} y$$

Distance from the neutral axis to the external fibre $y = \frac{2b}{2} = b$,

$$\sigma_b = \frac{F(L-b)}{\frac{(2b)^4}{12}} \times b \qquad \text{For square section } I = \frac{b^4}{12}$$

$$= \frac{12F(L-b)}{16b^3} = \frac{3F(L-b)}{4b^3}$$

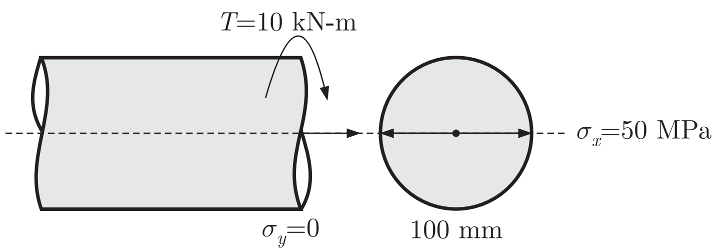
and direct stress,

$$\sigma_d = \frac{F}{(2b)^2} = \frac{F}{4b^2} = \frac{F}{4b^2} \times \frac{b}{b} = \frac{Fb}{4b^3}$$

Total axial stress at the corner point P is,

$$\sigma = \sigma_b + \sigma_d = \frac{3F(L-b)}{4b^3} + \frac{Fb}{4b^3} = \frac{F(3L-2b)}{4b^3}$$

SOL 3.26 Option (B) is correct.



The shaft is subjected to a torque of 10 kN-m and due to this shear stress is developed in the shaft,

$$\tau_{xy} = \frac{T}{J} \times r = \frac{10 \times 10^3}{\frac{\pi}{32} d^4} \times \frac{d}{2} \qquad \text{From Torsional equation}$$

$$= \frac{16 \times 10 \times 10^3}{\pi d^3} = \frac{16 \times 10^4}{3.14 \times (10^{-1})^3} = \frac{160}{3.14} = 50.95\,\text{MPa}$$

Maximum principal stress,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

Substitute the values, we get

$$\sigma_1 = \frac{50}{2} + \frac{1}{2}\sqrt{(50)^2 + 4 \times (50.95)^2} = 25 + \frac{1}{2}\sqrt{12883.61}$$

$$= 25 + \frac{113.50}{2} = 25 + 56.75 = 81.75\,\text{MPa} \simeq 82\,\text{MPa}$$

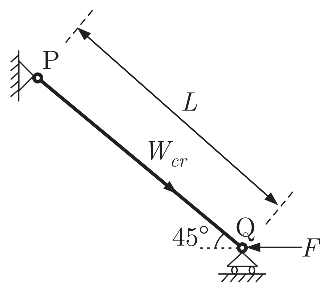
SOL 3.27 Option (B) is correct.

We know that according to Euler's theory, the crippling or buckling load (W_{cr}) under various end conditions is represented by the general equation,

$$W_{cr} = \frac{C\pi^2 EI}{L^2} \qquad \text{...(i)}$$

Where L = length of column

C = Constant, representing the end conditions of the column.



Here both ends are hinged, $C = 1$

From equation (i), $W_{cr} = \frac{\pi^2 EI}{L^2}$

Minimum force F required , $W_{cr} = F\cos 45°$

$$F = \frac{W_{cr}}{\cos 45°} = \frac{\sqrt{2}\,\pi^2 EI}{L^2}$$

SOL 3.28 Option (D) is correct.

We know that deflection in a compression spring is given by

$$\delta = \frac{64PR^3n}{d^4G} = \frac{8PD^3n}{d^4G}$$

Where n = number of active coils

D = Mean coil Diameter

d = Music wire Diameter

And $k = \frac{P}{\delta} = \frac{d^4G}{8D^3n}$

$$k \propto \frac{1}{D^3}$$

Given that mean coil diameter is reduced to 10 mm.

So, $D_1 = 20\text{ mm}$

$$D_2 = 20 - 10 = 10\text{ mm}$$

and $\frac{k_2}{k_1} = \left(\frac{D_1}{D_2}\right)^3 = \left(\frac{20}{10}\right)^3 = 8$

$$k_2 = 8k_1$$

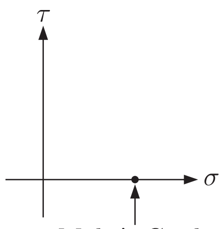
So, stiffness is increased by 8 times.

SOL 3.29 Option (B) is correct.

Pressure will remain uniform in all directions. So, hydrostatic load acts in all directions on the fluid element and Mohr's circle becomes a point on $\sigma - \tau$ axis and

$\sigma_x = \sigma_y$ and $\tau_{xy} = 0$

So, $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = 0$



SOL 3.30 Option (B) is correct.

Given : $R = 1\,\text{m}$, $t = 1\,\text{mm} = 10^{-3}\,\text{m}$

We know that axial or longitudinal stress for a thin cylinder is,

$$\sigma_x = \sigma_a = \frac{p \times D}{4t} = \frac{p \times 2R}{4t} \qquad ...(i)$$

Here, p = Pressure of the fluid inside the shell

So, pressure at 1 m depth is,

$$p = \rho g h = 1000 \times 10 \times 1 = 10^4\,\text{N/m}^2$$

From equation (i),

$$\sigma_a = \frac{10^4 \times 2 \times 1}{4 \times 10^{-3}} = 5 \times 10^6\,\text{N/m}^2 = 5\,\text{MPa}$$

and hoop or circumferential stress,

$$\sigma_y = \sigma_c = \frac{p \times D}{2t} = \frac{10^4 \times 2}{2 \times 10^{-3}} = 10 \times 10^6\,\text{N/m}^2 = 10\,\text{MPa}$$

SOL 3.31 Option (A) is correct.

Given : υ or $\frac{1}{m} = 0.3$, $E = 100\,\text{GPa} = 100 \times 10^9\,\text{Pa}$

Axial strain or longitudinal strain at mid – depth is,

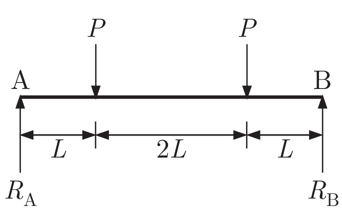
$$\sigma_a = \sigma_x = \frac{pD}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

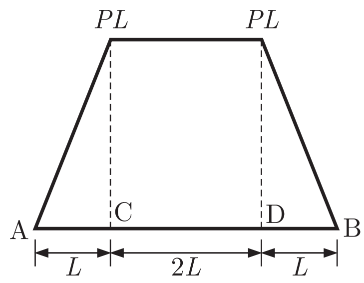
Substitute the values, we get

$$\sigma_a = \frac{10^4 \times 2 \times 1}{2 \times 10^{-3} \times 100 \times 10^9}\left(\frac{1}{2} - 0.3\right)$$

$$= \frac{10^4}{10^8}\left(\frac{1}{2} - 0.3\right) = 10^{-4} \times 0.2 = 2 \times 10^{-5}$$

SOL 3.32 Option (C) is correct.





B.M.D.

In equilibrium condition of forces,

$$R_A + R_B = 2P \qquad \text{...(i)}$$

Taking the moment about point A,

$$R_B \times 4L - P \times L - P \times 3L = 0$$

$$R_B \times 4L - 4PL = 0$$

$$R_B = \frac{4PL}{4L} = P$$

From equation (i), $R_A = 2P - P = P$

With the help of R_A and R_B, we have to make the Bending moment diagram of the given beam. From this B.M.D, at section AC and BD Bending moment varying with distance but at section CD, it is constant.

Now strain energy $U = \int \frac{M^2}{2EI} dx$

Where M is the bending moment of beam.

Total strain energy is given by

$$U = \underbrace{\int_0^L \frac{(Px)^2 dx}{2EI}}_{\{\text{for section } AC\}} + \underbrace{\frac{(PL)^2 2L}{2EI}}_{\{\text{for section } CD\}} + \underbrace{\int_0^L \frac{(Px)^2 dx}{2EI}}_{\{\text{for section } BD\}}$$

$$= 2\int_0^L \frac{(Px)^2 dx}{2EI} + \frac{P^2L^3}{EI} = \frac{P^2}{EI}\int_0^L x^2 dx + \frac{P^2L^3}{EI}$$

Integrating above equation, we get

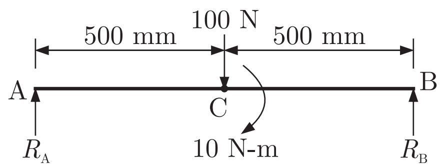
$$U = \frac{P^2}{EI}\left[\frac{x^3}{3}\right]_0^L + \frac{P^2L^3}{EI} = \frac{P^2L^3}{3EI} + \frac{P^2L^3}{EI} = \frac{4P^2L^3}{3EI}$$

SOL 3.33 Option (B) is correct.

Due to 100 N force, bending moment occurs at point C and magnitude of this bending moment is,

$$M_C = 100 \times (0.1) = 10 \text{ N–m} \qquad \text{(in clock wise direction)}$$

We have to make a free body diagram of the given beam,



Where R_A and R_B are the reactions acting at point A and B

For equilibrium of forces,

$$R_A + R_B = 100\text{ N} \quad ...(i)$$

Taking the moment about point A,

$$100 \times 0.5 + 10 = R_B \times 1 \qquad \Rightarrow R_B = 60 \text{ N}$$

From equation (i),

$$R_A = 100 - R_B = 100 - 60 = 40\text{ N}$$

Maximum bending moment occurs at point C,

$$M_C = R_A \times 0.5 + 10 = 40 \times 0.5 + 10 = 20 + 10 = 30\text{ N–m}$$

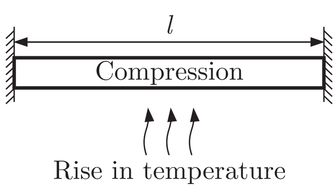
SOL 3.34 Option (C) is correct.

Let, l = original length of the bar

α = Co-efficient of linear expansion of the bar material

ΔT = Rise or drop in temperature of the bar

δl = Change in length which would have occurred due to difference of temperature if the ends of the bar were free to expand or contract.



$$\alpha = \frac{\delta l}{l \times \Delta T}$$

or,
$$\delta l = l \times \alpha \times \Delta T$$

And temperature strain,

$$\varepsilon = \frac{\delta l}{l} = \frac{l \times \alpha \times \Delta T}{l} = \alpha \times \Delta T$$

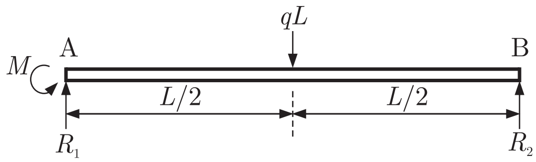
Basically temperature stress and strain are longitudinal (i.e. tensile or compressive) stress and strain

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon}$$

$$\sigma = E\varepsilon = E\alpha \Delta T$$

SOL 3.35 Option (A) is correct.

First of all, we have to make a FBD of the beam. We know that a UDL acting at the mid-point of the beam and its magnitude is equal to $(q \times L)$. So,



In equilibrium of forces,

$$R_1 + R_2 = qL \qquad ...(i)$$

This cantilever beam is subjected to two types of load.

First load is due to UDL and second load is due to point load at B. Due to this deflection occurs at B, which is equal in amount.

So, deflection occurs at B due to the UDL alone,

$$\delta_{UDL} = \frac{qL^4}{8EI}$$

Also, deflection at B due to point load,

$$\delta_{PL} = \frac{R_2 L^3}{3EI}$$

Deflections are equal at B,

$$\delta_{UDL} = \delta_{PL}$$

$$\frac{qL^4}{8EI} = \frac{R_2 L^3}{3EI} \qquad \Rightarrow \quad R_2 = \frac{3qL}{8}$$

And from equation (i), we have

$$R_1 = qL - R_2 = qL - \frac{3qL}{8} = \frac{5qL}{8}$$

For M, taking the moment about B,

$$-qL \times \frac{L}{2} + R_1 \times L - M = 0$$

$$-\frac{qL^2}{2} + \frac{5qL^2}{8} - M = 0$$

$$M = \frac{qL^2}{8}$$

Therefore, $R_1 = \frac{5qL}{8}$, $R_2 = \frac{3qL}{8}$ and $M = \frac{qL^2}{8}$

SOL 3.36 Option (B) is correct.

Given :

$$\nu = 200 \times 100 \times 50\,\text{mm}^3 = 10^6\,\text{mm}^3$$

$$p = 15\,\text{MPa} = 15 \times 10^6\,\text{N/m}^2 = 15\,\text{N/mm}^2$$

$$E = 200\,\text{GPa} = 200 \times 10^3\,\text{N/mm}^2$$

$$\left(\upsilon \text{ or } \frac{1}{m}\right) = 0.3$$

We know the relation between volumetric strain, young's modulus and Poisson's ration is given by,

$$\frac{\Delta\nu}{\nu} = \frac{3p}{E}(1-2\upsilon)$$

Substitute the values, we get

$$\frac{\Delta\nu}{10^6} = \frac{3 \times 15}{200 \times 10^3}(1 - 2 \times 0.3)$$

$$\Delta\nu = \frac{45 \times 10}{2}(1-0.6) = 225 \times 0.4 = 90\,\text{mm}^3$$

SOL 3.37 Option (C) is correct.

Given : $T = 10\,\text{N-m} = 10^4\,\text{N-mm}$, $G = 80\,\text{GPa} = 80 \times 10^3\,\text{N/mm}^2$

$L_1 = L_2 = 100\,\text{mm}$, $d_1 = 50\,\text{mm}$, $d_2 = 25\,\text{mm}$

We know that for a shaft of length l and polar moment of inertia J, subjected to a torque T with an angle of twist θ. The expression of strain energy,

$$U = \frac{1}{2}\frac{T^2 l}{GJ} \qquad U = \frac{1}{2}T\theta, \text{ and } \theta = \frac{Tl}{GJ}$$

So Total strain energy,

$$U = \frac{T^2 L}{2GJ_1} + \frac{T^2 L}{2GJ_2} = \frac{T^2 L}{2G}\left[\frac{1}{J_1} + \frac{1}{J_2}\right] \qquad J = \frac{\pi}{32}d^4$$

Substitute the values, we get

$$U = \frac{(10^4)^2 \times 100}{2 \times 80 \times 10^3}\left[\frac{1}{\frac{\pi}{32}(50)^4} + \frac{1}{\frac{\pi}{32}(25)^4}\right]$$

$$= \frac{10^6}{16} \times \frac{32}{\pi}\left[\frac{1}{625 \times 10^4} + \frac{1}{390625}\right]$$

$$= \frac{10^6}{16 \times 10^4} \times \frac{32}{\pi}\left[\frac{1}{625} + \frac{1}{39.0625}\right]$$

$$= 63.69 \times [0.0016 + 0.0256] = 1.73\,\text{N-mm}$$

SOL 3.38 Option (A) is correct.

Given : $F = 600\,\text{N}$ (Parallel to Z-direction), $d = 30\,\text{mm}$

Normal stress at point P, from bending equation

$$\sigma = \frac{M}{I} \times y = \frac{600 \times 300}{\frac{\pi}{64}d^4} \times \frac{d}{2} \qquad \text{Here } M = \text{bending moment}$$

$$= \frac{18 \times 10^4 \times 32}{\pi d^3} = \frac{18 \times 10^4 \times 32}{3.14(30)^3} = 67.9\,\text{MPa}$$

And from Torsional equation, shear stress, $\frac{T}{J} = \frac{\tau}{r}$

$$\tau = \frac{T}{J} \times r = \frac{600 \times 500}{\frac{\pi}{32}d^4} \times \frac{d}{2} \qquad T = \text{Force} \times \text{Area length}$$

$$= \frac{16 \times 600 \times 500}{3.14 \times (30)^3} = 56.61\,\text{MPa}$$

SOL 3.39 Option (D) is correct.

Here : $\sigma_x = 0$, $\sigma_y = 67.9\,\text{MPa}$, $\tau_{xy} = 56.6\,\text{MPa}$

Maximum principal stress,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \qquad \sigma_x = ?$$

Substitute the values, we get

$$\sigma_1 = \frac{0 + 67.9}{2} + \frac{1}{2}\sqrt{(-67.9)^2 + 4 \times (56.6)^2}$$

$$= 33.95 + \frac{1}{2}\sqrt{17424.65} = 33.95 + 66$$

$$= 99.95 \simeq 100\,\text{MPa}$$

And $$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

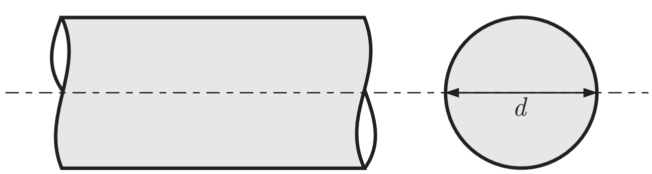
Substitute the values, we get

$$\tan 2\theta = \frac{2 \times 56.6}{0 - 67.9} = -1.667$$

$$2\theta = -59.04$$

$$\theta = -\frac{59.04}{2} = -29.52^\circ$$

SOL 3.40 Option (C) is correct.



From the Torsional equation

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}$$

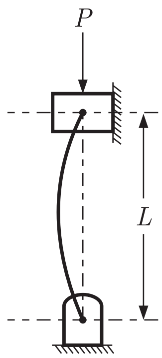
Take first two terms,

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\frac{T}{\frac{\pi}{32}d^4} = \frac{\tau}{\frac{d}{2}} \qquad J = \text{Polar moment of inertia}$$

$$\tau_{max} = \frac{16T}{\pi d^3}$$

SOL 3.41 Option (D) is correct.



According to Euler's theory, the crippling or buckling load (P_{cr}) under various end conditions is represented by a general equation,

$$P_{cr} = \frac{C\pi^2 EI}{L^2} \quad \text{...(i)}$$

Where, E = Modulus of elasticity

I = Mass-moment of inertia

L = Length of column

C = constant, representing the end conditions of the column or end fixity coefficient.

Here both ends are hinged, $C = 1$

Substitute in equation (i), we get $P_{cr} = \frac{\pi^2 EI}{L^2}$

SOL 3.42 Option (C) is correct.

According to "VON MISES - HENKY THEORY", the elastic failure of a material occurs when the distortion energy of the material reaches the distortion energy at the elastic limit in simple tension.

Shear strain energy due to the principle stresses σ_1, σ_2 and σ_3

$$\Delta E = \frac{1+\upsilon}{6E}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

$$= \frac{1+\upsilon}{6E}\left[2(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

$$= \frac{1+\upsilon}{3E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)]$$

SOL 3.43 Option (A) is correct.

Given : $A = (40)^2 = 1600\text{ mm}^2$, $P = -200\text{ kN}$ (Compressive)

$L = 2\text{ m} = 2000\text{ mm}$, $E = 200\text{ GPa} = 200 \times 10^3\text{ N/mm}^2$

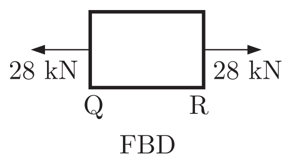
Elongation of the bar,

$$\Delta L = \frac{PL}{AE} = \frac{-200 \times 10^3 \times 2000}{1600 \times 200 \times 10^3} = -1.25 \text{ mm} \qquad \text{Compressive}$$

In magnitude, $\Delta L = 1.25 \text{ mm}$

SOL 3.44 Option (A) is correct.

The FBD of segment QR is shown below :



Given : $A = 700 \text{ mm}^2$

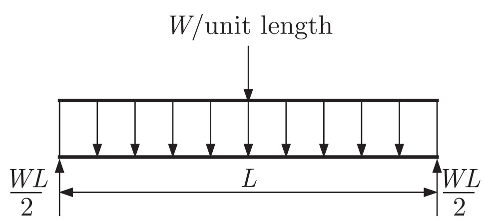
From the free body diagram of the segment QR,

Force acting on QR, $P = 28 \text{ kN}$ (Tensile)

Stress in segment QR is given by,

$$\sigma = \frac{P}{\text{Area}} = \frac{28 \times 10^3}{700 \times 10^{-6}} = 40 \text{ MPa}$$

SOL 3.45 Option none of these is correct.



Given : $L = 6 \text{ m}$, $W = 1.5 \text{ kN/m}$, $d = 75 \text{ mm}$

We know that for a uniformly distributed load, maximum bending moment at the centre is given by,

$$B.M. = \frac{WL^2}{8} = \frac{1.5 \times 10^3 \times (6)^2}{8}$$

$$B.M. = 6750 \text{ N–m} = 6.75 \text{ kN–m}$$

SOL 3.46 Option (A) is correct.

From the bending equation,

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

Where M = Bending moment acting at the given section $= 6.75 \text{ kN–m}$

I = Moment of inertia $= \frac{\pi}{64} d^4$

y = Distance from the neutral axis to the external fibre $= \frac{d}{2}$

σ_b = Bending stress

So, $\sigma_b = \frac{M}{I} \times y$

Substitute the values, we get

$$\sigma_b = \frac{6.75 \times 10^6}{\frac{\pi}{64}(75)^4} \times \frac{75}{2} = \frac{32400}{\pi \times 2 \times (75)^4} \times 10^6$$

$$= 1.6305 \times 10^{-4} \times 10^6 = 163.05\,\text{MPa} \simeq 162.98\,\text{MPa}$$

SOL 3.47 Option (A) is correct.

We know that due to temperature changes, dimensions of the material change. If these changes in the dimensions are prevented partially or fully, stresses are generated in the material and if the changes in the dimensions are not prevented, there will be no stress set up. (Zero stresses).

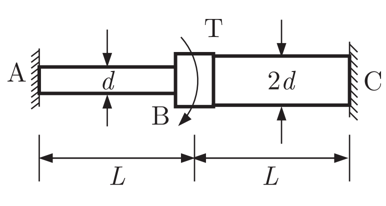
Hence cylindrical rod is allowed to expand or contract freely.

So, $\sigma_r = 0$ and $\sigma_z = 0$

SOL 3.48 Option (A) is correct.

From the figure, we can say that load P applies a force on upper cantilever and the reaction force also applied on upper cantilever by the rigid roller. Due to this, deflections are occur in both the cantilever, which are equal in amount. But because of different forces applied by the P and rigid roller, the slopes are unequal.

SOL 3.49 Option (C) is correct.



Here both the shafts AB and BC are in parallel connection.

So, deflection in both the shafts are equal.

$$\theta_{AB} = \theta_{BC} \qquad \text{...(i)}$$

From Torsional formula,

$$\frac{T}{J} = \frac{G\theta}{L} \qquad \Rightarrow \theta = \frac{TL}{GJ}$$

From equation (i),

$$\frac{T_A L}{GJ_{AB}} = \frac{T_C L}{GJ_{BC}}$$

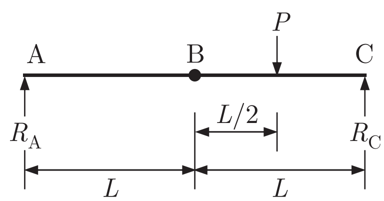
$$\frac{T_A \times L}{G \times \frac{\pi}{32} d^4} = \frac{T_C \times L}{G \times \frac{\pi}{32}(2d)^4}$$

$$\frac{T_A}{d^4} = \frac{T_C}{16d^4} \qquad \text{For same material, } G_{AB} = G_{BC}$$

$$T_C = 16T_A$$

SOL 3.50 Option (B) is correct.

First of all we have to make a Free body diagram of the given beam.



Where, R_A and R_B are the reactions acting at point A and B

The point B is a point of contraflexure or point of inflexion or a virtual hinge. The characteristic of the point of contraflexure is that, about this point moment equal to zero.

For span BC, $\quad M_B = 0$

$$R_C \times L = P \times \frac{L}{2}$$

$$R_C = \frac{P}{2}$$

For the equilibrium of forces on the beam,

$$R_A + R_C = P$$

$$R_A = P - \frac{P}{2} = \frac{P}{2}$$

Now for the bending moment about point A, take the moment about point A,

$$M_A + R_C \times 2L - P \times \left(L + \frac{L}{2}\right) = 0$$

$$M_A + \frac{P}{2} \times 2L - P \times \frac{3L}{2} = 0$$

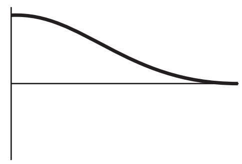
$$M_A = \frac{PL}{2}$$

SOL 3.51 Option (C) is correct.

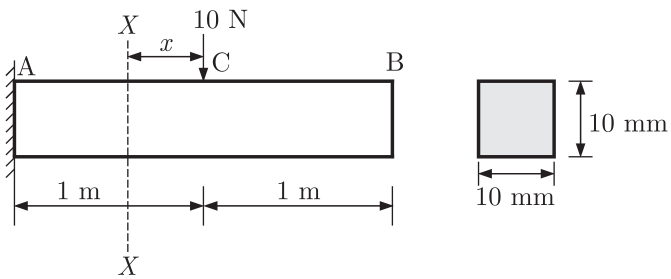
We know that, for a uniformly varying load bending moment will be cubic in nature.

(A) We see that there is no shear force at B, so the slope of BMD at right of B must be zero and similarly on left end A there is no shear force, so slope of BMD also zero.

(B) Now due to triangular shape of load intensity, when we move from right to left, the rate of increase of shear force decreases and maximum at the middle and therefore it reduces.



SOL 3.52 Option (A) is correct.



Taking a section XX on the beam.

Moment about this section XX

$$M_{XX} = 10 \times x = 10x \text{ N–m}$$

For a square section,

$$I = \frac{b^4}{12} = \frac{(10 \times 10^{-3})^4}{12} = \frac{10^{-8}}{12} \text{ m}^4$$

Using the bending equation,

$$\frac{M}{I} = \frac{\sigma}{y} \qquad \Rightarrow \sigma = \frac{M}{I} y$$

Substitute the values, we get

$$\sigma = \frac{10x}{\frac{10^{-8}}{12}} \times \frac{10^{-2}}{2} = 60 \times 10^6 x \qquad \text{...(i)}$$

From equation (i), Bending stress at point $A\,(x = 0)$,

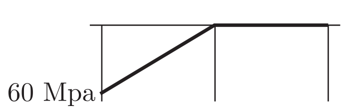
$$\sigma_A = 60 \times 10^6 \times 0 = 0$$

And at point C $(x = 1\,\text{m})$

$$\sigma_C = 60 \times 10^6 \times 1 = 60\,\text{MPa}$$

As no any forces are acting to the right of the point C.

So bending stress is constant after point C.



SOL 3.53 Option (C) is correct.

Maximum shear stress, $\tau = \frac{\sigma_{max} - \sigma_{min}}{2}$

Maximum shear stress at the elastic limit in simple tension (yield strength) $= \frac{\sigma_Y}{2}$

To prevent failure $\frac{\sigma_{max} - \sigma_{min}}{2} \leq \frac{\sigma_Y}{2}$

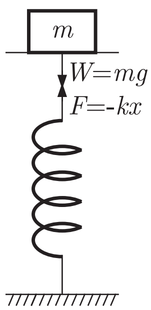
$$\sigma_{max} - \sigma_{min} = \sigma_Y$$

Here $\sigma_{max} = -10\,\text{MPa},\ \sigma_{min} = -100\,\text{MPa}$

So, $\sigma_Y = -10 - (-100) = 90\,\text{MPa}$

SOL 3.54 Option (B) is correct.

Initial length (un-deformed) of the spring $= L$ and spring stiffness $= k$



Let spring is deformed by an amount Δx, then Spring force, $F = k\Delta x$

For initial condition, $2g = k(L - 0.2)$ $W = mg$ ...(i)

After this a mass of 20 kg is placed on the 2 kg pan. So total mass becomes 22 kg and length becomes 100 mm.

For this condition, $(20 + 2)g = k(L - 0.1)$ ...(ii)

Dividing equation (ii) by equation (i),

$$\frac{22g}{2g} = \frac{k(L - 0.1)}{k(L - 0.2)}$$

$$11 = \frac{(L - 0.1)}{(L - 0.2)}$$

$$11L - 2.2 = L - 0.1$$

$$10L = 2.1$$

$$L = \frac{2.1}{10} = 0.21\,\text{m} = 210\,\text{mm}$$

And from equation (i),

$$2g = k(0.21 - 0.2)$$

$$k = \frac{2 \times 9.8}{0.01} = 1960\,\text{N/m}$$

So, $L = 210\,\text{mm}$, and $k = 1960\,\text{N/m}$

SOL 3.55 Option (A) is correct.

Relation between E, G and υ is given by,

$$E = 2G(1 + \upsilon)$$

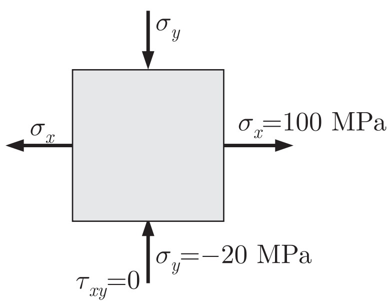
Where E = young's modulus

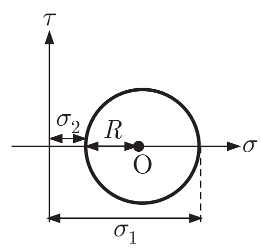
G = Shear Modulus

υ = Poisson's ratio

Now, $$\frac{E}{G} = 2(1 + \upsilon)$$

SOL 3.56 Option (C) is correct.





$\sigma_x = 100\,\text{MPa}$ (Tensile), $\sigma_y = -20\,\text{MPa}$ (Compressive)

We know that, $$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

From the figure, Radius of Mohr's circle,

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2} \times 2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Substitute the values, we get

$$R = \sqrt{\left[\frac{100 - (-20)}{2}\right]^2} = 60$$

SOL 3.57 Option (B) is correct.

Given : $T = 10\,\text{N-m}$, $k_{MN} = 20\,\text{N-m/rad}$, $k_{NO} = 30\,\text{N-m/rad}$, $k_{OP} = 60\,\text{N-m/rad}$

Angular deflection, $$\theta = \frac{T}{k}$$

For section MN, NO or OP, $\theta_{MN} = \frac{10}{20}$ rad, $\theta_{NO} = \frac{10}{30}$ rad, $\theta_{OP} = \frac{10}{60}$ rad

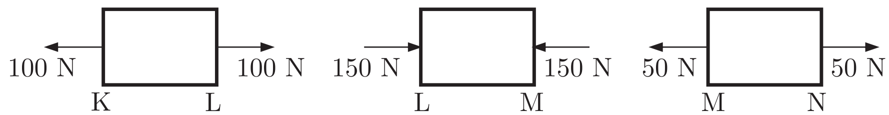
Since MN, NO and OP are connected in series combination. So angular deflection between the ends M and P of the shaft is,

$$\theta_{MP} = \theta_{MN} + \theta_{NO} + \theta_{OP} = \frac{10}{20} + \frac{10}{30} + \frac{10}{60} = 1\,\text{radian}$$

SOL 3.58 Option (B) is correct.

Given : $A = 25\text{ mm}^2$, $E_{steel} = 200\text{ GPa} = 200 \times 10^9\text{ N/m}^2 = 200 \times 10^3\text{ N/mm}^2$

First of all we have to make the F.B.D of the sections KL, LM and MN separately.



Now, From the F.B.D,

$$P_{KL} = 100\text{ N (Tensile)}$$
$$P_{LM} = -150\text{ N (Compressive)}$$
$$P_{MN} = 50\text{ N (Tensile)}$$

or $$L_{KL} = 500\text{ mm},\ L_{LM} = 800\text{ mm},\ L_{MN} = 400\text{ mm}$$

Total change in length,

$$\Delta L = \Delta L_{KL} + \Delta L_{LM} + \Delta L_{MN}$$
$$= \frac{P_{KL} L_{KL}}{AE} + \frac{P_{LM} L_{LM}}{AE} + \frac{P_{MN} L_{MN}}{AE} \qquad \Delta L = \frac{PL}{AE}$$

Substitute the values, we get

$$\Delta L = \frac{1}{25 \times 200 \times 10^3}[100 \times 500 - 150 \times 800 + 50 \times 400]$$
$$= \frac{1}{5000 \times 10^3}[-50000] = -10\ \mu\text{m}$$

SOL 3.59 Option (A) is correct.

Given : $d = 60\text{ mm}$, $T = 1600\text{ N–m}$

From the torsional formula,

$$\frac{T}{J} = \frac{\tau}{r} \qquad r = \frac{d}{2}\text{ and } J = \frac{\pi}{32}d^4$$

So, $$\tau_{\max} = \frac{T}{\frac{\pi}{32}d^4} \times \frac{d}{2} = \frac{16T}{\pi d^3}$$

Substitute the values, we get

$$\tau_{\max} = \frac{16 \times 1600}{3.14 \times (60 \times 10^{-3})^3} = \frac{8152.866}{(60)^3} \times 10^9$$
$$= 0.03774 \times 10^9\text{ Pa} = 37.74\text{ MPa} \simeq 37.72\text{ MPa}$$

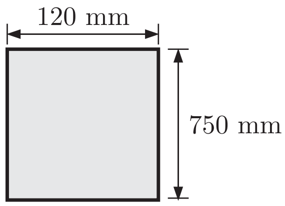
SOL 3.60 Option (A) is correct.

Given : $b = 120\text{ mm}$, $h = 750\text{ mm}$, $E_{steel} = 200\text{ GPa} = 200 \times 10^3\text{ N/mm}^2$, $W = 120\text{ kN/m}$, $L = 15\text{ m}$

It is a uniformly distributed load. For a uniformly distributed load, maximum bending moment at centre is given by,

$$B.M. = \frac{WL^2}{8} = \frac{120 \times 15 \times 15}{8} = 3375\text{ kN-m}$$

SOL 3.61 Option (A) is correct.



We know that maximum deflection at the centre of uniformly distributed load is given by,

$$\delta_{max} = \frac{5}{384} \times \frac{WL^4}{EI}$$

For rectangular cross-section,

$$I = \frac{bh^3}{12} = \frac{(120) \times (750)^3}{12} = 4.21875 \times 10^9 \text{ mm}^4 = 4.21875 \times 10^{-3} \text{ m}^4$$

So,
$$\delta_{max} = \frac{5}{384} \times \frac{120 \times 10^3 \times (15)^4}{200 \times 10^9 \times 4.21875 \times 10^{-3}}$$

$$= \frac{5}{384} \times 7200 \times 10^{-3} = 0.09375 \text{ m} = 93.75 \text{ mm}$$

SOL 3.62 Option (D) is correct.

We know that, moment of inertia is defined as the second moment of a plane area about an axis perpendicular to the area.

Polar moment of inertia perpendicular to the plane of paper,

$$J \text{ or } I_P = \frac{\pi D^4}{32}$$

By the "perpendicular axis" theorem,

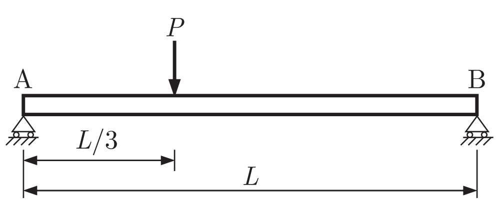
$$I_{XX} + I_{YY} = I_P \qquad \text{For circular section } I_{XX} = I_{YY}$$

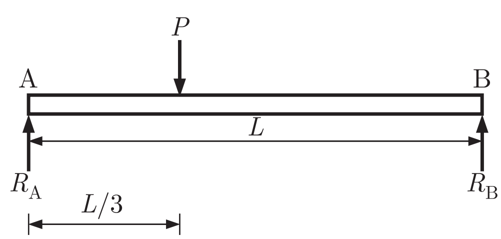
$$2I_{XX} = I_P$$

$$I_{XX} = \frac{I_P}{2} = \frac{\pi D^4}{64} = I_{YY}$$

SOL 3.63 Option (D) is correct.

We know that, the simplest form of the simply supported beams is the beam supported on rollers at ends. The simply supported beam and the *FBD* shown in the Figure.





Where, R_A and R_B are the reactions acting at the ends of the beam.
In equilibrium condition of forces,

$$P = R_A + R_B \quad \text{...(i)}$$

Taking the moment about point A,

$$R_B \times L = P \times \frac{L}{3}$$

$$R_B = \frac{P}{3}$$

From equation (i),

$$R_A = P - R_B = P - \frac{P}{3} = \frac{2P}{3}$$

Now bending moment at the point of application of the load

$$M = R_A \times \frac{L}{3} = \frac{2P}{3} \times \frac{L}{3} = \frac{2PL}{9}$$

Or, $$M = R_B \times \frac{2L}{3} = \frac{2PL}{9}$$

SOL 3.64 Option (C) is correct.
Given : $L_s = L_i$, $E_s = 206\,\text{GPa}$, $E_i = 100\,\text{GPa}$, $P_s = P_i$, $D_s = D_i$, $\Rightarrow A_s = A_i$
Where subscript s is for steel and i is for iron rod.
We know that elongation is given by,

$$\Delta L = \frac{PL}{AE}$$

Now, for steel or iron rod

$$\frac{\Delta L_s}{\Delta L_i} = \frac{P_s L_s}{A_s E_s} \times \frac{A_i E_i}{P_i L_i} = \frac{E_i}{E_s}$$

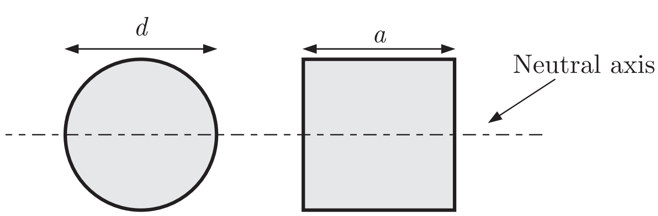
Substitute the values, we get

$$\frac{\Delta L_s}{\Delta L_i} = \frac{100}{206} = 0.485 < 1$$

or, $$\Delta L_s < \Delta L_i \quad \Rightarrow \quad \Delta L_i > \Delta L_s$$

So, cast iron rod elongates more than the mild steel rod.

SOL 3.65 Option (B) is correct.



Let, a = Side of square cross-section
d = diameter of circular cross-section
Using subscripts for the square and c for the circular cross section.

Given : $M_s = M_c;\ A_c = A_s$

So, $\frac{\pi}{4}d^2 = a^2$ ...(i)

From the bending equation,

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} \qquad \Rightarrow \qquad \sigma = \frac{M}{I} \times y$$

Where, y = Distance from the neutral axis to the external fibre.

σ = Bending stress

For square cross-section bending stress,

$$\sigma_s = \frac{M_s}{\frac{a^4}{12}} \times \frac{a}{2} = \frac{6M_s}{a^3} \qquad ...(ii)$$

And for circular cross-section,

$$\sigma_c = \frac{M_c}{\frac{\pi}{64}d^4} \times \frac{d}{2} = \frac{32M_c}{d^3} \qquad ...(iii)$$

On dividing equation (iii) by equation (ii), we get

$$\frac{\sigma_c}{\sigma_s} = \frac{32M_c}{d^3} \times \frac{a^3}{6M_s} = \frac{16}{3}\frac{a^3}{d^3} \qquad M_c = M_s \ ...(iv)$$

From equation (i),

$$\left(\frac{\pi}{4}d^2\right)^{3/2} = (a^2)^{3/2} = a^3$$

$$\frac{a^3}{d^3} = \left(\frac{\pi}{4}\right)^{3/2} = 0.695$$

Substitute this value in equation (iv), we get

$$\frac{\sigma_c}{\sigma_s} = \frac{16}{3} \times 0.695 = 3.706$$

$$\frac{\sigma_c}{\sigma_s} > 1 \qquad \Rightarrow \qquad \sigma_c > \sigma_s$$

So, Circular beam experience more bending stress than the square section.

SOL 3.66 Option (B) is correct.

Here k_1 and k_2 are in series combination and k_3 is in parallel combination with this series combination.

So, $$k_{eq} = \frac{k_1 \times k_2}{k_1 + k_2} + k_3 = \frac{k_1k_2 + k_2k_3 + k_1k_3}{k_1 + k_2}$$

Natural frequency of the torsional oscillation of the disc, $\omega_n = \sqrt{\frac{k_{eq}}{J}}$

Substitute the value of k_{eq}, we get $\omega_n = \sqrt{\frac{k_1k_2 + k_2k_3 + k_1k_3}{J(k_1 + k_2)}}$

SOL 3.67 Option (C) is correct.

Given : $\tau_1 = \tau_{max} = 240\,\text{MPa}$

Let, diameter of solid shaft $d_1 = d$, And Final diameter $d_2 = 2d$ (Given)

From the Torsional Formula,

$$\frac{T}{J} = \frac{\tau}{r} \quad \Rightarrow \quad T = \frac{\tau}{r} \times J$$

where, $J =$ polar moment of inertia. Given that torque is same,

$$\frac{\tau_1}{r_1} \times J_1 = \frac{\tau_2}{r_2} \times J_2$$

$$\frac{2\tau_1}{d_1} \times J_1 = \frac{2\tau_2}{d_2} \times J_2 \qquad J = \frac{\pi}{32} d^4$$

$$\frac{\tau_1}{d_1} \times \frac{\pi}{32} d_1^4 = \frac{\tau_2}{d_2} \times \frac{\pi}{32} d_2^4$$

$$\tau_1 \times d_1^3 = \tau_2 \times d_2^3 \quad \Rightarrow \quad \tau_2 = \tau_1 \times \frac{d_1^3}{d_2^3}$$

Substitute the values, we get

$$\tau_2 = 240 \times \left(\frac{d}{2d}\right)^3 = 240 \times \frac{1}{8} = 30\,\text{MPa}$$

Alternative Method :

From the Torsional Formula,

$$\tau = \frac{T\,r}{J} \qquad r = \frac{d}{2} \text{ and } J = \frac{\pi}{32} d^4$$

So, maximum shear stress,

$$\tau_{\max} = \frac{16T}{\pi d^3} = 240\ \text{MPa}$$

Given Torque is same and Shaft diameter is doubled then,

$$\tau'_{\max} = \frac{16T}{\pi (2d)^3} = \frac{16T}{8\pi d^3}$$

$$= \frac{\tau_{\max}}{8} = \frac{240}{8} = 30\,\text{MPa}$$

SOL 3.68 Option (A) is correct.

We know, differential equation of flexure for the beam is,

$$EI\frac{d^2y}{dx^2} = M \quad \Rightarrow \frac{d^2y}{dx^2} = \frac{M}{EI}$$

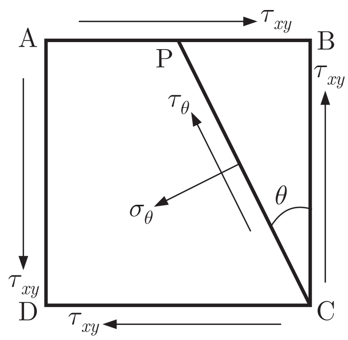
Integrating both sides, $\dfrac{dy}{dx} = \dfrac{1}{EI}\int M dx = \dfrac{1}{EI} Mx + c_1$

Again integrating, $y = \dfrac{1}{EI}\left(\dfrac{Mx^2}{2}\right) + c_1 x + c_2$ ...(i)

where, y gives the deflection at the given point. It is easily shown from the equation (i), If we increase the value of E and I, then deflection reduces.

SOL 3.69 Option (D) is correct.

Given figure shows stresses on an element subjected to pure shear.



Let consider a element to which shear stress have been applied to the sides AB and DC. Complementary stress of equal value but of opposite effect are then setup on sides AD and BC in order to prevent rotation of the element. So, applied and complementary shears are represented by symbol τ_{xy}.
Consider the equilibrium of portion PBC. Resolving normal to PC assuming unit depth.

$$\sigma_\theta \times PC = \tau_{xy} \times BC \sin\theta + \tau_{xy} \times PB \cos\theta$$

$$= \tau_{xy} \times PC \cos\theta + \tau_{xy} \times PC \sin\theta \cos\theta$$

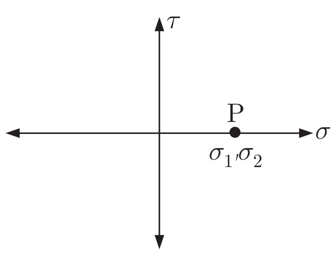
$$= \tau_{xy}(2\sin\theta\cos\theta) \times PC$$

$$\sigma_\theta = 2\tau_{xy}\sin\theta\cos\theta$$

The maximum value of σ_θ is τ_{xy} when $\theta = 45^\circ$.

$$\sigma_\theta = 2\tau\sin 45^\circ \cos 45^\circ \qquad \text{Given } (\tau_{xy} = \tau)$$

SOL 3.70 Option (B) is correct.



Given, Mohr's circle is a point located at 175 MPa on the positive Normal stress (at point P)

So, $\sigma_1 = \sigma_2 = 175$ MPa, and $\tau_{max} = 0$

So, both maximum and minimum principal stresses are equal.

Alternate Method :

$$\sigma_x = 175 \text{ MPa} \quad \sigma_y = 175 \text{ MPa and } \tau_{xy} = 0$$

Maximum principal stress

$$\sigma_1 = \frac{1}{2}\left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y) + 4\tau_{xy}^2}\right] = \frac{1}{2}[(175 + 175) + 0] = 175 \text{ MPa}$$

Minimum principal stress

$$\sigma_2 = \frac{1}{2}\left[(\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y) + 4\tau_{xy}^2}\right] = \frac{1}{2}\left[(175 + 175) - 0\right] = 175\,\text{MPa}$$

SOL 3.71 Option (D) is correct.

Mohr's circle is a point, and a point will move in every direction. So, the directions of maximum and minimum principal stresses at point P is in all directions.

Every value of θ will give the same result of $175\,\text{MPa}$ in all directions.

SOL 3.72 Option (D) is correct.

Mild steel is ductile in nature and it elongates appreciable before fracture.

The stress-strain curve of a specimen tested upto failure under tension is a measure of toughness.

SOL 3.73 Option (C) is correct.

3 dimensional stress tensor is defined as

$$z_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

There are 9 components of the stress tensor. But due to complementary nature of shear stresses,

$$\tau_{xy} = \tau_{yx},\ \tau_{xz} = \tau_{zx} \text{ and } \tau_{yz} = \tau_{zy}$$

So, we can say that the number of components in a stress tensor for defining stress at a point is 6 i.e. σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{zx}.

SOL 3.74 Option (A) is correct.

We know the volumetric strain is, $\varepsilon_\nu = \dfrac{(1-2\upsilon)}{E}(\sigma_1 + \sigma_2 + \sigma_3)$

Put $\sigma_1 = \sigma_2 = \sigma_3 = -\sigma$,

$$\varepsilon_\nu = \frac{1-2\upsilon}{E}(-3\sigma) = \frac{3(1-2\upsilon)}{E}\sigma \qquad \text{(in magnitude)}$$

The above equation gives the volumetric strain when the elemental volume is subjected to a compressive stress of σ from all sides. Negative sign indicates a compressive volumetric strain.

So, $$\frac{\varepsilon_\nu}{\sigma} = \frac{3(1-2\upsilon)}{E} \quad \Rightarrow \quad \frac{\sigma}{\varepsilon_\nu} = \frac{E}{3(1-2\upsilon)}$$

But $$\frac{\sigma}{\varepsilon_\nu} = K \text{ (Bulk modulus)}$$

Hence, $$E = 3K(1-2\upsilon)$$

SOL 3.75 Option (B) is correct.

According to Euler's theory, the crippling or buckling load (W_{cr}) under various end conditions is given by,

$$W_{cr} = \frac{C\pi^2 EA}{L^2}$$

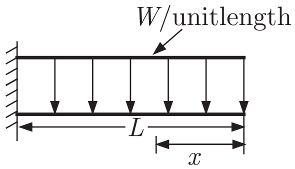
Where $C =$ constant, representing the end conditions of the column.

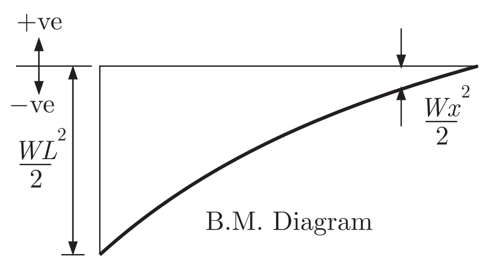
All parameters are same. So, $W_{cr} \propto C$

(i) For both ends fixed, $C = 4$

(ii) For both ends hinged, $C = 1$, so, $\frac{W_{(i)}}{W(ii)} = \frac{4}{1} = 4$

SOL 3.76 Option (D) is correct.

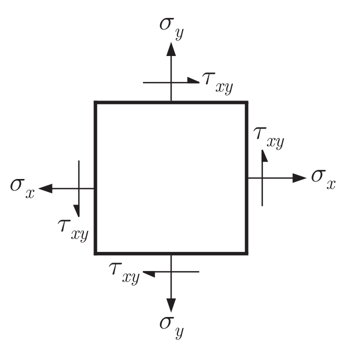




$$M_x = -Wx \times \frac{x}{2} = -\frac{Wx^2}{2}$$

The equation for M_x gives parabolic variations for $B.M.$ Maximum $B.M.$ occurs at $x = L$ and is equal to $WL^2/2$. (in magnitude)

SOL 3.77 Option (B) is correct.



For stress state the maximum principal stress is given by,

$$\sigma_1 = \frac{1}{2}\left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]$$

Here $\sigma_x = \sigma$, $\sigma_y = \sigma$ and $z_{xy} = \sigma$

Hence, $$\sigma_1 = \frac{1}{2}\left[(\sigma + \sigma) + \sqrt{0 + 4\sigma^2}\right] = \frac{1}{2}[2\sigma + 2\sigma] = 2\sigma$$

****

CHAPTER 4

THEORY OF MACHINES

YEAR 2012 ONE MARK

MCQ 4.1 The following are the data for two crossed helical gears used for speed reduction :

Gear I : Pitch circle diameter in the plane of rotation 80 mm and helix angle 30°.

Gear II : Pitch circle diameter in the plane of rotation 120 mm and helix angle 22.5°.

If the input speed is 1440 rpm, the output speed in rpm is

(A) 1200 (B) 900

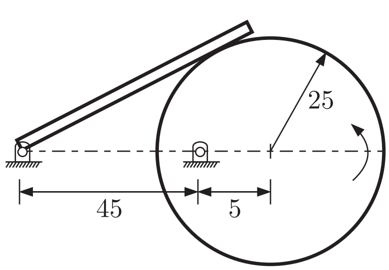
(C) 875 (D) 720

MCQ 4.2 A solid disc of radius r rolls without slipping on a horizontal floor with angular velocity ω and angular acceleration α. The magnitude of the acceleration of the point of contact on the disc is

(A) zero (B) $r\alpha$

(C) $\sqrt{(r\alpha)^2 + (r\omega^2)^2}$ (D) $r\omega^2$

MCQ 4.3 In the mechanism given below, if the angular velocity of the eccentric circular disc is 1 rad/s, the angular velocity (rad/s) of the follower link for the instant shown in the figure is (Note. All dimensions are in mm).



(A) 0.05 (B) 0.1

(C) 5.0 (D) 10.0

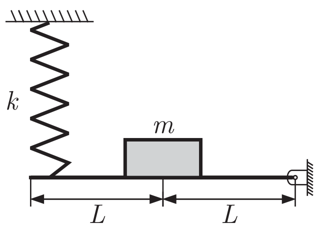
MCQ 4.4 A circular solid disc of uniform thickness 20 mm, radius 200 mm and mass 20 kg, is used as a flywheel. If it rotates at 600 rpm, the kinetic energy of the flywheel, in Joules is

(A) 395 (B) 790

(C) 1580 (D) 3160

YEAR 2012 TWO MARKS

MCQ 4.5 A concentrated mass m is attached at the centre of a rod of length $2L$ as shown in the figure. The rod is kept in a horizontal equilibrium position by a spring of stiffness k. For very small amplitude of vibration, neglecting the weights of the rod and spring, the undamped natural frequency of the system is

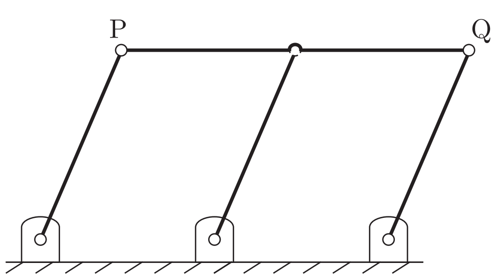


(A) $\sqrt{\frac{k}{m}}$ (B) $\sqrt{\frac{2k}{m}}$

(C) $\sqrt{\frac{k}{2m}}$ (D) $\sqrt{\frac{4k}{m}}$

YEAR 2011 ONE MARK

MCQ 4.6 A double-parallelogram mechanism is shown in the figure. Note that PQ is a single link. The mobility of the mechanism is

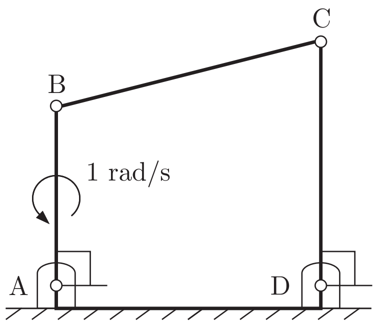


(A) -1 (B) 0

(C) 1 (D) 2

YEAR 2011 TWO MARKS

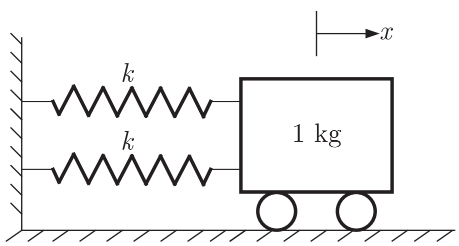
MCQ 4.7 For the four-bar linkage shown in the figure, the angular velocity of link AB is 1 rad/s. The length of link CD is 1.5 times the length of link AB. In the configuration shown, the angular velocity of link CD in rad/s is



(A) 3 (B) $\frac{3}{2}$

(C) 1 (D) $\frac{2}{3}$

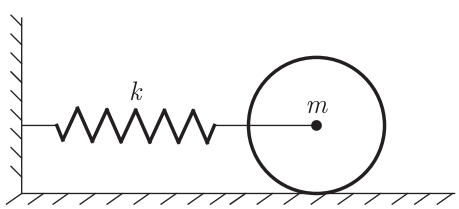
MCQ 4.8 A mass of 1 kg is attached to two identical springs each with stiffness $k = 20$ kN/m as shown in the figure. Under the frictionless conditions, the natural frequency of the system in Hz is close to



(A) 32 (B) 23

(C) 16 (D) 11

MCQ 4.9 A disc of mass m is attached to a spring of stiffness k as shown in the figure The disc rolls without slipping on a horizontal surface. The natural frequency of vibration of the system is



(A) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{2k}{3m}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{3k}{2m}}$

YEAR 2010 ONE MARK

MCQ 4.10 Mobility of a statically indeterminate structure is

(A) ≤ -1 (B) 0

(C) 1 (D) ≥ 2

MCQ 4.11 There are two points P and Q on a planar rigid body. The relative velocity between the two points

(A) should always be along PQ

(B) can be oriented along any direction

(C) should always be perpendicular to PQ

(D) should be along QP when the body undergoes pure translation

MCQ 4.12 Which of the following statements is INCORRECT ?

(A) Grashof's rule states that for a planar crank-rocker four bar mechanism, the sum of the shortest and longest link lengths cannot be less than the sum of the remaining two link lengths

(B) Inversions of a mechanism are created by fixing different links one at a time

(C) Geneva mechanism is an intermittent motion device

(D) Gruebler's criterion assumes mobility of a planar mechanism to be one

MCQ 4.13 The natural frequency of a spring-mass system on earth is ω_n. The natural frequency of this system on the moon ($g_{\text{moon}} = g_{\text{earth}}/6$) is

(A) ω_n

(B) $0.408\omega_n$

(C) $0.204\omega_n$

(D) $0.167\omega_n$

MCQ 4.14 Tooth interference in an external involute spur gear pair can be reduced by

(A) decreasing center distance between gear pair

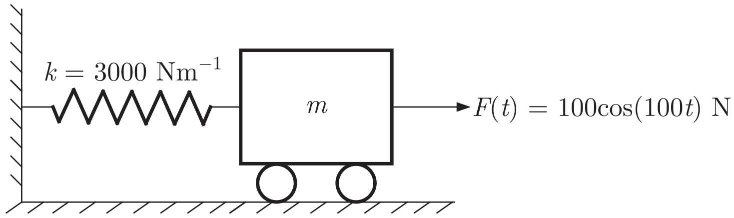
(B) decreasing module

(C) decreasing pressure angle

(D) increasing number of gear teeth

YEAR 2010 TWO MARKS

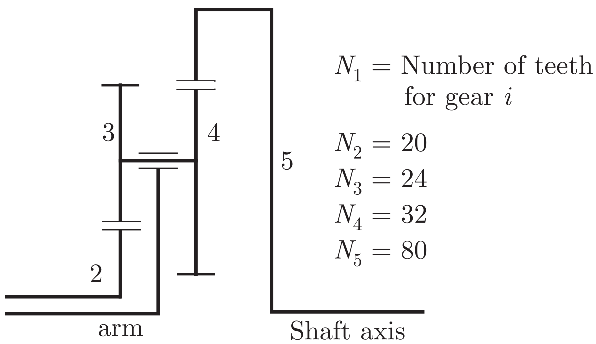
MCQ 4.15 A mass m attached to a spring is subjected to a harmonic force as shown in figure The amplitude of the forced motion is observed to be 50 mm. The value of m (in kg) is



(A) 0.1 (B) 1.0

(C) 0.3 (D) 0.5

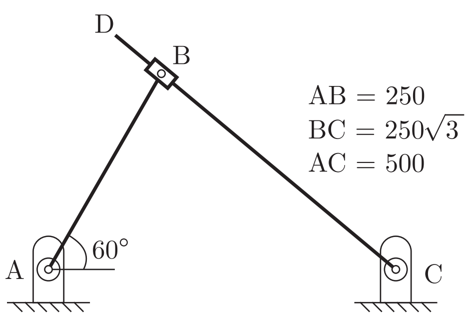
MCQ 4.16 For the epicyclic gear arrangement shown in the figure $\omega_2 = 100$ rad/s clockwise (CW) and $\omega_{arm} = 80$ rad/s counter clockwise (CCW). The angular velocity ω_5 (in rad/s) is



(A) 0 (B) 70 CW

(B) 140 CCW (D) 140 CW

MCQ 4.17 For the configuration shown, the angular velocity of link AB is 10 rad/s counterclockwise. The magnitude of the relative sliding velocity (in ms^{-1}) of slider B with respect to rigid link CD is

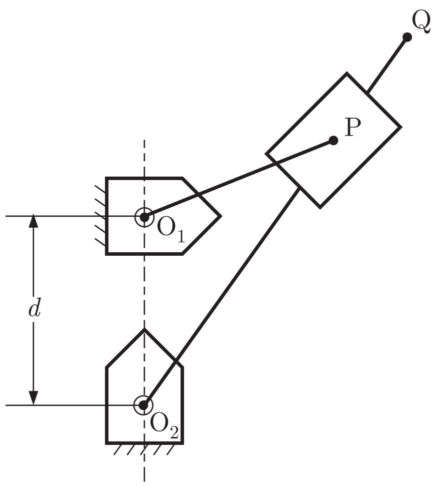


(A) 0 (B) 0.86

(C) 1.25 (D) 2.50

YEAR 2009 ONE MARK

MCQ 4.18 A simple quick return mechanism is shown in the figure. The forward to return ratio of the quick return mechanism is 2:1. If the radius of crank O_1P is 125 mm, then the distance 'd' (in mm) between the crank centre to lever pivot centre point should be



(A) 144.3 (B) 216.5

(C) 240.0 (D) 250.0

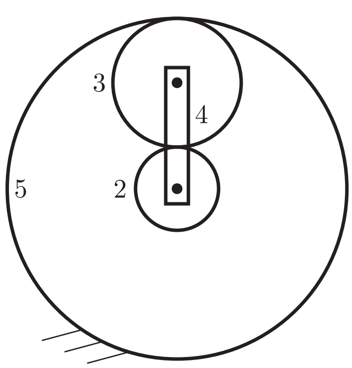
MCQ 4.19 The rotor shaft of a large electric motor supported between short bearings at both the ends shows a deflection of 1.8 mm in the middle of the rotor. Assuming the rotor to be perfectly balanced and supported at knife edges at both the ends, the likely critical speed (in rpm) of the shaft is

(A) 350 (B) 705

(C) 2810 (D) 4430

YEAR 2009 TWO MARKS

MCQ 4.20 An epicyclic gear train in shown schematically in the given figure. The run gear 2 on the input shaft is a 20 teeth external gear. The planet gear 3 is a 40 teeth external gear. The ring gear 5 is a 100 teeth internal gear. The ring gear 5 is fixed and the gear 2 is rotating at 60 rpm CCW (CCW=counter-clockwise and CW=clockwise).



The arm 4 attached to the output shaft will rotate at

(A) 10 rpm CCW (B) 10 rpm CW

(C) 12 rpm CW (D) 12 rpm CCW

MCQ 4.21 An automotive engine weighing $240\,\text{kg}$ is supported on four springs with linear characteristics. Each of the front two springs have a stiffness of $16\,\text{MN/m}$ while the stiffness of each rear spring is $32\,\text{MN/m}$. The engine speed (in rpm), at which resonance is likely to occur, is

(A) 6040 (B) 3020

(C) 1424 (D) 955

MCQ 4.22 A vehicle suspension system consists of a spring and a damper. The stiffness of the spring is $3.6\,\text{kN/m}$ and the damping constant of the damper is $400\,\text{Ns/m}$. If the mass is $50\,\text{kg}$, then the damping factor (d) and damped natural frequency (f_n), respectively, are

(A) 0.471 and 1.19 Hz (B) 0.471 and 7.48 Hz

(C) 0.666 and 1.35 Hz (D) 0.666 and 8.50 Hz

MCQ 4.23 Match the approaches given below to perform stated kinematics/dynamics analysis of machine.

	Analysis		Approach
P.	Continuous relative rotation	1.	D' Alembert's principle
Q.	Velocity and acceleration	2.	Grubler's criterion
R.	Mobility	3.	Grashoff's law
S.	Dynamic-static analysis	4.	Kennedy's theorem

(A) P-1, Q-2, R-3, S-4 (B) P-3, Q-4, R-2, S-1

(C) P-2, Q-3, R-4, S-1 (D) P-4, Q-2, R-1, S-3

YEAR 2008 ONE MARK

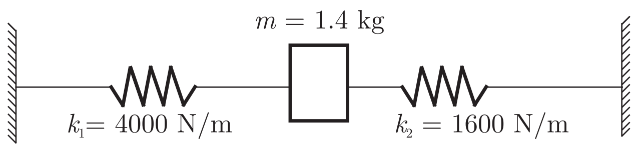
MCQ 4.24 A planar mechanism has 8 links and 10 rotary joints. The number of degrees of freedom of the mechanism, using Gruebler's criterion, is

(A) 0 (B) 1

(C) 2 (D) 3

YEAR 2008 TWO MARKS

MCQ 4.25 The natural frequency of the spring mass system shown in the figure is closest to



(A) 8 Hz (B) 10 Hz

(C) 12 Hz (D) 14 Hz

MCQ 4.26 In a cam design, the rise motion is given by a simple harmonic motion $(SHM)\, s = h/2(1-\cos(\pi\theta/\beta))$ where h is total rise, θ is camshaft angle, β is the total angle of the rise interval . The jerk is given by

(A) $\frac{h}{2}\left(1-\cos\frac{\pi\theta}{\beta}\right)$ (B) $\frac{\pi}{\beta}\frac{h}{2}\sin\left(\frac{\pi\theta}{\beta}\right)$

(C) $\frac{\pi^2}{\beta^2}\frac{h}{2}\cos\left(\frac{\pi\theta}{\beta}\right)$ (D) $-\frac{\pi^3}{\beta^3}\frac{h}{2}\sin\left(\frac{\pi\theta}{\beta}\right)$

MCQ 4.27 A uniform rigid rod of mass $m = 1$ kg and length $L = 1$ m is hinged at its centre and laterally supported at one end by a spring of spring constant $k = 300$ N/m. The natural frequency ω_n in rad/s is

(A) 10 (B) 20

(C) 30 (D) 40

YEAR 2007 ONE MARK

MCQ 4.28 For an under damped harmonic oscillator, resonance

(A) occurs when excitation frequency is greater than undamped natural frequency

(B) occurs when excitation frequency is less than undamped natural frequency

(C) occurs when excitation frequency is equal to undamped natural frequency

(D) never occurs

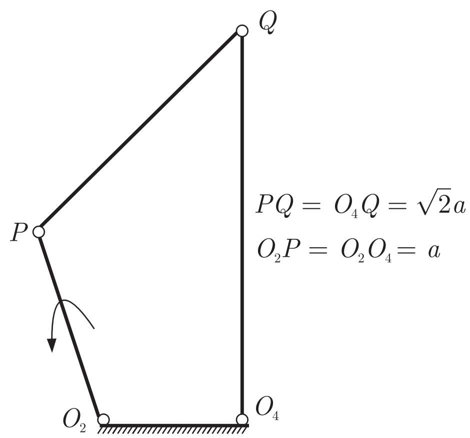
YEAR 2007 TWO MARKS

MCQ 4.29 The speed of an engine varies from 210 rad/s to 190 rad/s. During the cycle the change in kinetic energy is found to be 400 Nm. The inertia of the flywheel in kg/m^2 is

(A) 0.10 (B) 0.20

(C) 0.30 (D) 0.40

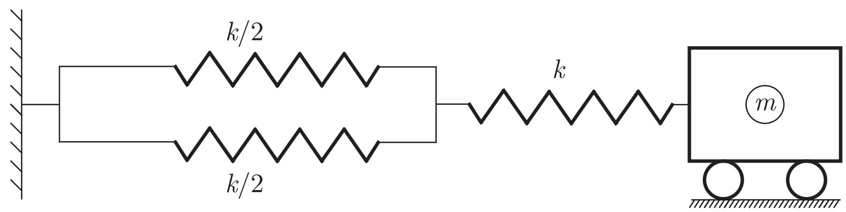
MCQ 4.30 The input link O_2P of a four bar linkage is rotated at 2 rad/s in counter clockwise direction as shown below. The angular velocity of the coupler PQ in rad/s, at an instant when $\angle O_4O_2P = 180^\circ$, is



(A) 4 (B) $2\sqrt{2}$

(C)1 (D) $\frac{1}{\sqrt{2}}$

MCQ 4.31 The natural frequency of the system shown below is



(A) $\sqrt{\frac{k}{2m}}$ (B) $\sqrt{\frac{k}{m}}$

(C) $\sqrt{\frac{2k}{m}}$ (D) $\sqrt{\frac{3k}{m}}$

MCQ 4.32 The equation of motion of a harmonic oscillator is given by

$$\frac{d^2x}{dt^2} + 2\xi\omega_n\frac{dx}{dt} + \omega_n^2 x = 0$$

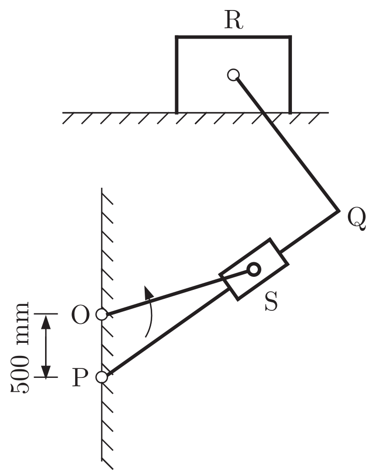
and the initial conditions at $t = 0$ are $x(0) = X, \frac{dx}{dt}(0) = 0$. The amplitude of $x(t)$ after n complete cycles is

(A) $Xe^{-2n\pi\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)}$ (B) $Xe^{2n\pi\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)}$

(C) $Xe^{-2n\pi\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)}$ (D) X

- Common Data For Q.33 Q. 34

A quick return mechanism is shown below. The crank OS is driven at 2 rev/s in counter-clockwise direction.



MCQ 4.33 If the quick return ratio is 1 : 2, then the length of the crank in mm is

(A) 250 (B) $250\sqrt{3}$

(C) 500 (D) $500\sqrt{3}$

MCQ 4.34 The angular speed of PQ in rev/s when the block R attains maximum speed during forward stroke (stroke with slower speed) is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) 2 (D) 3

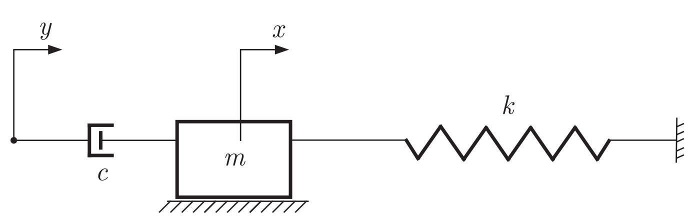
YEAR 2006 ONE MARK

MCQ 4.35 For a four-bar linkage in toggle position, the value of mechanical advantage is

(A) 0.0 (B) 0.5

(C) 1.0 (D) ∞

MCQ 4.36 The differential equation governing the vibrating system is



(A) $m\ddot{x} + c\dot{x} + k(x - y) = 0$

(B) $m(\ddot{x} - \ddot{y}) + c(\dot{x} - \dot{y}) + kx = 0$

(C) $m\ddot{x} + c(\dot{x} - \dot{y}) + kx = 0$

(D) $m(\ddot{x} - \ddot{y}) + c(\dot{x} - \dot{y}) + k(x - y) = 0$

MCQ 4.37 The number of inversion for a slider crank mechanism is

(A) 6 (B) 5

(C) 4 (D) 3

YEAR 2006 TWO MARKS

MCQ 4.38 Match the item in columns I and II

	Column I		Column II
P.	Addendum	1.	Cam
Q.	Instantaneous centre of velocity	2.	Beam
R.	Section modulus	3.	Linkage
S.	Prime circle	4.	Gear

(A) P-4, Q-2, R-3, S-1 (B) P-4, Q-3, R-2, S-1

(C) P-3, Q-2, R-1, S-4 (D) P-3, Q-4, R-1, S-2

MCQ 4.39 If C_f is the coefficient of speed fluctuation of a flywheel then the ratio of $\omega_{max}/\omega_{min}$ will be

(A) $\frac{1-2C_f}{1+2C_f}$ (B) $\frac{2-C_f}{2+C_f}$

(C) $\frac{1+2C_f}{1-2C_f}$ (D) $\frac{2+C_f}{2-C_f}$

MCQ 4.40 Match the items in columns I and II

	Column I		ColumnII
P.	Higher Kinematic Pair	1.	Grubler's Equation
Q.	Lower Kinemation Pair	2.	Line contact
R.	Quick Return Mechanism	3.	Euler's Equation
S.	Mobility of a Linkage	4.	Planar
		5.	Shaper
		6.	Surface contact

(A) P-2, Q-6, R-4, S-3 (B) P-6, Q-2, R-4, S-1

(C) P-6, Q-2, R-5, S-3 (D) P-2, Q-6, R-5, S-1

MCQ 4.41 A machine of 250 kg mass is supported on springs of total stiffness 100 kN/m . Machine has an unbalanced rotating force of 350 N at speed of 3600 rpm . Assuming a damping factor of 0.15, the value of transmissibility ratio is

(A) 0.0531 (B) 0.9922

(C) 0.0162 (D) 0.0028

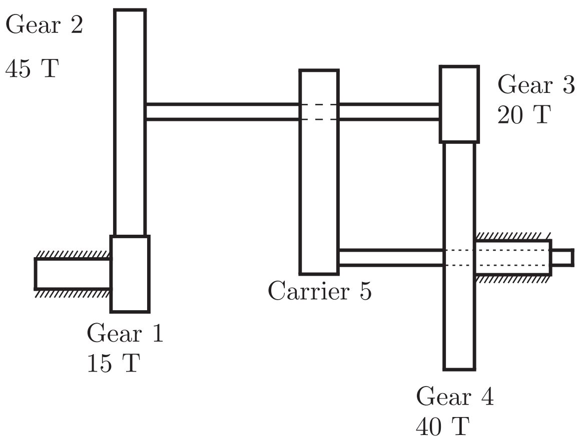
MCQ 4.42 In a four-bar linkage, S denotes the shortest link length, L is the longest link length, P and Q are the lengths of other two links. At least one of the three moving links will rotate by $360°$ if

(A) $S + L \le P + Q$ (B) $S + L > P + Q$

(C) $S + P \le L + Q$ (D) $S + P > L + Q$

• Common Data For Q. 43 and Q. 44

A planetary gear train has four gears and one carrier. Angular velocities of the gears are $\omega_1, \omega_2, \omega_3$ and ω_4, respectively. The carrier rotates with angular velocity ω_5.



MCQ 4.43 What is the relation between the angular velocities of Gear 1 and Gear 4 ?

(A) $\dfrac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = 6$ (B) $\dfrac{\omega_4 - \omega_5}{\omega_1 - \omega_5} = 6$

(C) $\dfrac{\omega_1 - \omega_2}{\omega_4 - \omega_5} = -\left(\dfrac{2}{3}\right)$ (D) $\dfrac{\omega_2 - \omega_5}{\omega_4 - \omega_5} = \dfrac{8}{9}$

MCQ 4.44 For $\omega_1 = 60$ rpm clockwise (CW) when looked from the left, what is the angular velocity of the carrier and its direction so that Gear 4 rotates in counterclockwise (CCW)direction at twice the angular velocity of Gear 1 when looked from the left ?

(A) 130 rpm, CW (B) 223 rpm, CCW

(C) 256 rpm, CW (D) 156 rpm, CCW

• Common Data For Q. 45 and Q. 46 :

A vibratory system consists of a mass 12.5 kg, a spring of stiffness 1000 N/m , and a dash-pot with damping coefficient of 15 Ns/m.

MCQ 4.45 The value of critical damping of the system is

(A) 0.223 Ns/m (B) 17.88 Ns/m

(C) 71.4 Ns/m (D) 223.6 Ns/m

MCQ 4.46 The value of logarithmic decrement is
(A) 1.35 (B) 1.32
(C) 0.68 (D) 0.66

YEAR 2005 ONE MARK

MCQ 4.47 The number of degrees of freedom of a planar linkage with 8 links and 9 simple revolute joints is
(A) 1 (B) 2
(C) 3 (D) 4

MCQ 4.48 There are four samples P, Q, R and S, with natural frequencies 64, 96, 128 and 256 Hz, respectively. They are mounted on test setups for conducting vibration experiments. If a loud pure note of frequency 144 Hz is produced by some instrument, which of the samples will show the most perceptible induced vibration?
(A) P (B) Q
(C) R (D) S

YEAR 2005 TWO MARKS

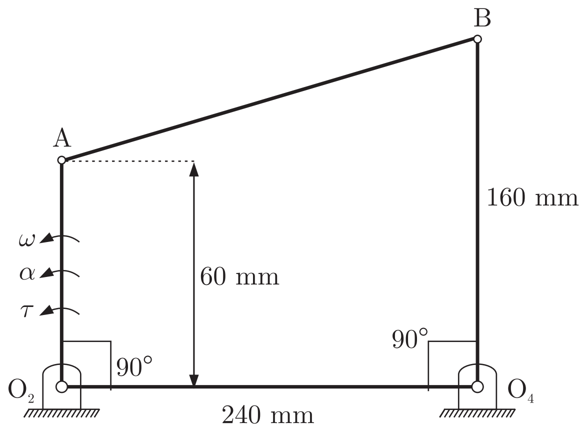
MCQ 4.49 In a cam-follower mechanism, the follower needs to rise through 20 mm during 60° of cam rotation, the first 30° with a constant acceleration and then with a deceleration of the same magnitude. The initial and final speeds of the follower are zero. The cam rotates at a uniform speed of 300 rpm. The maximum speed of the follower is
(A) 0.60 m/s (B) 1.20 m/s
(C) 1.68 m/s (D) 2.40 m/s

MCQ 4.50 A rotating disc of 1 m diameter has two eccentric masses of 0.5 kg each at radii of 50 mm and 60 mm at angular positions of 0° and 150°, respectively. A balancing mass of 0.1 kg is to be used to balance the rotor. What is the radial position of the balancing mass ?
(A) 50 mm (B) 120 mm
(C) 150 mm (D) 280 mm

MCQ 4.51 In a spring-mass system, the mass is 0.1 kg and the stiffness of the spring is 1 kN/m. By introducing a damper, the frequency of oscillation is found to be 90% of the original value. What is the damping coefficient of the damper ?
(A) 1.2 Ns/m (B) 3.4 Ns/m
(C) 8.7 Ns/m (D) 12.0 Ns/m

• Common Data For Q. 52, 53, and Q. 54

An instantaneous configuration of a four-bar mechanism, whose plane is horizontal is shown in the figure below. At this instant, the angular velocity and angular acceleration of link O_2 A are $\omega = 8$ rad/s and $\alpha = 0$, respectively, and the driving torque (τ) is zero. The link O_2 A is balanced so that its centre of mass falls at O_2.



MCQ 4.52 Which kind of 4-bar mechanism is O_2ABO_4 ?

(A) Double-crank mechanism
(B) Crank-rocker mechanism
(C) Double-rocker mechanism
(D) Parallelogram mechanism

MCQ 4.53 At the instant considered, what is the magnitude of the angular velocity of O_4B?

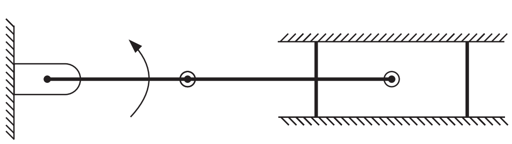
(A) 1 rad/s
(B) 3 rad/s
(C) 8 rad/s
(D) $\frac{64}{3}$ rad/s

MCQ 4.54 At the same instant, if the component of the force at joint A along AB is 30 N, then the magnitude of the joint reaction at O_2

(A) is zero
(B) is 30 N
(C) is 78 N
(D) cannot be determined from the given data

YEAR 2004 ONE MARK

MCQ 4.55 For a mechanism shown below, the mechanical advantage for the given configuration is



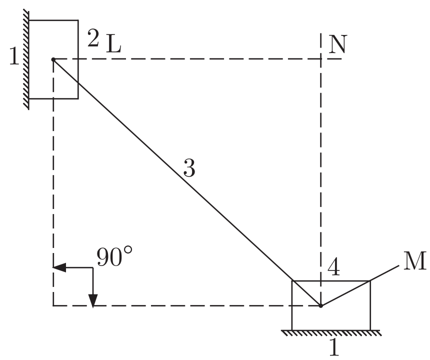
(A) 0
(B) 0.5
(C) 1.0
(D) ∞

MCQ 4.56 A vibrating machine is isolated from the floor using springs. If the ratio of excitation frequency of vibration of machine to the natural frequency of the isolation system is equal to 0.5, then transmissibility ratio of isolation is

(A) 1/2
(B) 3/4
(C) 4/3
(D) 2

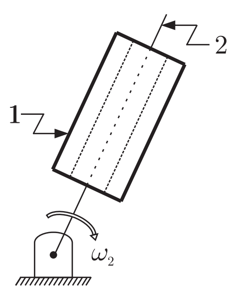
YEAR 2004 TWO MARKS

MCQ 4.57 The figure below shows a planar mechanism with single degree of freedom. The instant centre 24 for the given configuration is located at a position



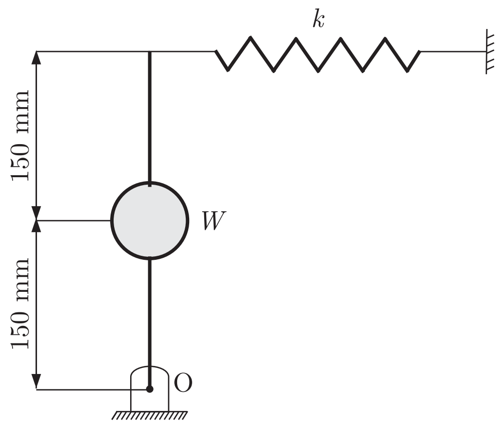
(A) L
(B) M
(C) N
(D) ∞

MCQ 4.58 In the figure shown, the relative velocity of link 1 with respect to link 2 is 12 m/sec. Link 2 rotates at a constant speed of 120 rpm. The magnitude of Coriolis component of acceleration of link 1 is



(A) 302 m/s^2
(B) 604 m/s^2
(C) 906 m/s^2
(D) 1208 m/s^2

MCQ 4.59 A uniform stiff rod of length 300 mm and having a weight of 300 N is pivoted at one end and connected to a spring at the other end. For keeping the rod vertical in a stable position the minimum value of spring constant k needed is



(A) 300 N/m (B) 400 N/m

(C) 500 N/m (D) 1000 N/m

MCQ 4.60 Match the following

	Type of Mechanism		Motion achieved
P.	Scott-Russel Mechanism	1.	Intermittent Motion
Q.	Geneva Mechanism	2.	Quick return Motion
R.	Off-set slider-crank Mechanism	3.	Simple Harmonic Motion
S.	Scotch Yoke Mechanism	4.	Straight Line Motion

(A) P-2 Q-3 R-1 S-4 (B) P-3 Q-2 R-4 S-1

(C) P-4 Q-1 R-2 S-3 (D) P-4 Q-3 R-1 S-2

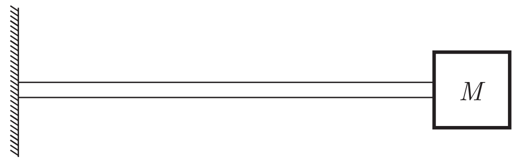
MCQ 4.61 Match the following with respect to spatial mechanisms.

	Types of Joint		Degree of constraints
P.	Revolute	1.	Three
Q.	Cylindrical	2.	Five
R.	Spherical	3.	Four
		4.	Two
		5.	Zero

(A) P-1 Q-3 R-3 (B) P-5 Q-4 R-3

(C) P-2 Q-3 R-1 (D) P-4 Q-5 R-3

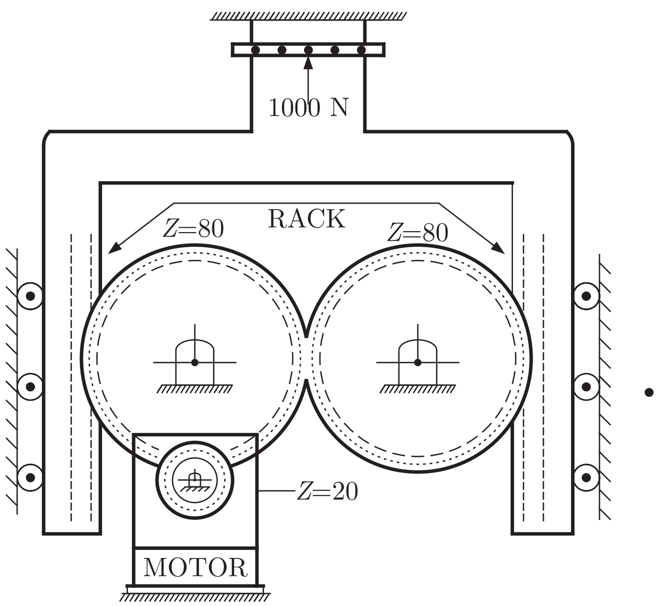
MCQ 4.62 A mass M, of 20 kg is attached to the free end of a steel cantilever beam of length 1000 mm having a cross-section of 25×25 mm. Assume the mass of the cantilever to be negligible and $E_{steel} = 200$ GPa. If the lateral vibration of this system is critically damped using a viscous damper, then damping constant of the damper is



(A) 1250 Ns/m
(B) 625 Ns/m
(C) 312.50 Ns/m
(D) 156.25 Ns/m

• Common Data For Q. 63 and Q. 64

A compacting machine shown in the figure below is used to create a desired thrust force by using a rack and pinion arrangement. The input gear is mounted on the motor shaft. The gears have involute teeth of 2 mm module.



MCQ 4.63 If the drive efficiency is 80%, the torque required on the input shaft to create 1000 N output thrust is

(A) 20 Nm
(B) 25 Nm
(C) 32 Nm
(D) 50 Nm

MCQ 4.64 If the pressure angle of the rack is 20°, then force acting along the line of action between the rack and the gear teeth is

(A) 250 N
(B) 342 N
(C) 532 N
(D) 600 N

YEAR 2003 ONE MARK

MCQ 4.65 The mechanism used in a shaping machine is
(A) a closed 4-bar chain having 4 revolute pairs
(B) a closed 6-bar chain having 6 revolute pairs
(C) a closed 4-bar chain having 2 revolute and 2 sliding pairs
(D) an inversion of the single slider-crank chain

MCQ 4.66 The lengths of the links of a 4-bar linkage with revolute pairs are p, q, r, and s units. given that $p < q < r < s$. Which of these links should be the fixed one, for obtaining a "double crank" mechanism ?
(A) link of length p
(B) link of length q
(C) link of length r
(D) link of length s

MCQ 4.67 When a cylinder is located in a Vee-block, the number of degrees of freedom which are arrested is
(A) 2
(B) 4
(C) 7
(D) 8

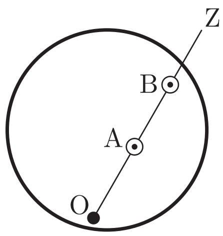
YEAR 2003 TWO MARKS

MCQ 4.68 For a certain engine having an average speed of 1200 rpm, a flywheel approximated as a solid disc, is required for keeping the fluctuation of speed within 2% about the average speed. The fluctuation of kinetic energy per cycle is found to be 2 kJ. What is the least possible mass of the flywheel if its diameter is not to exceed 1 m ?
(A) 40 kg
(B) 51 kg
(C) 62 kg
(D) 73 kg

MCQ 4.69 A flexible rotor-shaft system comprises of a 10 kg rotor disc placed in the middle of a mass-less shaft of diameter 30 mm and length 500 mm between bearings (shaft is being taken mass-less as the equivalent mass of the shaft is included in the rotor mass) mounted at the ends. The bearings are assumed to simulate simply supported boundary conditions. The shaft is made of steel for which the value of E 2.1×10^{11} Pa. What is the critical speed of rotation of the shaft ?
(A) 60 Hz
(B) 90 Hz
(C) 135 Hz
(D) 180 Hz

• Common Data For Q. 70 and Q. 71 :

The circular disc shown in its plan view in the figure rotates in a plane parallel to the horizontal plane about the point O at a uniform angular velocity ω. Two other points A and B are located on the line OZ at distances r_A and r_B from O respectively.



MCQ 4.70 The velocity of Point B with respect to point A is a vector of magnitude

(A) 0

(B) $\omega(r_B - r_A)$ and direction opposite to the direction of motion of point B

(C) $\omega(r_B - r_A)$ and direction same as the direction of motion of point B

(D) $\omega(r_B - r_A)$ and direction being from O to Z

MCQ 4.71 The acceleration of point B with respect to point A is a vector of magnitude

(A) 0

(B) $\omega(r_B - r_A)$ and direction same as the direction of motion of point B

(C) $\omega^2(r_B - r_A)$ and direction opposite to be direction of motion of point B

(D) $\omega^2(r_B - r_A)$ and direction being from Z to O

MCQ 4.72 The undamped natural frequency of oscillations of the bar about the hinge point is

(A) 42.43 rad/s (B) 30 rad/s

(C) 17.32 rad/s (D) 14.14 rad/s

MCQ 4.73 The damping coefficient in the vibration equation is given by

(A) 500 Nms/rad (B) 500 N/(m/s)

(C) 80 Nms/rad (D) 80 N/(m/s)

YEAR 2002 ONE MARK

MCQ 4.74 The minimum number of links in a single degree-of-freedom planar mechanism with both higher and lower kinematic pairs is

(A) 2 (B) 3

(C) 4 (D) 5

MCQ 4.75 The Coriolis component of acceleration is present in

(A) 4 bar mechanisms with 4 turning pairs

(B) shape mechanism

(C) slider-crank mechanism (D) scotch yoke mechanism

YEAR 2002 TWO MARKS

MCQ 4.76 If the length of the cantilever beam is halved, the natural frequency of the mass M at the end of this cantilever beam of negligible mass is increased by a factor of

(A) 2 (B) 4

(C) $\sqrt{8}$ (D) 8

YEAR 2001 ONE MARK

MCQ 4.77 For a spring-loaded roller follower driven with a disc cam,

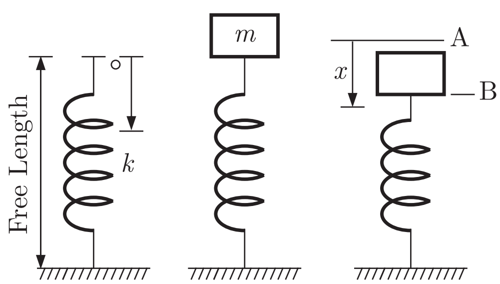
(A) the pressure angle should be larger during rise than that during return for ease of transmitting motion.

(B) the pressure angle should be smaller during rise than that during return for ease of transmitting motion.

(C) the pressure angle should be large during rise as well as during return for ease of transmitting motion.

(D) the pressure angle does not affect the ease of transmitting motion.

MCQ 4.78 In the figure shown, the spring deflects by δ to position A (the equilibrium position) when a mass m is kept on it. During free vibration, the mass is at position B at some instant. The charge in potential energy of the spring mass system from position A to position B is



(A) $\frac{1}{2}kx^2$ (B) $\frac{1}{2}kx^2 - mgx$

(C) $\frac{1}{2}k(x+\delta)^2$ (D) $\frac{1}{2}kx^2 + mgx$

MCQ 4.79 Which of the following statements is correct ?

(A) Flywheel reduces speed fluctuations during a cycle for a constant load, but flywheel does not control the mean speed of the engine, if the load changes.

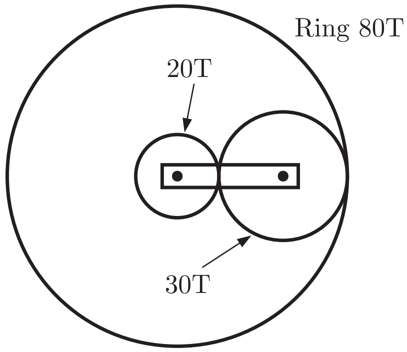
(B) Flywheel does not reduce speed fluctuation during a cycle for a constant load, but flywheel does not control the mean speed of the engine, if the load changes.

(C) Governor controls speed fluctuations during a cycle for a constant load, but governor does not control the mean speed of the engine, if the load changes.

(D) Governor controls speed fluctuations during a cycle for a constant load, and governor also controls the mean speed of the engine, if the load changes.

YEAR 2001 TWO MARKS

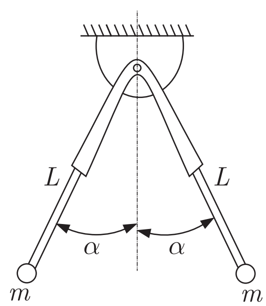
MCQ 4.80 The sun gear in the figure is driven clockwise at 100 rpm. The ring gear is held stationary. For the number of teeth shown on the gears, the arm rotates at



(A) zero (B) 20 rpm

(C) 33.33 rpm (D) 66.67 rpm

MCQ 4.81 The assembly shown in the figure is composed of two massless rods of length L with two particles, each of mass m. The natural frequency of this assembly for small oscillations is



(A) $\sqrt{\frac{g}{L}}$ (B) $\sqrt{\frac{2g}{(L\cos\alpha)}}$

(C) $\sqrt{\frac{g}{(L\cos\alpha)}}$ (D) $\sqrt{\frac{(g\cos\alpha)}{L}}$

****

SOLUTION

SOL 4.1 Option (B) is correct.

For helical gears, speed ratio is given by

$$\frac{N_1}{N_2} = \frac{D_2}{D_1} \times \frac{\cos \beta_2}{\cos \beta_1} \qquad ...(i)$$

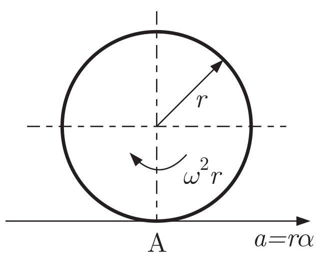
$$N_1 = 1440 \text{ rpm}, D_1 = 80 \text{ mm}, \ D_2 = 120 \text{ mm}, \ \beta_1 = 30^\circ, \ \beta_2 = 22.5^\circ$$

Hence from Eq. (i)

$$N_2 = \frac{D_1}{D_2} \times \frac{\cos \beta_1}{\cos \beta_2} \times N_1 = \frac{80}{120} \times \frac{\cos 30^\circ}{\cos 22.5^\circ} \times 1440$$

$$= 899.88 \simeq 900 \text{ rpm}$$

SOL 4.2 Option (D) is correct.



For A solid disc of radius (r) as given in figure, rolls without slipping on a horizontal floor with angular velocity ω and angular acceleration α.

The magnitude of the acceleration of the point of contact (A) on the disc is only by centripetal acceleration because of no slip condition.

$$v = \omega r \qquad ...(i)$$

Differentiating Eq. (1) w.r.t. (t)

$$\frac{dv}{dt} = r\frac{d\omega}{dt} = r \cdot \alpha \qquad \left(\frac{d\omega}{dt} = \alpha, \frac{dv}{dt} = a\right)$$

or, $$a = r \cdot \alpha$$

Instantaneous velocity of point A is zero

So at point A, Instantaneous tangential acceleration = zero

Therefore only centripetal acceleration is there at point A.

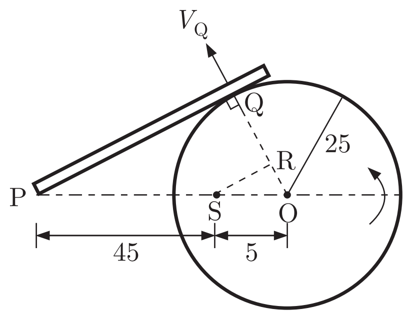
Centripetal acceleration $= r\omega^2$

SOL 4.3 Option (B) is correct.

From similar ΔPQO and ΔSRO

$$\frac{PQ}{SR} = \frac{PO}{SO} \qquad ...(i)$$

$$PQ = \sqrt{(50)^2 - (25)^2} = 43.3\text{ mm}$$



From Eq. (i) $\quad \frac{43.3}{SR} = \frac{50}{5}$

$$SR = \frac{43.5 \times 5}{50} = 4.33\text{ mm}$$

Velocity of Q = Velocity of R (situated at the same link)

$$V_Q = V_R = SR \times \omega = 4.33 \times 1 = 4.33\text{ m/s}$$

Angular velocity of PQ. $\quad \omega_{PQ} = \frac{V_Q}{PQ} = \frac{4.33}{43.3} = 0.1\text{ rad/s}$

SOL 4.4 Option (B) is correct.

For flywheel $\quad K.E = \frac{1}{2} I\omega^2$

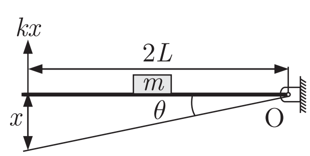
$$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 600}{60} = 62.83\text{ rad/s}$$

$$I \text{ (for solid circular disk)} = \frac{1}{2} mR^2 = \frac{1}{2} \times 20 \times (0.2)^2 = 0.4\text{ kg - m}^2$$

Hence, $\quad K.E = \frac{1}{2} \times (0.4) \times (62.83)^2 = 789.6 \simeq 790\text{ Joules}.$

SOL 4.5 Option (D) is correct.

For a very small amplitude of vibration.



From above figure change in length of spring

$$x = 2L\sin\theta = 2L\theta \qquad \text{(is very small so } \sin\theta \simeq \theta)$$

Mass moment of inertia of mass (m) about O is

$$I = mL^2$$

As no internal force acting on the system. So governing equation of motion from Newton's law of motion is,

$$I\ddot{\theta} + kx \times 2L = 0$$

or, $$mL^2\ddot{\theta} + k2L\theta \times 2L = 0$$

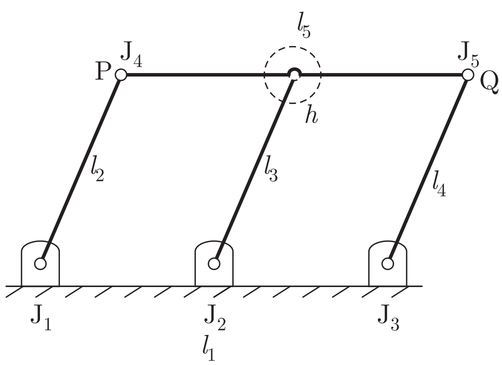
$$\ddot{\theta} + \frac{4kL^2\theta}{mL^2} = 0$$

or $$\ddot{\theta} + \frac{4k\theta}{m} = 0$$

Comparing general equation $\ddot{\theta} + \omega_n^2\theta = 0$ we have

$$\omega_n^2 = \frac{4k}{m} \Rightarrow \omega_n = \sqrt{\frac{4k}{m}}$$

SOL 4.6 Option (C) is correct.



Given that PQ is a single link.
Hence : $l = 5,\ j = 5,\ h = 1$
It has been assumed that slipping is possible between the link l_5 & l_1. From the kutzbach criterion for a plane mechanism,
Numbers of degree of freedom or movability.

$$n = 3(l-1) - 2j - h = 3(5-1) - 2 \times 5 - 1 = 1$$

SOL 4.7 Option (D) is correct.

Given $\omega_{AB} = 1\,\text{rad/sec},\ l_{CD} = 1.5 l_{AB}$ $\Rightarrow \frac{l_{CD}}{l_{AB}} = 1.5$
Let angular velocity of link CD is ω_{CD}
From angular velocity ratio theorem,

$$\frac{\omega_{AB}}{\omega_{CD}} = \frac{l_{CD}}{l_{AB}}$$

$$\omega_{CD} = \omega_{AB} \times \frac{l_{AB}}{l_{CD}} = 1 \times \frac{1}{1.5} = \frac{2}{3}\,\text{rad/sec}$$

SOL 4.8 Option (A) is correct.
Given $k = 20\,\text{kN/m},\ m = 1\,\text{kg}$
From the Given spring mass system, springs are in parallel combination. So,

$$k_{eq} = k + k = 2k$$

Natural Frequency of spring mass system is,

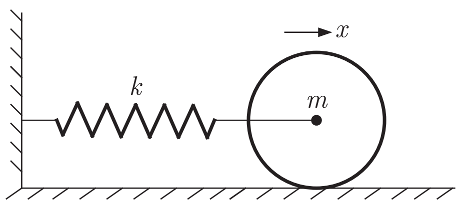
$$\omega_n = \sqrt{\frac{k_{eq}}{m}}$$

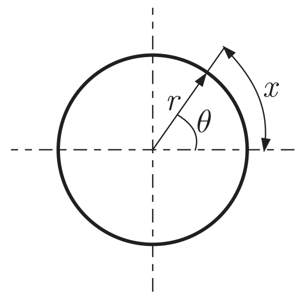
$$2\pi f_n = \sqrt{\frac{k_{eq}}{m}} \qquad f_n = \text{Natural Frequency in Hz.}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \frac{1}{2\times 3.14}\sqrt{\frac{2\times 20\times 1000}{1}}$$

$$= \frac{200}{6.28} = 31.84\text{ Hz} \simeq 32\text{ Hz}$$

SOL 4.9 Option (C) is correct.





$$\theta = \frac{x}{r} \Rightarrow x = r\theta \qquad ...(i)$$

Total energy of the system remains constant.

So, T.E. = K.E. due to translatory motion + K.E. due to rotary motion + P.E. of spring

$$\text{T.E.} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}kx^2$$

$$= \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}kr^2\theta^2 \qquad \text{From equation (i) } \dot{x} = r\dot{\theta}$$

$$= \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{2}\times\frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{2}kr^2\theta^2 \qquad \text{For a disc } I = \frac{mr^2}{2}$$

$$= \frac{3}{4}mr^2\dot{\theta}^2 + \frac{1}{2}kr^2\theta^2 = \text{Constant}$$

On differentiating above equation w.r.t. t, we get

$$\frac{3}{4}mr^2\times(2\dot{\theta}\ddot{\theta}) + \frac{1}{2}kr^2(2\theta\dot{\theta}) = 0$$

$$\frac{3}{2}mr^2\ddot{\theta} + kr^2\theta = 0$$

$$\ddot{\theta} + \frac{2k}{3m}\theta = 0$$

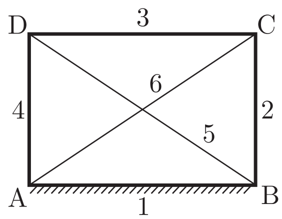
$$\omega_n^2 = \frac{2k}{3m} \Rightarrow \omega_n = \sqrt{\frac{2k}{3m}}$$

Therefore, natural frequency of vibration of the system is,

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2k}{3m}}$$

SOL 4.10 Option (A) is correct.

Given figure shows the six bar mechanism.



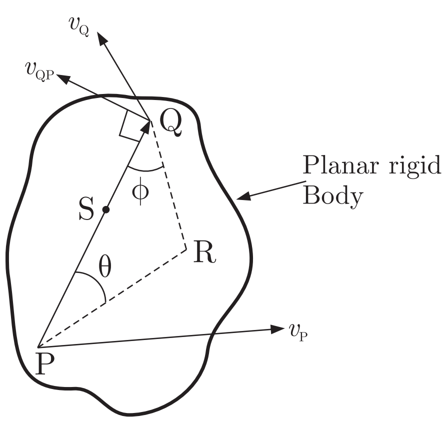
We know movability or degree of freedom is $n = 3(l-1) - 2j - h$

The mechanism shown in figure has six links and eight binary joints (because there are four ternary joints A, B, C & D, i.e. $l = 6$, $j = 8$ $h = 0$

So, $$n = 3(6-1) - 2 \times 8 = -1$$

Therefore, when $n = -1$ or less, then there are redundant constraints in the chain, and it forms a statically indeterminate structure. So, From the Given options (A) satisfy the statically indeterminate structure $n \leq -1$

SOL 4.11 Option (C) is correct.



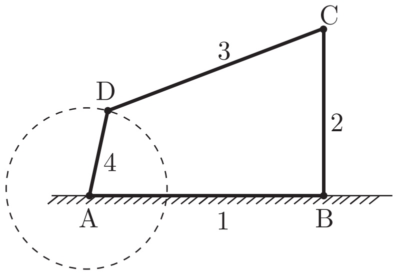
Velocity of any point on a link with respect to another point (relative velocity) on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.

$$v_{QP} = \text{Relative velocity between P \& Q}$$
$$= v_P - v_Q \text{ always perpendicular to PQ.}$$

SOL 4.12 Option (A) is correct.

According to Grashof's law "For a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of remaining two link lengths if there is to be continuous relative motion between the two links.

$$l_4 + l_2 \not> l_1 + l_3$$



SOL 4.13 Option (A) is correct.

We know natural frequency of a spring mass system is,

$$\omega_n = \sqrt{\frac{k}{m}} \qquad \text{...(i)}$$

This equation (i) does not depend on the g and weight $(W = mg)$

So, the natural frequency of a spring mass system is unchanged on the moon.

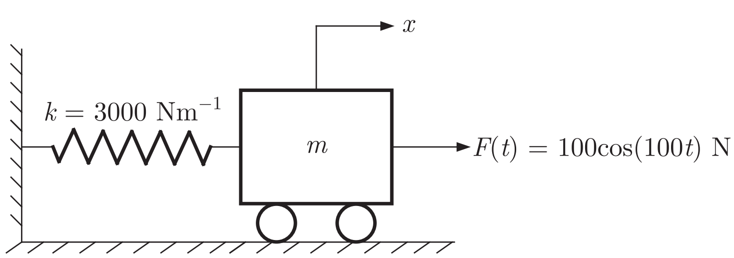
Hence, it will remain ω_n, i.e. $\omega_{moon} = \omega_n$

SOL 4.14 Option (D) is correct.

When gear teeth are produced by a generating process, interference is automatically eliminated because the cutting tool removes the interfering portion of the flank. This effect is called undercutting. By undercutting the undercut tooth can be considerably weakened.

So, interference can be reduced by using more teeth on the gear. However, if the gears are to transmit a given amount of power, more teeth can be used only by increasing the pitch diameter.

SOL 4.15 Option (A) is correct.



Given $k = 3000\,\text{N/m}$, $c = 0$, $A = 50\,\text{mm}$, $F(t) = 100\cos(100t)\,\text{N}$

$$\omega t = 100t$$

$$\omega = 100$$ It is a forced vibratory system.

From the Newton's law,

$$m\ddot{x} + kx = F \qquad \text{...(i)}$$

And its general solution will be,

$$x = A\cos\omega t$$

$$\frac{dx}{dt} = \dot{x} = -A\omega\sin\omega t \qquad \text{where } \omega = \sqrt{\frac{k}{m}}$$

$$\frac{d^2x}{dt^2} = \ddot{x} = -A\omega^2\cos\omega t$$

Substitute these values in equation (i), we get

$$-mA\omega^2\cos\omega t + kA\cos\omega t = 100\cos(\omega t)$$

$$-mA\omega^2 + kA = 100$$

Now substitute $k = 3000\ \text{N/m}$, $A = 0.05\ \text{m}$, in above equation, we get

$$-m\times 0.05\times(100)^2 + 3000\times 0.05 = 100$$

$$-5m + 1.5 = 1$$

$$m = 0.1\ \text{kg}$$

Alternate Method:

We know that, in forced vibration amplitude is given by :

$$A = \frac{F_O}{\sqrt{(k - m\omega)^2 + (c\omega)^2}} \qquad ...(i)$$

Here, $F(t) = 100\cos(100t)$, $F_O = 100\ \text{N}$, $A = 50\ \text{mm} = 50\times 10^{-3}\ \text{m}$

$\omega = 100\ \text{rad/sec}$, $k = 3000\ \text{Nm}^{-1}$, $c = 0$

So, from equation (i), we get

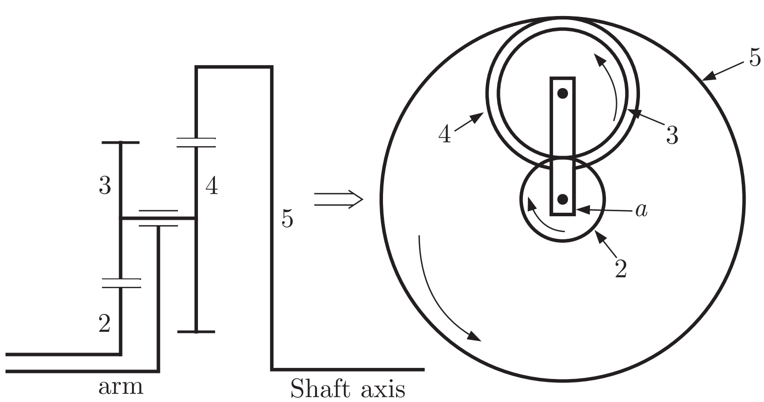
$$A = \frac{F_O}{k - m\omega^2}$$

$$k - m\omega^2 = \frac{F_O}{A}$$

$$3000 - m\times(100)^2 = \frac{100}{50\times 10^{-3}}$$

$$10000m = 1000 \qquad \Rightarrow m = 0.1\ \text{kg}$$

SOL 4.16 Option (C) is correct.



Given $N_i =$ No. of teeth for gear i,

$N_2 = 20,\ N_3 = 24,\ N_4 = 32,\ N_5 = 80,\ \omega_2 = 100$ rad/sec (CW)

$\omega_{arm} = 80$ rad/sec (CCW) $= -80$ rad/sec

The table of the motion given below :

Take CCW $= -$ ve and CW $= +$ ve

S. No.	Condition of Motion	Revolution of elements			
		Arm	Gear 2 ω_2	Compound Gear 3 - 4, $\omega_3 = \omega_4$	Gear 5 ω_5
1.	Arm 'a' is fixed & Gear 2 rotates through $+1$ revolution (CW)	0	$+1$	$-\frac{N_2}{N_3}$	$-\frac{N_2}{N_3} \times \frac{N_4}{N_5}$
2.	Gear 2 rotates through $+x$ revolution (CW)	0	$+x$	$-x\frac{N_2}{N_3}$	$-x\frac{N_2}{N_3} \times \frac{N_4}{N_5}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion.	$+y$	$x+y$	$y - x\frac{N_2}{N_3}$	$y - x\frac{N_2}{N_3} \times \frac{N_4}{N_5}$

Note. Speed ratio $= \dfrac{\text{Speed of driver}}{\text{Speed of driven}} = \dfrac{\text{No.of teeth on driven}}{\text{No. of teeth on driver}}$

i.e. $\dfrac{\omega_1}{\omega_2} = \dfrac{N_2}{N_1}$

Gear 3 & 4 mounted on same shaft, So $\omega_3 = \omega_4$

And $\omega_{arm} = y$ From the table

$y = -80$ rad/sec (CCW)

$x + y = \omega_2 = 100$ From the table

$x = 100 - (-80) = 180$ rad/sec (CW)

And $\omega_5 = y - x \times \dfrac{N_2}{N_3} \times \dfrac{N_4}{N_5}$ From the table

$$= -80 - 180 \times \frac{20}{24} \times \frac{32}{80} = -140 \text{ rad/sec}$$

Negative sign shows the counter clockwise direction.

SOL 4.17 Option (D) is correct.

Let, v_B is the velocity of slider B relative to link CD

The crank length AB $= 250$ mm and velocity of slider B with respect to rigid link CD is simply velocity of B (because C is a fixed point).

Hence, $v_B = (AB) \times \omega_{AB} = 250 \times 10^{-3} \times 10 = 2.5$ m/sec

Alternate Method :

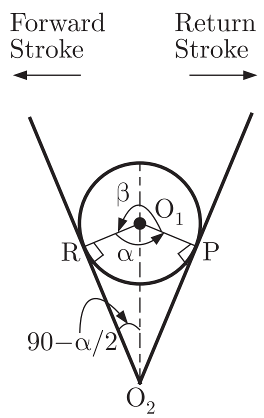
From the given figure, direction of velocity of CD is perpendicular to link AB & direction of velocity of AB is parallel to link CD.

So, direction of relative velocity of slider B with respect to C is in line with link BC.

Hence $v_C = 0$

Or $v_{BC} = v_B - v_C = AB \times \omega_{AB} - 0 = 0.025 \times 10 = 2.5\ \text{m/sec}$

SOL 4.18 Option (D) is correct.



Given $O_1P = r = 125\ \text{mm}$

Forward to return ratio $= 2:1$

We know that, $\dfrac{\text{Time of cutting (forward) stroke}}{\text{Time of return stroke}} = \dfrac{\beta}{\alpha} = \dfrac{360 - \alpha}{\alpha}$

Substitute the value of Forward to return ratio, we have

$$\frac{2}{1} = \frac{360 - \alpha}{\alpha}$$

$$2\alpha = 360 - \alpha \qquad \Rightarrow \alpha = 120^\circ$$

And angle $\angle RO_1O_2 = \dfrac{\alpha}{2} = \dfrac{120^\circ}{2} = 60^\circ$

Now we are to find the distance 'd' between the crank centre to lever pivot centre point (O_1O_2). From the ΔRO_2O_1

$$\sin\left(90^\circ - \frac{\alpha}{2}\right) = \frac{O_1R}{O_1O_2} = \frac{r}{O_1O_2}$$

$$\sin(90^\circ - 60^\circ) = \frac{r}{O_1O_2}$$

$$O_1O_2 = \frac{r}{\sin 30^\circ} = \frac{125}{1/2} = 250\ \text{mm}$$

SOL 4.19 Option (B) is correct.

Given $\delta = 1.8\ \text{mm} = 0.0018\ \text{m}$

The critical or whirling speed is given by,

$$\omega_c = \sqrt{\frac{g}{\delta}}$$

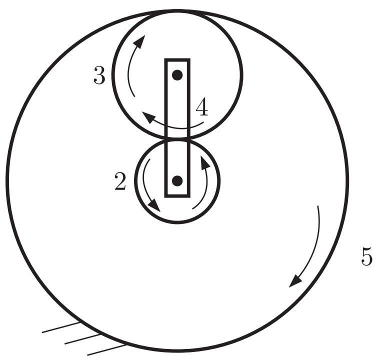
$$\frac{2\pi N_c}{60} = \sqrt{\frac{g}{\delta}} \qquad N_C = \text{Critical speed in rpm}$$

$$N_c = \frac{60}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{60}{2 \times 3.14}\sqrt{\frac{9.81}{0.0018}}$$

$$= 9.55\sqrt{5450} = 704.981 \simeq 705 \text{ rpm}$$

SOL 4.20 Option (A) is correct.

Given $Z_2 = 20$ Teeth, $Z_3 = 40$ Teeth, $Z_5 = 100$ Teeth, $N_5 = 0$, $N_2 = 60$ rpm (CCW)



If gear 2 rotates in the CCW direction, then gear 3 rotates in the clockwise direction. Let, Arm 4 will rotate at N_4 rpm. The table of motions is given below. Take CCW $= +$ ve, CW $= -$ ve

S. No.	Condition of Motion	Revolution of elements			
		Sun Gear 2	Planet Gear 3	Arm 4	Ring Gear 5
		N_2	N_3	N_4	N_5
1.	Arm fixed and sun gear 2 rotates $+1$ rpm (CCW)	$+1$	$-\frac{Z_2}{Z_3}$	0	$-\frac{Z_2}{Z_3} \times \frac{Z_3}{Z_5}$
2.	Give $+x$ rpm to gear 2 (CCW)	$+x$	$-\frac{Z_2}{Z_3}x$	0	$-x\frac{Z_2}{Z_5}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion.	$y + x$	$y - x\frac{Z_2}{Z_3}$	$+y$	$y - x\frac{Z_2}{Z_5}$

Note : Speed ratio $= \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on dirven}}{\text{No. of teeth on driver}}$

Ring gear 5 is fixed. So,

$$N_5 = 0$$

$$y - x\frac{Z_2}{Z_5} = 0 \quad \text{From the table}$$

$$y = \frac{Z_2}{Z_5}x = \frac{20}{100}x = \frac{x}{5} \quad \text{...(i)}$$

Given, $N_2 = 60\,\text{rpm}\,(\text{CCW})$

$$y + x = 60 \quad \text{From table}$$

$$\frac{x}{5} + x = 60$$

$$x = 10 \times 5 = 50\,\text{rpm}$$

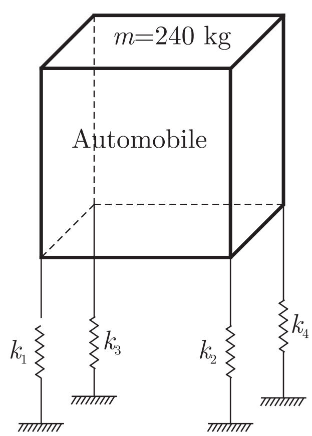
And from equation (i),

$$y = \frac{50}{5} = 10\,\text{rpm}\,(\text{CCW})$$

From the table the arm will rotate at

$$N_4 = y = 10\,\text{rpm}\,(\text{CCW})$$

SOL 4.21 Option (A) is correct.



Given $k_1 = k_2 = 16\,\text{MN/m},\ k_3 = k_4 = 32\,\text{MN/m},\ m = 240\,\text{kg}$

Here, k_1 & k_2 are the front two springs or k_3 and k_4 are the rear two springs. These 4 springs are parallel, So equivalent stiffness

$$k_{eq} = k_1 + k_2 + k_3 + k_4 = 16 + 16 + 32 + 32 = 96\,\text{MN/m}^2$$

We know at resonance

$$\omega = \omega_n = \sqrt{\frac{k}{m}}$$

$$\frac{2\pi N}{60} = \sqrt{\frac{k_{eq}}{m}} \quad N = \text{Engine speed in rpm}$$

$$N = \frac{60}{2\pi}\sqrt{\frac{k_{eq}}{m}} = \frac{60}{2\pi}\sqrt{\frac{96 \times 10^6}{240}}$$

$$= \frac{60}{2\pi} \times 10^2 \times \sqrt{40} = 6042.03 \simeq 6040 \text{ rpm}$$

SOL 4.22 Option (A) is correct.

Given $k = 3.6 \text{ kN/m}$, $c = 400 \text{ Ns/m}$, $m = 50 \text{ kg}$

We know that, Natural Frequency

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3.6 \times 1000}{50}} = 8.485 \text{ rad/sec} \quad \text{...(i)}$$

And damping factor is given by,

$$d \text{ or } \varepsilon = \frac{c}{c_c} = \frac{c}{2\sqrt{km}} = \frac{400}{2 \times \sqrt{3.6 \times 1000 \times 50}}$$

$$= \frac{400}{2 \times 424.26} = 0.471$$

Damping Natural frequency,

$$\omega_d = \sqrt{1 - \varepsilon^2}\,\omega_n$$

$$2\pi f_d = \sqrt{1 - \varepsilon^2}\,\omega_n \qquad f_d = \frac{\omega_d}{2\pi}$$

$$f_d = \frac{\omega_n}{2\pi} \times \sqrt{1 - \varepsilon^2} = \frac{8.485}{2 \times 3.14} \times \sqrt{1 - (0.471)^2} = 1.19 \text{ Hz}$$

SOL 4.23 Option (B) is correct.

	Analysis		Approach
P.	Continuous relative rotation	3.	Grashoff law
Q.	Velocity and Acceleration	4.	Kennedy's Theorem
R.	Mobility	2.	Grubler's Criterion
S.	Dynamic-static Analysis	1.	D'Alembert's Principle

So, correct pairs are P-3, Q-4, R-2, S-1

SOL 4.24 Option (B) is correct.

From Gruebler's criterion, the equation for degree of freedom is given by,

$$n = 3(l - 1) - 2j - h \quad \text{...(i)}$$

Given $l = 8$ and $j = 10$, $h = 0$

$$n = 3(8 - 1) - 2 \times 10 = 1 \quad \text{from equation(i)}$$

SOL 4.25 Option (B) is correct.

Given $m = 1.4 \text{ kg}$, $k_1 = 4000 \text{ N/m}$, $k_2 = 1600 \text{ N/m}$

In the given system k_1 & k_2 are in parallel combination

So, $k_{eq} = k_1 + k_2 = 4000 + 1600 = 5600 \text{ N/m}$

Natural frequency of spring mass system is given by,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m}} = \frac{1}{2\pi}\sqrt{\frac{5600}{1.4}} = \frac{1}{2\pi} \times 63.245 = 10.07 \simeq 10 \text{ Hz}$$

SOL 4.26 Option (D) is correct.

Jerk is given by triple differentiation of s w.r.t. t,

$$\text{Jerk} = \frac{d^3s}{dt^3}$$

Given $$s = \frac{h}{2}\left(1 - \cos\frac{\pi\theta}{\beta}\right) = \frac{h}{2}\left[1 - \cos\frac{\pi(\omega t)}{\beta}\right] \qquad \theta = \omega t$$

Differentiating above equation w.r.t. t, we get

$$\frac{ds}{dt} = \frac{h}{2}\left[-\frac{\pi\omega}{\beta}\left\{-\sin\frac{\pi(\omega t)}{\beta}\right\}\right]$$

Again Differentiating w.r.t. t,

$$\frac{d^2s}{dt^2} = \frac{h}{2}\frac{\pi^2\omega^2}{\beta^2}\left[\cos\frac{\pi(\omega t)}{\beta}\right]$$

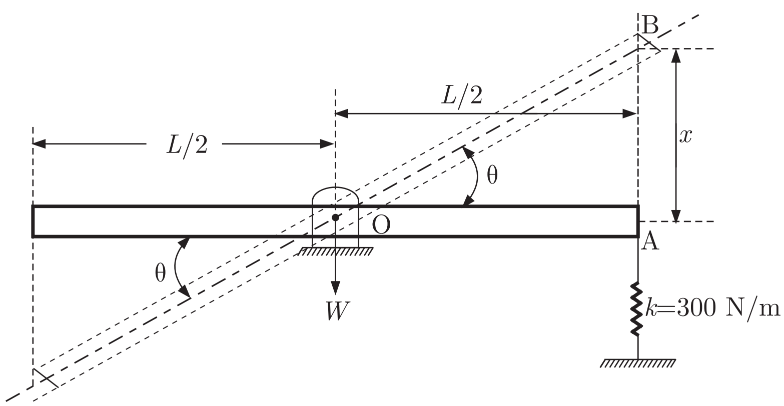
Again Differentiating w.r.t. t,

$$\frac{d^3s}{dt^3} = -\frac{h}{2}\frac{\pi^3\omega^3}{\beta^3}\sin\frac{\pi\theta}{\beta}$$

Let $\omega = 1\,\text{rad/sec}$

$$\frac{d^3s}{dt^3} = -\frac{h}{2}\frac{\pi^3}{\beta^3}\sin\left(\frac{\pi\theta}{\beta}\right)$$

SOL 4.27 Option (C) is correct.



Given $m = 1\,\text{kg}$, $L = 1\,\text{m}$, $k = 300\,\text{N/m}$

We have to turn the rigid rod at an angle θ about its hinged point, then rod moves upward at a distance x and also deflect in the opposite direction with the same amount. Let θ is very very small and take $\tan\theta \backsimeq \theta$

From ΔAOB, $$\theta = \frac{x}{L/2} \Rightarrow x = \frac{L}{2}\theta \qquad \text{...(i)}$$

and $$\theta = \omega t \Rightarrow \dot{\theta} = \omega \qquad \text{...(ii)}$$

By using the principal of energy conservation,

$$\frac{1}{2}I\omega^2 + \frac{1}{2}kx^2 = \text{Constant}$$

$$\frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}k\left(\frac{L}{2}\theta\right)^2 = c \qquad \text{From equation (i) and (ii)}$$

$$\frac{1}{2}I\dot{\theta}^2 + \frac{1}{8}L^2k\theta^2 = c$$

On differentiating w.r.t. t, we get

$$\frac{1}{2}I \times 2\dot{\theta}\ddot{\theta} + \frac{kL^2}{8} \times 2\theta\dot{\theta} = 0 \qquad \text{...(iii)}$$

For a rigid rod of length L & mass m, hinged at its centre, the moment of inertia,

$$I = \frac{mL^2}{12}$$

Substitute I in equation (iii), we get

$$\frac{1}{2} \times \frac{mL^2}{12} \times 2\dot{\theta}\ddot{\theta} + \frac{kL^2}{4}\theta\dot{\theta} = 0$$

$$\ddot{\theta} + \frac{3k}{m}\theta = 0 \qquad \text{...(iv)}$$

Compare equation (iv) with the general equation,

$$\ddot{\theta} + \omega_n^2\theta = 0$$

So, we have $\omega_n^2 = \frac{3k}{m}$

$$\omega_n = \sqrt{\frac{3k}{m}} = \sqrt{\frac{3 \times 300}{1}} = 30 \text{ rad/ sec}$$

SOL 4.28 Option (C) is correct.

For an under damped harmonic oscillator resonance occurs when excitation frequency is equal to the undamped natural frequency

$$\omega_d = \omega_n$$

SOL 4.29 Option (A) is correct.

Given $\omega_1 = 210$ rad/ sec, $\omega_2 = 190$ rad/ sec, $\Delta E = 400$ Nm

As the speed of flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

$$\Delta E = \frac{1}{2}I[(\omega_1)^2 - (\omega_2)^2]$$

$$I = \frac{2\Delta E}{[(\omega_1)^2 - (\omega_2)^2]} = \frac{2 \times 400}{[(210)^2 - (190)^2]} = \frac{800}{400 \times 20} = 0.10 \text{ kgm}^2$$

SOL 4.30 Option (C) is correct.

Given, $\angle O_4O_2P = 180^\circ$, $\omega_{O_2P} = 2$ rad/sec

The instantaneous centre diagram is given below,

Let, velocity of point P on link O_2P is V_P,

$$V_P = \omega_{O_2P} \times O_2P = \omega_{O_2P} \times (I_{12}I_{23}) = 2a \qquad \text{...(i)}$$

And P is also a point on link QP,

So, $$V_P = \omega_{PQ} \times O_4P = \omega_{PQ} \times (I_{13}I_{23})$$

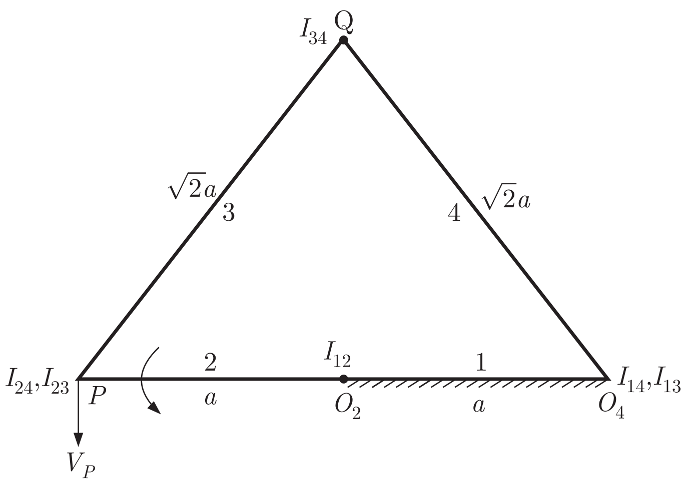
$$= \omega_{PQ} \times 2a \qquad ...(ii)$$

Both the links O_2P and QP are runs at the same speed

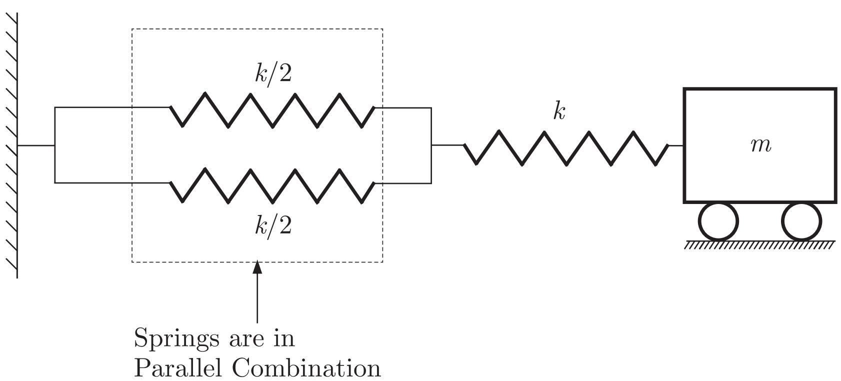
From equation (i) and (ii), we get

$$2a = \omega_{PQ} \times 2a$$

or, $$\omega_{PQ} = 1 \text{ rad/sec}$$



SOL 4.31 Option (A) is correct.



The springs, with stiffness $\frac{k}{2}$ & $\frac{k}{2}$ are in parallel combination. So their resultant stiffness will be,

$$k_1 = \frac{k}{2} + \frac{k}{2} = k$$

As k_1 & k are in series, so the resultant stiffness will be,

$$k_{eq} = \frac{k \times k}{k + k} = \frac{k^2}{2k} = \frac{k}{2}$$

The general equation of motion for undamped free vibration is given as,

$$m\ddot{x} + k_{eq}x = 0$$
$$m\ddot{x} + \frac{k}{2}x = 0$$
$$\ddot{x} + \frac{k}{2m}x = 0$$

Compare above equation with general equation $\ddot{x} + \omega_n^2 x = 0$, we get
Natural frequency of the system is,

$$\omega_n^2 = \frac{k}{2m} \Rightarrow \omega_n = \sqrt{\frac{k}{2m}}$$

Alternative :

$$k_{eq} = \frac{k}{2}$$

We know, for a spring mass system,

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k/2}{m}} = \sqrt{\frac{k}{2m}}$$

SOL 4.32 Option (A) is correct.
Given The equation of motion of a harmonic oscillator is

$$\frac{d^2x}{dt^2} + 2\xi\omega_n\frac{dx}{dt} + \omega_n^2 x = 0 \quad ...(i)$$
$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = 0$$

Compare equation (i) with the general equation,

$$m\ddot{x} + c\dot{x} + kx = 0$$
$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

We get,
$$\frac{c}{m} = 2\xi\omega_n \quad ...(ii)$$
$$\frac{k}{m} = \omega_n^2, \quad \Rightarrow \quad \omega_n = \sqrt{\frac{k}{m}} \quad ...(iii)$$

From equation (ii) & (iii), $\xi = \dfrac{c}{2m \times \sqrt{\frac{k}{m}}} = \dfrac{c}{2\sqrt{km}}$

Logarithmic decrement,

$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi c}{\sqrt{c_c^2 - c^2}} \quad ...(iv)$$
$$= \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi \times 2\xi\sqrt{km}}{(2\sqrt{km})^2 - (2\xi\sqrt{km})^2} = \frac{4\pi\xi\sqrt{km}}{\sqrt{4km - 4\xi^2 km}}$$
$$= \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$
$$\frac{x_1}{x_2} = e^{\frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$

If system executes n cycles, the logarithmic decrement δ can be written as

$$\delta = \frac{1}{n}\log_e\frac{x_1}{x_{n+1}}$$

$$e^{n\delta} = \frac{x_1}{x_{n+1}}$$

Where x_1 = amplitude at the starting position.

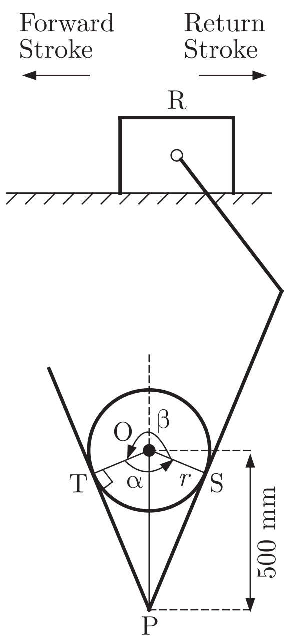
x_{n+1} = Amplitude after n cycles

The amplitude of $x(t)$ after n complete cycles is,

$$e^{n\delta} = \frac{X}{x(t)}$$

$$x(t) = e^{-n\delta}\times X = Xe^{-\frac{n2\pi\xi}{\sqrt{1-\xi^2}}}$$ From equation (iv)

SOL 4.33 Option (A) is correct.



Given Quick return ratio $= 1:2$, $OP = 500\,\text{mm}$

Here $OT =$ Length of the crank. We see that the angle β made by the forward stroke is greater than the angle α described by the return stroke. Since the crank has uniform angular speed, therefore

$$\text{Quick return ratio} = \frac{\text{Time of return stroke}}{\text{Time of cutting stroke}}$$

$$\frac{1}{2} = \frac{\alpha}{\beta} = \frac{\alpha}{360-\alpha}$$

$$360-\alpha = 2\alpha$$

$$3\alpha = 360$$

$$\alpha = 120^\circ$$

and Angle $\angle TOP = \frac{\alpha}{2} = \frac{120}{2} = 60^\circ$

From the ΔTOP, $\cos\frac{\alpha}{2} = \frac{OT}{OP} = \frac{r}{500}$ OT $= r$

$$\cos 60^\circ = \frac{r}{500}$$

$$r = 500 \times \frac{1}{2} = 250\text{ mm}$$

SOL 4.34 Option (B) is correct.

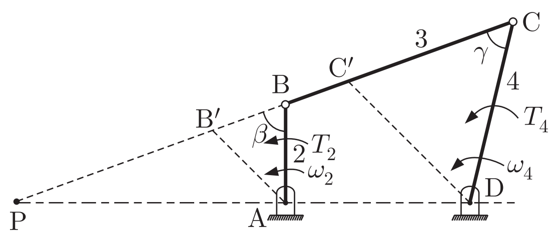
We know that maximum speed during forward stroke occur when QR & QP are perpendicular.

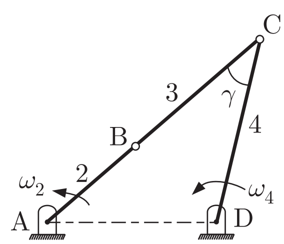
So, $V = OS \times \omega_{OS} = PQ \times \omega_{PQ}$ $V = r\omega$

$$250 \times 2 = 750 \times \omega_{PQ}$$

$$\omega_{PQ} = \frac{500}{750} = \frac{2}{3}\text{ rad/sec}$$

SOL 4.35 Option (D) is correct.





$M.A = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4} = \frac{R_{PD}}{R_{PA}}$ from angular velocity ratio theorem

Construct $B'A$ and $C'D$ perpendicular to the line PBC. Also, assign lables β and γ to the acute angles made by the coupler.

$$\frac{R_{PD}}{R_{PA}} = \frac{R_{C'D}}{R_{B'A}} = \frac{R_{CD}\sin\gamma}{R_{BA}\sin\beta}$$

So, $M.A. = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4} = \frac{R_{CD}\sin\gamma}{R_{BA}\sin\beta}$

When the mechanism is toggle,then $\beta = 0^\circ$ and 180°.

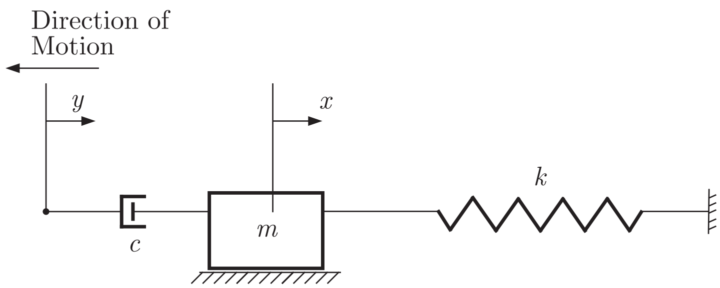
So $M.A = \infty$

SOL 4.36 Option (C) is correct.

Assume any arbitrary relationship between the coordinates and their first derivatives, say $x > y$ and $\dot{x} > \dot{y}$. Also assume $x > 0$ and $\dot{x} > 0$.

A small displacement gives to the system towards the left direction. Mass m is fixed, so only damper moves for both the variable x and y.

Note that these forces are acting in the negative direction.



Differential equation governing the above system is,

$$\sum F = -m\frac{d^2x}{dt^2} - c\left(\frac{dx}{dt} - \frac{dy}{dt}\right) - kx = 0$$

$$m\ddot{x} + c(\dot{x} - \dot{y}) + kx = 0$$

SOL 4.37 Option (C) is correct.

For a 4 bar slider crank mechanism, there are the number of links or inversions are 4. These different inversions are obtained by fixing different links once at a time for one inversion. Hence, the number of inversions for a slider crank mechanism is 4.

SOL 4.38 Option (B) is correct.

	Column I		Column II
P.	Addendum	4.	Gear
Q.	Instantaneous centre of velocity	3.	Linkage
R.	Section modulus	2.	Beam
S.	Prime circle	1.	Cam

So correct pairs are, P-4, Q-3, R-2, S-1

SOL 4.39 Option (D) is correct.

The ratio of the maximum fluctuation of speed to the mean speed is called the coefficient of fluctuation of speed (C_f).

Let, N_1 & N_2 = Maximum & Minimum speeds in r.p.m. during the cycle

$$N = \text{Mean speed in r.p.m.} = \frac{N_1 + N_2}{2} \quad \text{...(i)}$$

Therefore, $$C_f = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2} \quad \text{from equation (i)}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}$$

$$C_f = \frac{2(\omega_{max} - \omega_{min})}{\omega_{max} + \omega_{min}} \quad \omega_1 = \omega_{max},$$

$$\omega_2 = \omega_{min}$$

$$C_f\omega_{max} + C_f\omega_{min} = 2\omega_{max} - 2\omega_{min}$$

$$\omega_{max}(C_f - 2) = \omega_{min}(-2 - C_f)$$

Hence, $$\frac{\omega_{max}}{\omega_{min}} = -\frac{(2 + C_f)}{C_f - 2} = \frac{2 + C_f}{2 - C_f}$$

SOL 4.40 Option (D) is correct.

In this question pair or mechanism is related to contact & machine related to it.

	Column I		Column II
P.	Higher Kinematic Pair	2.	Line Contact
Q.	Lower Kinematic Pair	6.	Surface Contact
R.	Quick Return Mechanism	5.	Shaper
S.	Mobility of a Linkage	1.	Grubler's Equation

So correct pairs are, P-2, Q-6, R-5, S-1

SOL 4.41 Option (C) is correct.

Given $m = 250$ kg, $k = 100$ kN/m, $N = 3600$ rpm, $\varepsilon = \frac{c}{c_c} = 0.15$

$$\omega = \frac{2\pi N}{60} = \frac{2 \times 3.14 \times 3600}{60} = 376.8 \text{ rad/sec}$$

Natural frequency of spring mass system,

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 \times 1000}{250}} = 20 \text{ rad/sec}$$

So, $$\frac{\omega}{\omega_n} = \frac{376.8}{20} = 18.84$$

$$T.R. = \frac{F_T}{F} = \sqrt{\frac{1 + \left(2\varepsilon\frac{\omega}{\omega_n}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\varepsilon\frac{\omega}{\omega_n}\right]^2}}$$

$$= \sqrt{\frac{1 + (2 \times 0.15 \times 18.84)^2}{[1 - (18.84)^2]^2 + [2 \times 0.15 \times 18.84]^2}}$$

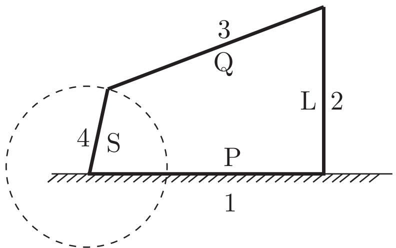
$$= \sqrt{\frac{1 + 31.945}{[1 - 354.945]^2 + 31.945}} = \sqrt{\frac{32.945}{125309}} = 0.0162$$

SOL 4.42 Option (A) is correct.

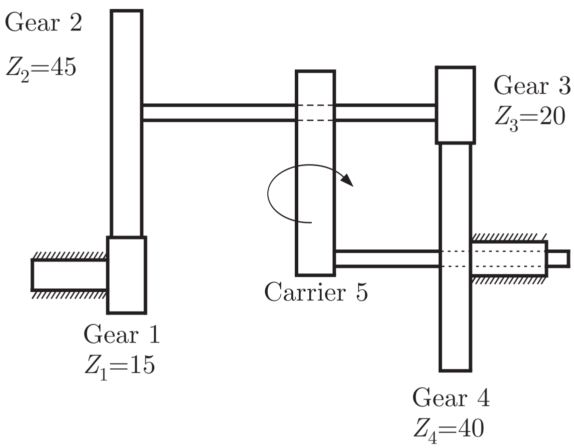
Here $P, Q, R,$ & S are the lengths of the links.

According to Grashof's law : "For a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of remaining two link lengths, if there is to be continuous relative motion between the two links

$$S + L \leqslant P + Q$$



SOL 4.43 Option (A) is correct.



The table of motions is given below. Take CW $=+ve$, CCW $=-ve$

S. No.	Condition of Motion	Revolution of elements			
		Gear 1 N_1	Compound Gear 2-3, $N_2 = N_3$	Gear 4 N_4	Carrier N_5
1.	Carrier 5 is fixed & Gear 1 rotates +1 rpm (CW)	$+1$	$-\frac{Z_1}{Z_2}$	$\frac{Z_1}{Z_2} \times \frac{Z_3}{Z_4}$	0
2.	Gear 1 rotates through $+x$ rpm (CW)	$+x$	$-x\frac{Z_1}{Z_2}$	$x\frac{Z_1 Z_3}{Z_2 Z_4}$	0
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion.	$x + y$	$y - x\frac{Z_1}{Z_2}$	$y + x \times \frac{Z_1 Z_3}{Z_2 Z_4}$	$+y$

Note

(i) $$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No.of teeth on driver}}$$

i.e. $$\frac{N_1}{N_2} = \frac{Z_2}{Z_1}$$

CCW = Counter clock wise direction (– ve)

CW = Clock wise direction (+ ve)

(ii) Gear 2 & Gear 3 mounted on the same shaft (Compound Gears)

So, $$N_2 = N_3$$

We know, $$\omega = \frac{2\pi N}{60}, \;\Rightarrow \omega \propto N$$

Hence, $$\frac{N_1 - N_5}{N_4 - N_5} = \frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = \frac{(x+y) - y}{y + x \times \frac{Z_1 Z_3}{Z_2 Z_4} - y}$$

$$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = \frac{x}{x \times \frac{Z_1 Z_3}{Z_2 Z_4}} = \frac{Z_2 Z_4}{Z_1 Z_3}$$

$$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = \frac{45 \times 40}{15 \times 20} = 3 \times 2 = 6$$

SOL 4.44 Option (D) is correct.

Given $\omega_1 = 60\text{ rpm}$ (CW), $\omega_4 = -2 \times 60$ (CCW) $= -120\text{ rpm}$

From the previous part,

$$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = 6$$

$$\frac{60 - \omega_5}{-120 - \omega_5} = 6$$

$$60 - \omega_5 = -720 - 6\omega_5$$

$$\omega_5 = -\frac{780}{5} = -156\text{ rpm}$$

Negative sign show the counter clock wise direction.

So, $$\omega_5 = 156\text{ rpm, CCW}$$

SOL 4.45 Option (D) is correct.

Given $m = 12.5\text{ kg}$, $k = 1000\text{ N/m}$, $c = 15\text{ Ns/m}$

Critical Damping,

$$c_c = 2m\sqrt{\frac{k}{m}} = 2\sqrt{km}$$

On substituting the values, we get

$$c_c = 2\sqrt{1000 \times 12.5} = 223.6\text{ Ns/m}$$

SOL 4.46 None of these

We know logarithmic decrement,

$$\delta = \frac{2\pi\varepsilon}{\sqrt{1 - \varepsilon^2}} \qquad \text{...(i)}$$

And $\varepsilon = \frac{c}{c_c} = \frac{15}{223.6} = 0.0671$ $c_c = 223.6$ Ns/m

Now, from equation (i), we get

$$\delta = \frac{2 \times 3.14 \times 0.0671}{\sqrt{1-(0.0671)^2}} = 0.422$$

SOL 4.47 Option (C) is correct.

Given $l = 8$, $j = 9$

We know that, Degree of freedom,

$$n = 3(l-1) - 2j = 3(8-1) - 2 \times 9 = 3$$

SOL 4.48 Option (C) is correct.

The speed of sound in air $= 332$ m/s

For frequency of instrument of 144 Hz, length of sound wave

$$L_I = \frac{332}{144} = 2.30 \text{ m}$$

For sample P of 64 Hz,

$$L_P = \frac{332}{64} = 5.1875 \text{ m}$$

Q of 96 Hz $L_Q = \frac{332}{96} = 3.458$ m

R of 128 Hz $L_R = \frac{332}{128} = 2.593$ m

S of 250 Hz $L_S = \frac{332}{256} = 1.2968$ m

Here, the length of sound wave of sample R ($L_R = 2.593$ m) is most close to the length of sound wave of Instrument ($L_I = 2.30$ m). Hence, sample R produce most perceptible induced vibration.

SOL 4.49 Option (B) is correct.

Given $N = 300$ r.p.m

Angular velocity of cam,

$$\omega = \frac{2\pi N}{60} = 10\pi \text{ rad/sec}$$

Time taken to move $30°$ is,

$$t = \frac{\frac{\pi}{180} \times 30}{10\pi} = \frac{\frac{1}{6}}{10} = \frac{1}{60} \text{ sec}$$

Now, Cam moves $30°$ with a constant acceleration & then with a deceleration, so maximum speed of the follower is at the end of first $30°$ rotation of the cam and during this $30°$ rotation the distance covered is 10 mm, with initial velocity $u = 0$.

From Newton's second law of motion,

$$S = ut + \frac{1}{2}at^2$$

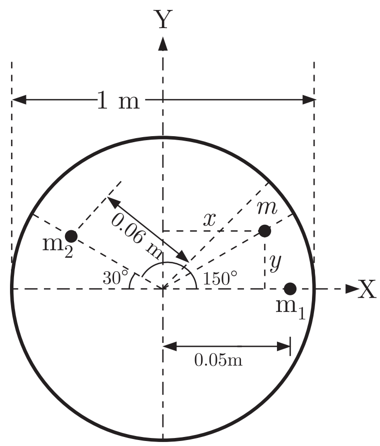
$$0.01 = 0 + \frac{1}{2} \times a \times \left(\frac{1}{60}\right)^2$$

$$a = 0.01 \times 2 \times (60)^2 = 72 \text{ m/sec}^2$$

Maximum velocity,

$$v_{max} = u + at = 72 \times \frac{1}{60} = 1.2 \text{ m/sec}$$

SOL 4.50 Option (C) is correct.



Given $m_1 = m_2 = 0.5$ kg, $r_1 = 0.05$ m, $r_2 = 0.06$ m

Balancing mass $m = 0.1$ kg

Let disc rotates with uniform angular velocity ω and x & y is the position of balancing mass along X & Y axis.

Resolving the forces in the x-direction, we get

$$\Sigma F_x = 0$$

$$0.5[-0.06\cos 30^\circ + 0.05\cos 0^\circ]\omega^2 = 0.1 \times x \times \omega^2$$

$$0.5 \times (-0.00196) = 0.1x \qquad F_c = mr\omega^2$$

$$x = -0.0098 \text{ m} = -9.8 \text{ mm}$$

Similarly in y-direction, $\Sigma F_y = 0$

$$0.5(0.06 \times \sin 30^\circ + 0.05 \times \sin 0)\omega^2 = 0.1 \times y \times \omega^2$$

$$0.5 \times 0.03 = 0.1 \times y$$

$$y = 0.15 \text{ m} = 150 \text{ mm}$$

Position of balancing mass is given by, $r = \sqrt{x^2 + y^2} = \sqrt{(-9.8)^2 + (150)^2}$

$$= 150.31 \text{ mm} \simeq 150 \text{ mm}$$

SOL 4.51 Option (C) is correct.

Given $m = 0.1$ kg, $k = 1$ kN/m

Let, ω_d be the frequency of damped vibration & ω_n be the natural frequency of spring mass system.

Hence, $\omega_d = 90\%$ of $\omega_n = 0.9\omega_n$ (Given) ...(i)

Frequency of damped vibration

$$\omega_d = \sqrt{(1-\varepsilon^2)}\omega_n \quad ...(ii)$$

From equation (i) and equation (ii), we get

$$\sqrt{(1-\varepsilon^2)}\omega_n = 0.9\omega_n$$

On squaring both the sides, we get

$$1-\varepsilon^2 = (0.9)^2 = 0.81$$

$$\varepsilon^2 = 1 - 0.81 = 0.19$$

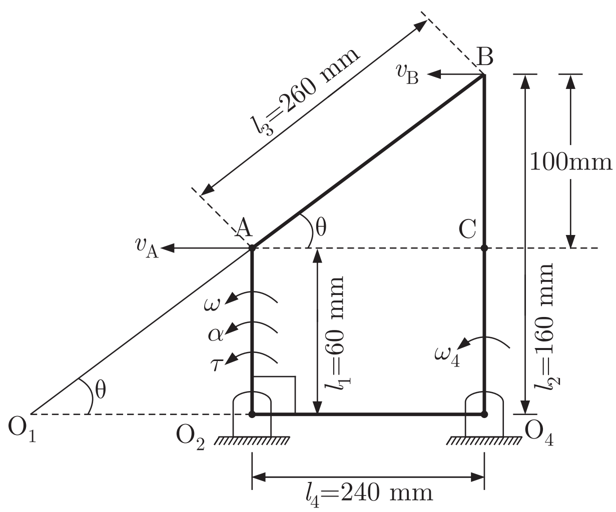
$$\varepsilon = \sqrt{0.19} = 0.436$$

And Damping ratio is given by,

$$\varepsilon = \frac{c}{c_c} = \frac{c}{2\sqrt{km}}$$

$$c = 2\sqrt{km} \times \varepsilon = 2\sqrt{1000 \times 0.1} \times 0.436 = 8.72\,\text{Ns/m} \backsimeq 8.7\,\text{Ns/m}$$

SOL 4.52 Option (B) is correct.



From Triangle ABC,

$$AB = \sqrt{(100)^2 + (240)^2} = \sqrt{67600} = 260\,\text{mm}$$

Length of shortest link $l_1 = 60$ mm

Length of longest link $l_3 = 260$ mm

From the Grashof's law,

$$l_1 + l_3 \ngtr l_2 + l_4$$

$$60 + 260 \ngtr 160 + 240$$

$$320 \ngtr 400$$

So, $l_1 + l_3 < l_2 + l_4$

Also, when the shortest link O_2A will make a complete revolution relative to other three links, if it satisfies the Grashof's law. Such a link is known as crank. The link O_4B which makes a partial rotation or oscillates is known as rocker. So, crank rocker mechanism is obtained.

Here, $O_2A = l_1 = 60\text{ mm}$ is crank (fixed link)

Adjacent link, $O_2O_4 = 240\text{ mm}$ is fixed

So, crank rocker mechanism will be obtained.

SOL 4.53 Option (B) is correct.

Let, ω_4 is the angular velocity of link O_4B

From the triangle ABC,

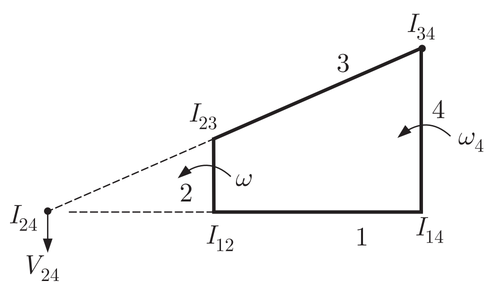
$$\tan\theta = \frac{100}{240} = \frac{5}{12} \quad \text{...(i)}$$

$$\theta = \tan^{-1}\left(\frac{5}{12}\right) = 22.62^\circ$$

Also from the triangle O_1O_2A,

$$\tan\theta = \frac{O_2A}{O_1O_2}$$

$$O_1O_2 = \frac{O_2A}{\tan\theta} = \frac{60}{\frac{5}{12}} = 144\text{ mm}$$



From the angular velocity ratio theorem.

$$V_{24} = \omega_4 \times I_{24}I_{14} = \omega \times I_{24}I_{12}$$

$$\omega_4 = \frac{I_{24}I_{12}}{I_{24}I_{14}} \times \omega = \frac{144}{(240+144)} \times 8 = \frac{144}{384} \times 8 = 3\text{ rad/sec}$$

SOL 4.54 Option (D) is correct.

From the given data the component of force at joint A along AO_2 is necessary to find the joint reaction at O_2. So, it is not possible to find the magnitude of the joint reaction at O_2.

SOL 4.55 Option (D) is correct.

Mechanical advantage in the form of torque is given by,

$$M.A. = \frac{T_{output}}{T_{input}} = \frac{\omega_{input}}{\omega_{output}}$$

Here output link is a slider, So, $\omega_{output} = 0$

Therefore, $M.A. = \infty$

SOL 4.56 Option (C) is correct.

Given $\frac{\omega}{\omega_n} = r = 0.5$

And due to isolation damping ratio,

$$\varepsilon = \frac{c}{c_c} = 0 \qquad \text{For isolation } c = 0$$

We know the transmissibility ratio of isolation is given by,

$$T.R. = \frac{\sqrt{1 + \left(2\varepsilon\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\varepsilon\frac{\omega}{\omega_n}\right]^2}} = \frac{\sqrt{1+0}}{\sqrt{[1-(0.5)^2]^2 + 0}} = \frac{1}{0.75} = \frac{4}{3}$$

SOL 4.57 Option (D) is correct.

Given planar mechanism has degree of freedom, $N = 1$ and two infinite parallel lines meet at infinity. So, the instantaneous centre I_{24} will be at N, but for single degree of freedom, system moves only in one direction.

Hence, I_{24} is located at infinity(∞).

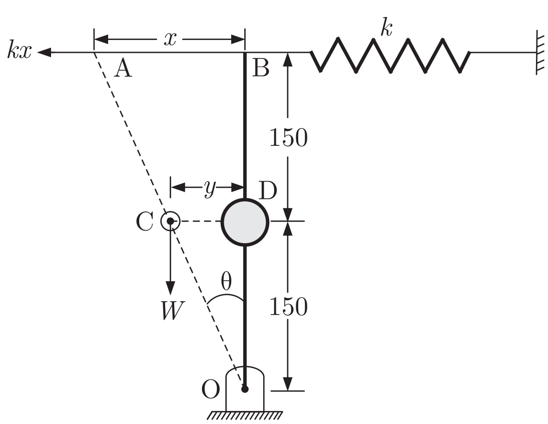
SOL 4.58 Option (A) is correct.

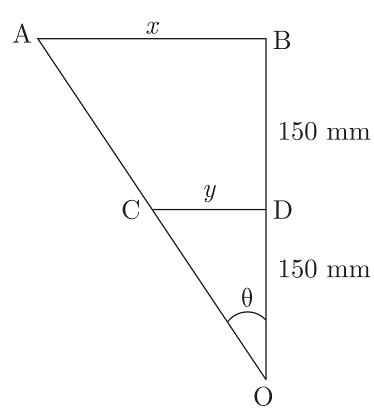
Given $N_2 = 120\text{ rpm}$, $v_1 = 12\text{ m/sec}$

So, coriolis component of the acceleration of link 1 is,

$$a_{12}^c = 2\omega_2 v_1 = 2 \times \frac{2\pi \times 120}{60} \times 12 = 301.44\text{ m/s}^2 \simeq 302\text{ m/s}^2$$

SOL 4.59 Option (C) is correct.





Given $l = 300\text{ mm} = 0.3\text{ m}$, $W = 300\text{ N}$

Let, rod is twisted to the left, through an angle θ.

From the similar triangle OCD & OAB,

$$\tan\theta = \frac{y}{0.15} = \frac{x}{0.30}$$

If θ is very very small, then $\tan\theta \simeq \theta = \frac{y}{0.15} = \frac{x}{0.30}$

$x = 0.30\theta$ and $y = 0.15\theta$ ..(i)

On taking moment about the hinged point O

$kx \times 300 + W \times y = 0$

$$k = -\frac{Wy}{300x} = -\frac{300}{300} \times \left(\frac{y}{x}\right) = -\frac{1}{2} = -0.5\text{ N/mm}$$

From equation (i) $\frac{y}{x} = \frac{0.15\theta}{0.30\theta} = -500\text{ N/m}$

Negative sign shows that the spring tends to move to the point B.

In magnitude, $k = 500\text{ N/m}$

SOL 4.60 Option (C) is correct.

	Types of Mechanisms		Motion Achieved
P.	Scott-Russel Mechanism	4.	Straight Line Motion
Q.	Geneva Mechanism	1.	Intermittent Motion
R.	Off-set slider-crank Mechanism	2.	Quick Return Mechanism
S.	Scotch Yoke Mechanism	3.	Simple Harmonic Motion

So, correct pairs are, P-4, Q-1, R-2, S-3

SOL 4.61 Option (C) is correct.

	Types of Joint		Degree of constraints
P.	Revolute	2.	Five
Q.	Cylindrical	3.	Four
R.	Spherical	1.	Three

So, correct pairs are P-2, Q-3, R-1

SOL 4.62 Option (A) is correct.

Given $M = 20\text{ kg}$, $l = 1000\text{ mm} = 1\text{ m}$, $A = 25 \times 25\text{ mm}^2$

$E_{steel} = 200\text{ GPa} = 200 \times 10^9\text{ Pa}$

Mass moment of inertia of a square section is given by,

$$I = \frac{b^4}{12} = \frac{(25 \times 10^{-3})^4}{12} = 3.25 \times 10^{-8}\text{ m}^4$$

Deflection of a cantilever, Loaded with a point load placed at the free end is,

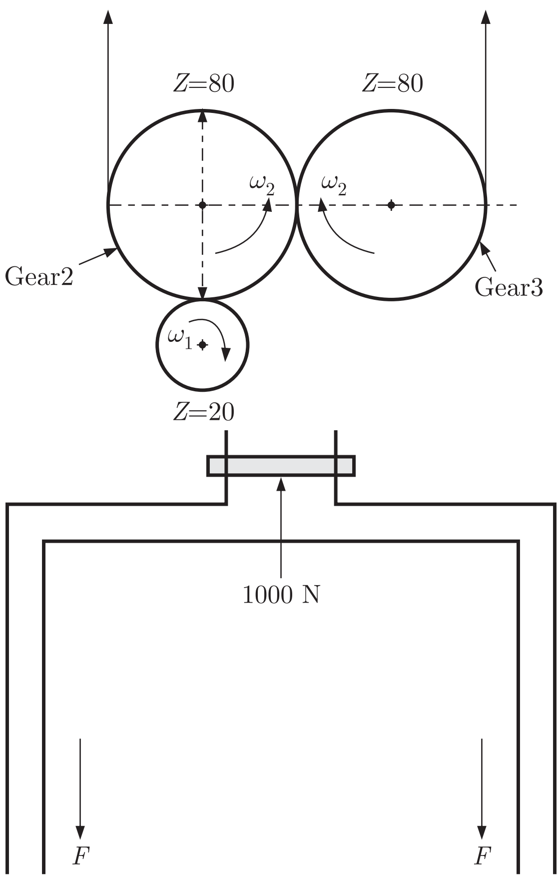
$$\delta = \frac{Wl^3}{3EI} = \frac{mgl^3}{3EI} = \frac{20 \times 9.81 \times (1)^3}{3 \times 200 \times 10^9 \times 3.25 \times 10^{-8}} = \frac{196.2}{19500} = 0.01 \text{ m}$$

$$\omega_n = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{9.81}{0.01}} = 31.32 \text{ rad/sec}$$

Therefore, critical damping constant

$$c_c = 2M\omega_n = 2 \times 20 \times 31.32$$
$$= 1252.8 \text{ Ns/m} \simeq 1250 \text{ Ns/m}$$

SOL 4.63 Option (B) is correct.



Let, Z is the number of teeth and motor rotates with an angular velocity ω_1 in clockwise direction & develops a torque T_1.

Due to the rotation of motor, the gear 2 rotates in anti-clockwise direction & gear 3 rotates in clock wise direction with the same angular speed.

Let, T_2 is the torque developed by gear.

Now, for two equal size big gears,

Module $$m = \frac{D}{Z} = \frac{\text{(Pitch circle diameter)}}{\text{(No.of teeths)}}$$

$$D = mZ = 2 \times 80 = 160 \text{ mm}$$

(Due to rotation of gear 2 & gear 3 an equal force (F) is generated in the downward direction because teeth are same for both the gears)

For equilibrium condition, we have

$$\text{Downward force} = \text{upward force}$$

$$F + F = 1000$$

$$F = 500\text{ N}$$

And $$\eta = \frac{\text{Power Output}}{\text{Power Input}} = \frac{2 \times T_2\omega_2}{T_1\omega_1}$$

Output power is generated by the two gears

$$= \frac{2 \times \left(F \times \frac{D}{2}\right)\omega_2}{T_1\omega_1} \qquad \text{...(i)}$$

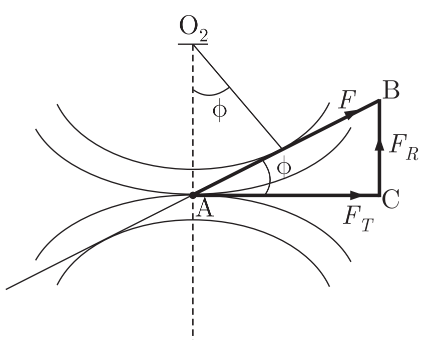
We know velocity ratio is given by

$$\frac{N_1}{N_2} = \frac{\omega_1}{\omega_2} = \frac{Z_2}{Z_1} \qquad \omega = \frac{2\pi N}{60}$$

From equation (i), $$\eta = \frac{2 \times \left(F \times \frac{D}{2}\right)}{T_1} \times \frac{Z_1}{Z_2}$$

$$T_1 = \frac{F \times D}{\eta} \times \left(\frac{Z_1}{Z_2}\right) = \frac{500 \times 0.160}{0.8} \times \frac{20}{80} = 25\text{ N-m}$$

SOL 4.64 Option (C) is correct.



Given pressure angle $\phi = 20^\circ$, $F_T = 500\text{ N}$ from previous question.

From the given figure we easily see that force action along the line of action is F.

From the triangle ABC,

$$\cos\phi = \frac{F_T}{F}$$

$$F = \frac{F_T}{\cos\phi} = \frac{500}{\cos 20^\circ} = 532\text{ N}$$

SOL 4.65 Option (D) is correct.

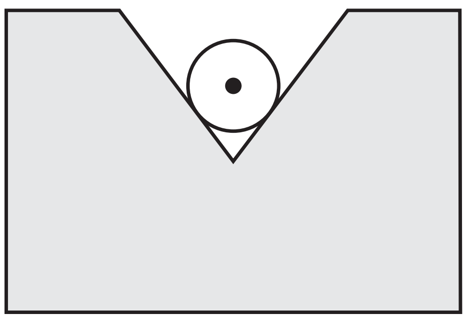
A single slider crank chain is a modification of the basic four bar chain. It is find, that four inversions of a single slider crank chain are possible. From these four inversions, crank and slotted lever quick return motion mechanism is used in shaping machines, slotting machines and in rotary internal combustion engines.

SOL 4.66 Option (A) is correct.

Given $p < q < r < s$

"Double crank" mechanism occurs, when the shortest link is fixed. From the given pairs p is the shortest link. So, link of length p should be fixed.

SOL 4.67 Option (B) is correct.



We clearly see from the figure that cylinder can either revolve about x-axis or slide along x-axis & all the motions are restricted.

Hence, Number of degrees of freedom $= 2$ & movability includes the six degrees of freedom of the device as a whole, as the ground link were not fixed. So, 4 degrees of freedom are constrained or arrested.

SOL 4.68 Option (B) is correct.

Given $N = 1200\text{ rpm}$, $\Delta E = 2\text{ kJ} = 2000\text{ J}$, $D = 1\text{ m}$, $C_s = 0.02$

Mean angular speed of engine,

$$\omega = \frac{2\pi N}{60} = \frac{2 \times 3.14 \times 1200}{60} = 125.66\text{ rad/sec}$$

Fluctuation of energy of the flywheel is given by,

$$\Delta E = I\omega^2 C_s = \frac{1}{2} m R^2 \omega^2 C_s \qquad \text{For solid disc } I = \frac{mR^2}{2}$$

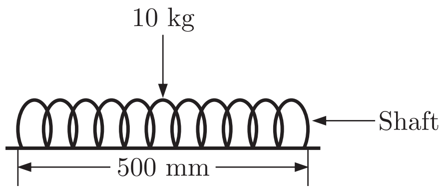
$$m = \frac{2\Delta E}{R^2 \omega^2 C_s} = \frac{2 \times 2000}{\left(\frac{1}{2}\right)^2 \times (125.66)^2 \times 0.02}$$

$$= \frac{4 \times 2 \times 2000}{(125.66)^2 \times 0.02} = 50.66\text{ kg} \simeq 51\text{ kg}$$

SOL 4.69 Option (B) is correct.

Given $m = 10\text{ kg}$, $d = 30\text{ mm} = 0.03\text{ m}$, $l = 500\text{ mm} = 0.5\text{ m}$,

$E_{shaft} = 2.1 \times 10^{11}$ Pa



We know that, static deflection due to 10 kg of Mass at the centre is given by,

$$\delta = \frac{Wl^3}{48EI} = \frac{mgl^3}{48EI} \qquad \text{...(i)}$$

The moment of inertia of the shaft,

$$I = \frac{\pi}{64}d^4 = \frac{\pi}{64}(0.03)^4 = 3.974 \times 10^{-8}\,\text{m}^4 \qquad \text{...(ii)}$$

Substitute values in equation (i), we get

$$\delta = \frac{10 \times 9.81 \times (0.5)^3}{48 \times 2.1 \times 10^{11} \times 3.974 \times 10^{-8}}$$

$$= \frac{12.2625}{400.58 \times 10^3} = 3.06 \times 10^{-5}\,\text{m}$$

If ω_c is the critical or whirling speed in r.p.s. then,

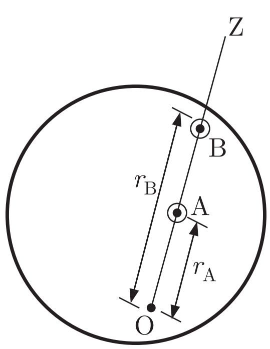
$$\omega_c = \sqrt{\frac{g}{\delta}} \qquad \Rightarrow 2\pi f_c = \sqrt{\frac{g}{\delta}}$$

$$f_c = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{1}{2 \times 3.14}\sqrt{\frac{9.81}{3.06 \times 10^{-5}}}$$

$$= \frac{1}{6.28}\sqrt{\frac{9.81}{30.6 \times 10^{-6}}} = 90.16\,\text{Hz} \simeq 90\,\text{Hz}$$

SOL 4.70 Option (C) is correct.

Given, the circular disc rotates about the point O at a uniform angular velocity ω.



Let v_A is the linear velocity of point A & v_B is the linear velocity of point B.

$v_A = \omega r_A$ and $v_B = \omega r_B$.

Velocity of point B with respect to point A is given by,

$$v_{BA} = v_B - v_A = \omega r_B - \omega r_A = \omega(r_B - r_A)$$

From the given figure,

$$r_B > r_A$$

So, $$\omega r_B > \omega r_A$$

$$v_B > v_A$$

Therefore, relative velocity $\omega(r_B - r_A)$ in the direction of point B.

SOL 4.71 Option (D) is correct.

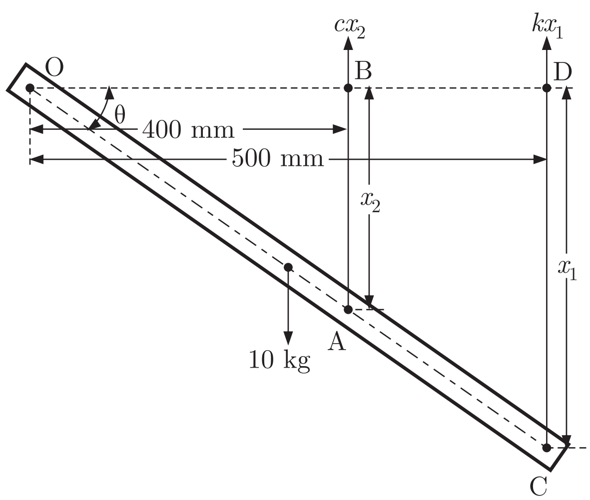
Acceleration of point B with respect to point A is given by,

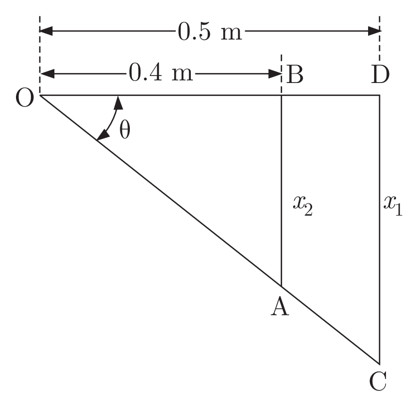
$$a_{BA} = \omega v_{BA} = \omega \times \omega(r_B - r_A) = \omega^2(r_B - r_A) \quad ...(i)$$

This equation (i) gives the value of centripetal acceleration which acts always towards the centre of rotation.

So, a_{BA} acts towards to O i.e. its direction from Z to O

SOL 4.72 Option (A) is correct.





Given $m = 10\,\text{kg}$, $k = 2\,\text{kN/m}$, $c = 500\,\text{Ns/m}$, $k_\theta = 1\,\text{kN/m/rad}$

$l_1 = 0.5\,\text{m}$, $l_2 = 0.4\,\text{m}$

Let, the rigid slender bar twist downward at the angle θ. Now spring & damper exert a force kx_1 & cx_2 on the rigid bar in the upward direction.

From similar triangle OAB & OCD,

$$\tan\theta = \frac{x_2}{0.4} = \frac{x_1}{0.5}$$

Let θ be very very small, then $\tan\theta \simeq \theta$,

$$\theta = \frac{x_2}{0.4} = \frac{x_1}{0.5}$$

$$x_2 = 0.4\theta \text{ or } x_1 = 0.5\theta \quad ...(i)$$

On differentiating the above equation, we get

$$\dot{x}_2 = 0.4\dot{\theta} \text{ or } \dot{x}_1 = 0.5\dot{\theta} \qquad ...(ii)$$

We know, the moment of inertia of the bar hinged at the one end is,

$$I = \frac{ml_1^2}{3} = \frac{10 \times (0.5)^2}{3} = 0.833 \text{ kg - m}^2$$

As no external force acting on the system. So, governing equation of motion from the Newton's law of motion is,

$$I\ddot{\theta} + c\dot{x}_2 l_2 + kx_1 l_1 + k_\theta \theta = 0$$

$$0.833\ddot{\theta} + 500 \times 0.4\dot{x}_2 + 2000 \times (0.5)x_1 + 1000\theta = 0$$

$$0.833\ddot{\theta} + 200\dot{x}_2 + 1000x_1 + 1000\theta = 0 \qquad ...(iii)$$

$$0.833\ddot{\theta} + 200 \times 0.4\dot{\theta} + 1000 \times 0.5\theta + 1000\theta = 0$$

$$0.833\ddot{\theta} + 80\dot{\theta} + 1500\theta = 0 \qquad ...(iv)$$

On comparing equation (iv) with its general equation,

$$I\ddot{\theta} + c\dot{\theta} + k\theta = 0$$

We get, $I = 0.833$, $c = 80$, $k = 1500$

So, undamped natural frequency of oscillations is given by

$$\omega_n = \sqrt{\frac{k}{I}} = \sqrt{\frac{1500}{0.833}} = \sqrt{1800.72} = 42.43 \text{ rad/sec}$$

SOL 4.73 Option (C) is correct.

From the previous part of the question

Damping coefficient, $c = 80$ Nms/rad

SOL 4.74 Option (C) is correct.

From the Kutzbach criterion the degree of freedom,

$$n = 3(l-1) - 2j - h$$

For single degree of Freedom ($n = 1$),

$$1 = 3(l-1) - 2j - h$$

$$3l - 2j - 4 - h = 0 \qquad ...(i)$$

The simplest possible mechanisms of single degree of freedom is four-bar mechanism. For this mechanism $j = 4$, $h = 0$

From equation (i), we have

$$3l - 2 \times 4 - 4 - 0 = 0 \Rightarrow l = 4$$

SOL 4.75 Option (B) is correct.

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the coriolis component of the acceleration must be calculated. Quick return motion mechanism is used in shaping machines, slotting machines and in rotary internal combustion engines.

SOL 4.76 Option (C) is correct.

The deflection of a cantilever beam loaded at the free end is given by,

$$\delta = \frac{MgL^3}{3EI}$$

And natural frequency,

$$\omega_n = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{3EI}{ML^3}} \quad \text{...(i)}$$

If the length of the cantilever beam is halved, then

$$\omega_n' = \sqrt{\frac{3EI}{M \times \left(\frac{L}{2}\right)^3}} = \sqrt{8\left(\frac{3EI}{ML^3}\right)}$$

From equation (i)

$$\omega_n' = \sqrt{8}\,\omega_n$$

So, natural frequency is increased by a factor $\sqrt{8}$.

SOL 4.77 Option (C) is correct.

For a spring loaded roller follower driven with a disc cam, the pressure angle should be large during rise as well as during return for ease of transmitting motion.

If pressure angle is large, then side thrust will be minimum. Pressure angles of up to about 30° to 35° are about the largest that can be used without causing difficulties.

SOL 4.78 Option (B) is correct.

Let initial length of the spring $= L$

Potential energy at A,

$$PE_A = mg(L - \delta)$$

and at B,

$$PE_B = mg[L - (\delta + x)] + \frac{1}{2}kx^2$$

So, change in potential energy from position A to position B is

$$\Delta PE_{AB} = PE_B - PE_A$$

$$= mgL - mg\delta - mgx + \frac{1}{2}kx^2 - mgL + mg\delta$$

$$\Delta PE_{AB} = \frac{1}{2}kx^2 - mgx$$

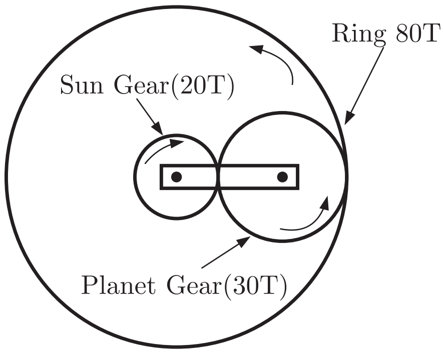
SOL 4.79 Option (A) is correct.

The mean speed of the engine is controlled by the governor. If load increases then fluid supply increases by the governor and vice-versa.

Flywheel stores the extra energy and delivers it when needed. So, Flywheel reduces speed fluctuations.

Flywheel reduce speed fluctuations during a cycle for a constant load, but Flywheel does not control the mean speed $\left(N = \frac{N_1 + N_2}{2}\right)$ of the engine.

SOL 4.80 Option (B) is correct.
First make the table for the motion of the gears.
Take CW $=+ve$, CCW $=-ve$



S. No.	Condition of Motion	Arm	Sun Gear N_S	Planet Gear N_P	Ring Gear N_G
(i)	Arm is fixed & sun gear rotates +1 rpm (CW)	0	$+1$	$-\frac{Z_S}{Z_P}$	$-\frac{Z_S}{Z_R}$
(ii)	Sun Gear rotates through $+x$ rpm (CW)	0	$+x$	$-x\frac{Z_S}{Z_P}$	$-x\frac{Z_S}{Z_R}$
(iii)	Add $+y$ revolution to all elements	$+y$	$+y$	$+y$	$+y$
(iv)	Total Motion	$+y$	$x+y$	$y-x\frac{Z_S}{Z_P}$	$y-x\frac{Z_S}{Z_R}$

Let Teethes and speed of the sum gear, planet gear and ring gear is represented by Z_G, Z_P, Z_R and N_G, N_P, N_R respectively.
Given sun gear is driven clockwise at 100 rpm. So, From the table

$$x + y = 100 \qquad ...(i)$$

Ring gear is held stationary. From the table

$$y - x\frac{Z_S}{Z_P} = 0$$

$$y = x \times \frac{20}{80}$$

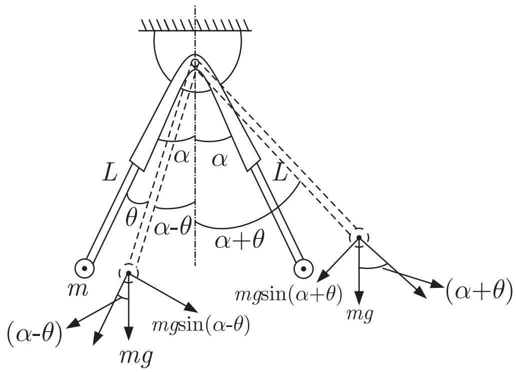
$$y = \frac{x}{4} \qquad \Rightarrow x = 4y \qquad ...(ii)$$

From equation on (i) and (ii)

$$4y + y = 100$$
$$y = 20\,\text{rpm}$$

SOL 4.81 Option (D) is correct.

Give a small displacement θ to the assembly. So assembly oscillates about its mean position.



From this a restoring torque is acts along the line of oscillation.

Net restoring torque,

$$T = mg\sin(\alpha+\theta)\times L - mg\sin(\alpha-\theta)\times L$$
$$T = mgL[\sin\alpha\cos\theta + \cos\alpha\sin\theta - \sin\alpha\cos\theta + \cos\alpha\sin\theta]$$
$$T = 2mgL\cos\alpha\sin\theta$$

For very small deflection θ,

$$\sin\theta \cong \theta$$
$$T = 2mgL\theta\cos\alpha$$

Now from newton's law,

$$I\ddot{\theta} + T = 0$$
$$I\ddot{\theta} + 2mgL\theta\cos\alpha = 0$$
$$2mL^2\frac{d^2\theta}{dt^2} + (2mgL\cos\alpha)\theta = 0 \qquad I = mL^2 + mL^2$$
$$\frac{d^2\theta}{dt^2} + \frac{g\cos\alpha}{L}\theta = 0$$

On comparing with $\ddot{\theta} + \omega_n^2\theta = 0$, we get

$$\omega_n^2 = \frac{g\cos\alpha}{L}$$
$$\omega_n = \sqrt{\frac{g\cos\alpha}{L}}$$

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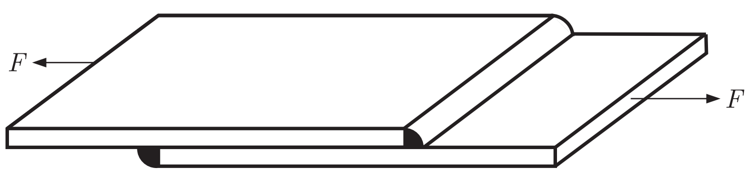
CHAPTER 5

MACHINE DESIGN

YEAR 2012 TWO MARKS

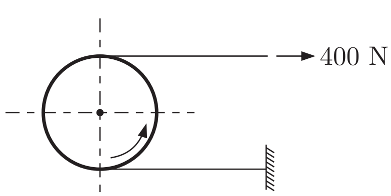
MCQ 5.1 A fillet welded joint is subjected to transverse loading F as shown in the figure. Both legs of the fillets are of 10 mm size and the weld length is 30 mm. If the allowable shear stress of the weld is 94 MPa, considering the minimum throat area of the weld, the maximum allowable transverse load in kN is



(A) 14.44 (B) 17.92

(C) 19.93 (D) 22.16

MCQ 5.2 A force of 400 N is applied to the brake drum of 0.5 m diameter in a band-brake system as shown in the figure, where the wrapping angle is 180°. If the coefficient of friction between the drum and the band is 0.25, the braking torque applied, in Nm is



(A) 100.6 (B) 54.4

(C) 22.1 (D) 15.7

MCQ 5.3 A solid circular shaft needs to be designed to transmit a torque of 50 Nm. If the allowable shear stress of the material is 140 MPa, assuming a factor of safety of 2, the minimum allowable design diameter is mm is

(A) 8 (B) 16

(C) 24 (D) 32

YEAR 2011 TWO MARKS

MCQ 5.4 Two identical ball bearings P and Q are operating at loads 30 kN and 45 kN respectively. The ratio of the life of bearing P to the life of bearing Q is

(A) $\frac{81}{16}$ (B) $\frac{27}{8}$

(C) $\frac{9}{4}$ (D) $\frac{3}{2}$

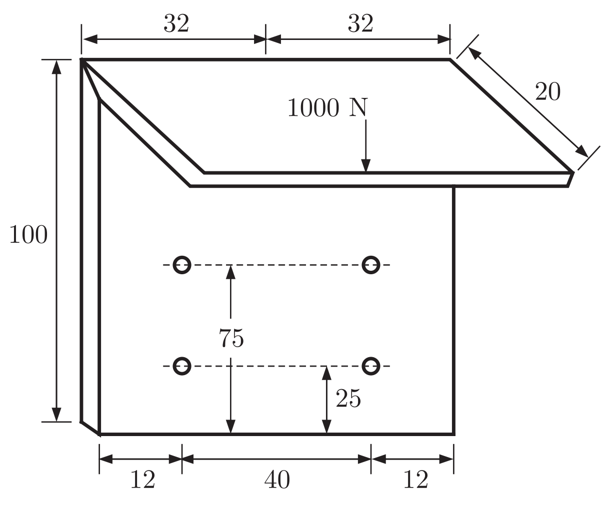
YEAR 2010 TWO MARKS

MCQ 5.5 A band brake having band-width of 80 mm, drum diameter of 250 mm, coefficient of friction of 0.25 and angle of wrap of 270 degrees is required to exert a friction torque of 1000 Nm. The maximum tension (in kN) developed in the band is

(A) 1.88 (B) 3.56

(C) 6.12 (D) 11.56

MCQ 5.6 A bracket (shown in figure) is rigidly mounted on wall using four rivets. Each rivet is 6 mm in diameter and has an effective length of 12 mm.



Direct shear stress (in MPa) in the most heavily loaded rivet is

(A) 4.4 (B) 8.8

(C) 17.6 (D) 35.2

MCQ 5.7 A lightly loaded full journal bearing has journal diameter of 50 mm, bush bore of 50.05 mm and bush length of 20 mm. If rotational speed of journal is 1200 rpm and average viscosity of liquid lubricant is 0.03 Pa s, the power loss (in W) will be

(A) 37 (B) 74

(C) 118 (D) 237

YEAR 2009 TWO MARKS

MCQ 5.8 A forged steel link with uniform diameter of 30 mm at the centre is subjected to an axial force that varies from 40 kN in compression to 160 kN in tension. The tensile (S_u), yield (S_y) and corrected endurance (S_e) strengths of the steel material are 600 MPa, 420 MPa and 240 MPa respectively. The factor of safety against fatigue endurance as per Soderberg's criterion is

(A) 1.26 (B) 1.37

(C) 1.45 (D) 2.00

- Common Data For Q.9 and Q.10

A 20° full depth involute spur pinion of 4 mm module and 21 teeth is to transmit 15 kW at 960 rpm. Its face width is 25 mm.

MCQ 5.9 The tangential force transmitted (in N) is

(A) 3552 (B) 2611

(C) 1776 (D) 1305

MCQ 5.10 Given that the tooth geometry factor is 0.32 and the combined effect of dynamic load and allied factors intensifying the stress is 1.5; the minimum allowable stress (in MPa) for the gear material is

(A) 242.0 (B) 166.5

(C) 121.0 (D) 74.0

YEAR 2008 TWO MARKS

MCQ 5.11 A journal bearing has a shaft diameter of 40 mm and a length of 40 mm. The shaft is rotating at 20 rad/s and the viscosity of the lubricant is 20 mPa-s . The clearance is 0.020 mm. The loss of torque due to the viscosity of the lubricant is approximately.

(A) 0.040 N-m (B) 0.252 N-m

(C) 0.400 N-m (D) 0.652 N-m

MCQ 5.12 A clutch has outer and inner diameters 100 mm and 40 mm respectively. Assuming a uniform pressure of 2 MPa and coefficient of friction of liner material is 0.4, the torque carrying capacity of the clutch is

(A) 148 N-m (B) 196 N-m

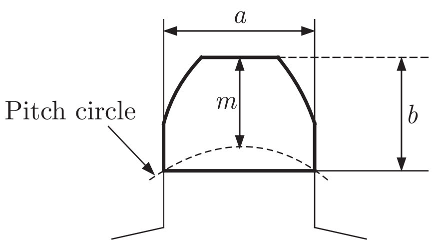
(C) 372 N-m (D) 490 N-m

MCQ 5.13 A spur gear has a module of 3 mm, number of teeth 16, a face width of 36 mm and a pressure angle of 20°. It is transmitting a power of 3 kW at 20 rev/s. Taking a velocity factor of 1.5 and a form factor of 0.3, the stress in the gear tooth is about.

(A) 32 MPa (B) 46 MPa

(C) 58 MPa (D) 70 MPa

MCQ 5.14 One tooth of a gear having 4 module and 32 teeth is shown in the figure. Assume that the gear tooth and the corresponding tooth space make equal intercepts on the pitch circumference. The dimensions '*a*' and '*b*', respectively, are closest to



(A) 6.08 mm, 4 mm (B) 6.48 mm, 4.2 mm

(C) 6.28 mm, 4.3 mm (D) 6.28 mm, 4.1 mm

MCQ 5.15 Match the type of gears with their most appropriate description.

Type of gear

P. Helical

Q. Spiral Bevel

C. Hypoid

S. Rack and pinion

Description

1. Axes non parallel and non intersecting
2. Axes parallel and teeth are inclined to the axis
3. Axes parallel and teeth are parallel to the axis
4. Axes are perpendicular and intersecting, and teeth are inclined to the axis.
5. Axes are perpendicular and used for large speed reduction

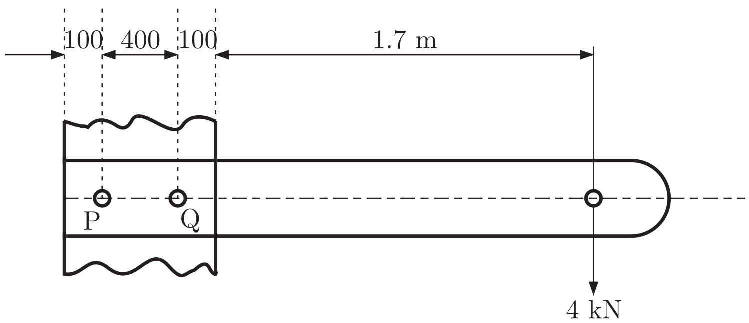
6. Axes parallel and one of the gears has infinite radius

(A) P-2, Q-4, R-1, S-6 (B) P-1, Q-4, R-5, S-6

(C) P-2, Q-6, R-4, S-2 (D) P-6, Q-3, R-1, S-5

• Common Data For Q.16 and Q.17

A steel bar of 10×50 mm is cantilevered with two M 12 bolts (P and Q) to support a static load of 4 kN as shown in the figure.



MCQ 5.16 The primary and secondary shear loads on bolt P, respectively, are

(A) 2 kN, 20 kN (B) 20 kN, 2 kN

(C) 20 kN, 0 kN (D) 0 kN, 20 kN

MCQ 5.17 The resultant shear stress on bolt P is closest to

(A) 132 MPa (B) 159 MPa

(C) 178 MPa (D) 195 MPa

YEAR 2007 ONE MARK

MCQ 5.18 A ball bearing operating at a load F has 8000 hours of life. The life of the bearing, in hours, when the load is doubled to $2F$ is

(A) 8000 (B) 6000

(C) 4000 (D) 1000

YEAR 2007 TWO MARKS

MCQ 5.19 A thin spherical pressure vessel of 200 mm diameter and 1 mm thickness is subjected to an internal pressure varying form 4 to 8 MPa. Assume that the yield, ultimate and endurance strength of material are 600, 800 and 400 MPa respectively. The factor of safety as per Goodman's relation is

(A) 2.0 (B) 1.6

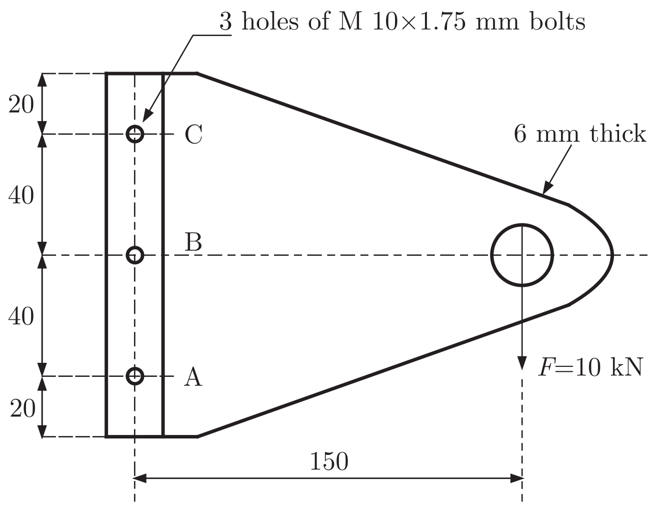
(C) 1.4 (D) 1.2

MCQ 5.20 A natural feed journal bearing of diameter 50 mm and length 50 mm operating at 20 revolution/ second carries a load of 2 kN. The lubricant used has a viscosity of 20 mPas. The radial clearance is 50 μm. The Sommerfeld number for the bearing is

(A) 0.062 (B) 0.125

(C) 0.250 (D) 0.785

MCQ 5.21 A bolted joint is shown below. The maximum shear stress, in MPa in the bolts at A and B , respectively are

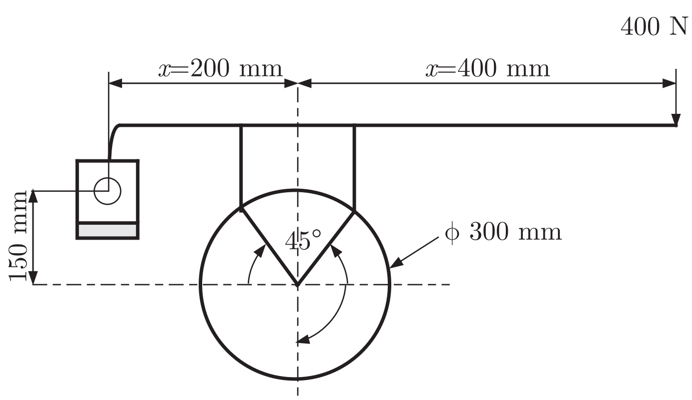


(all dimensions in the figure are in mm)

(A) 242.6, 42.5 (B) 42.5, 242.6

(C) 42.5, 42.5 (D) 18.75, 343.64

MCQ 5.22 A block-brake shown below has a face width of 300 mm and a mean co-efficient of friction of 0.25. For an activating force of 400 N, the braking torque in Nm is



(A) 30 (B) 40

(C) 45 (D) 60

MCQ 5.23 The piston rod of diameter 20 mm and length 700 mm in a hydraulic cylinder is subjected to a compressive force of 10 kN due to the internal pressure. The end conditions for the rod may be assumed as guided at the piston end and hinged at the other end. The Young's modulus is 200 GPa. The factor of safety for the piston rod is

(A) 0.68 (B) 2.75

(C) 5.62 (D) 11.0

• Common Data For Q.24 and Q.26

A gear set has a pinion with 20 teeth and a gear with 40 teeth. The pinion runs at 30 rev/s and transmits a power of 20 kW. The teeth are on the 20° full-depth system and have a module of 5 mm. The length of the line of action is 19 mm.

MCQ 5.24 The center distance for the above gear set in mm is

(A) 140 (B) 150

(C) 160 (D) 170

MCQ 5.25 The contact ratio of the contacting tooth is

(A) 1.21 (B) 1.25

(C) 1.29 (D) 1.33

MCQ 5.26 The resultant force on the contacting gear tooth in N is

(A) 77.23 (B) 212.20

(C) 2258.1 (D) 289.43

YEAR 2006 TWO MARKS

MCQ 5.27 A disc clutch is required to transmit 5 kW at 2000 rpm. The disk has a friction lining with coefficient of friction equal to 0.25. Bore radius of friction lining is equal to 25 mm. Assume uniform contact pressure of 1 MPa. The value of outside radius of the friction lining is

(A) 39.4 mm (B) 49.5 mm

(C) 97.9 mm (D) 142.9 mm

MCQ 5.28 Twenty degree full depth involute profiled 19 tooth pinion and 37 tooth gear are in mesh. If the module is 5 mm, the centre distance between the gear pair will be

(A) 140 mm (B) 150 mm

(C) 280 mm (D) 300 mm

MCQ 5.29 A cylindrical shaft is subjected to an alternating stress of $100\,\text{MPa}$. Fatigue strength to sustain 1000 cycles is $490\,\text{MPa}$. If the corrected endurance strength is $70\,\text{MPa}$, estimated shaft life will be

(A) 1071 cycles (B) 15000 cycles

(C) 281914 cycles (D) 928643 cycles

MCQ 5.30 A $60\,\text{mm}$ long and $6\,\text{mm}$ thick fillet weld carries a steady load of $15\,\text{kN}$ along the weld. The shear strength of the weld material is equal to $200\,\text{MPa}$. The factor of safety is

(A) 2.4 (B) 3.4

(C) 4.8 (D) 6.8

YEAR 2005 ONE MARK

MCQ 5.31 Which one of the following is criterion in the design of hydrodynamic journal bearings ?

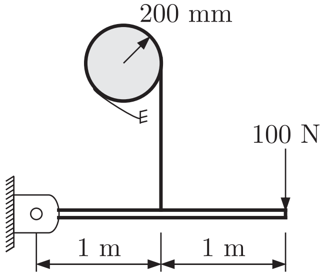
(A) Sommerfeld number (B) Rating life

(C) Specific dynamic capacity (D) Rotation factor

YEAR 2005 TWO MARKS

• Common Data For Q.32 and Q.33

A band brake consists of a lever attached to one end of the band. The other end of the band is fixed to the ground. The wheel has a radius of $200\,\text{mm}$ and the wrap angle of the band is 270°. The braking force applied to the lever is limited to $100\,\text{N}$ and the coefficient of friction between the band and the wheel is 0.5. No other information is given.



MCQ 5.32 The maximum tension that can be generated in the band during braking is

(A) 1200 N (B) 2110 N

(C) 3224 N (D) 4420 N

MCQ 5.33 The maximum wheel torque that can be completely braked is

(A) 200 Nm (B) 382 Nm

(C) 604 Nm (D) 844 Nm

YEAR 2004 ONE MARK

MCQ 5.34 Two mating spur gears have 40 and 120 teeth respectively. The pinion rotates at 1200 rpm and transmits a torque of 20 Nm. The torque transmitted by the gear is

(A) 6.6 Nm (B) 20 Nm

(C) 40 Nm (D) 60 Nm

MCQ 5.35 In terms of theoretical stress concentration factor (K_t) and fatigue stress concentration factor (K_f), the notch sensitivity 'q' is expressed as

(A) $\frac{(K_f - 1)}{(K_t - 1)}$ (B) $\frac{(K_f - 1)}{(K_t + 1)}$

(C) $\frac{(K_t - 1)}{(K_f - 1)}$ (D) $\frac{(K_f + 1)}{(K_t + 1)}$

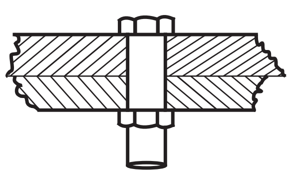
MCQ 5.36 The S-N curve for steel becomes asymptotic nearly at

(A) 10^3 cycles (B) 10^4 cycles

(C) 10^6 cycles (D) 10^9 cycles

YEAR 2004 TWO MARKS

MCQ 5.37 In a bolted joint two members are connected with an axial tightening force of 2200 N. If the bolt used has metric threads of 4 mm pitch, the torque required for achieving the tightening force is



(A) 0.7 Nm (B) 1.0 Nm

(C) 1.4 Nm (D) 2.8 Nm

MCQ 5.38 Match the following

	Type of gears		Arrangement of shafts
P.	Bevel gears	1.	Non-parallel off-set shafts
Q.	Worm gears	2.	Non-parallel intersecting shafts
R.	Herringbone gears	3.	Non-parallel, non-intersecting shafts
S.	Hypoid gears	4.	Parallel shafts

(A)	P-4	Q-2	R-1	S-3
(B)	P-2	Q-3	R-4	S-1
(C)	P-3	Q-2	R-1	S-4
(D)	P-1	Q-3	R-4	S-2

YEAR 2003 ONE MARK

MCQ 5.39 A wire rope is designated as 6×19 standard hoisting. The numbers 6×19 represent

(A) diameter in millimeter $\times$ length in meter

(B) diameter in centimeter $\times$ length in meter

(C) number of strands $\times$ numbers of wires in each strand

(D) number of wires in each strand $\times$ number of strands

YEAR 2003 TWO MARKS

MCQ 5.40 Square key of side "$d/4$" each and length 'l' is used to transmit torque "T" from the shaft of diameter "d" to the hub of a pulley. Assuming the length of the key to be equal to the thickness of pulley, the average shear stress developed in the key is given by

(A) $\frac{4T}{ld}$ (B) $\frac{16T}{ld^2}$

(C) $\frac{8T}{ld^2}$ (D) $\frac{16T}{\pi d^3}$

MCQ 5.41 In a band brake the ratio of tight side band tension to the tension on the slack side is 3. If the angle of overlap of band on the drum is $180°$, the coefficient of friction required between drum and the band is

(A) 0.20 (B) 0.25

(C) 0.30 (D) 0.35

• Common Data For Q.42 and Q.43

The overall gear ratio in a 2 stage speed reduction gear box (with all spur gears) is 12. The input and output shafts of the gear box are collinear. The counter shaft which is parallel to the input and output shafts has a gear (Z_2 teeth) and pinion ($Z_3 = 15$ teeth) to mesh with pinion ($Z_1 = 16$ teeth) on the input shaft and gear (Z_4 teeth) on the output shaft respectively. It was decided to use a gear ratio of 4 with 3 module in the first stage and 4 module in the second stage.

MCQ 5.42 Z_2 and Z_4 are

(A) 64 and 45 (B) 45 and 64

(C) 48 and 60 (D) 60 and 48

MCQ 5.43 The centre distance in the second stage is

(A) 90 mm (B) 120 mm

(C) 160 mm (D) 240 mm

YEAR 2002 ONE MARK

MCQ 5.44 The minimum number of teeth on the pinion to operate without interference in standard full height involute teeth gear mechanism with 20° pressure angle is

(A) 14 (B) 12

(C) 18 (D) 32

YEAR 2002 TWO MARKS

MCQ 5.45 The coupling used to connect two shafts with large angular misalignment is

(A) a flange coupling

(B) an Oldham's coupling

(C) a flexible bush coupling

(D) a Hooke's joint

MCQ 5.46 A static load is mounted at the centre of a shaft rotating at uniform angular velocity. This shaft will be designed for

(A) the maximum compressive stress (static)

(B) the maximum tensile (static)

(C) the maximum bending moment (static)

(D) fatigue loading

MCQ 5.47 Large speed reductions (greater than 20) in one stage of a gear train are possible through

(A) spur gearing

(B) worm gearing

(C) bevel gearing

(D) helical gearing

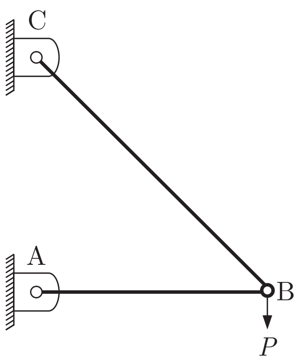
MCQ 5.48 If the wire diameter of a closed coil helical spring subjected to compressive load is increased from 1 cm to 2 cm, other parameters remaining same, the deflection will decrease by a factor of

(A) 16 (B) 8

(C) 4 (D) 2

YEAR 2001 ONE MARK

MCQ 5.49 Bars AB and BC, each of negligible mass, support load P as shown in the figure. In this arrangement,



(A) bar AB is subjected to bending but bar BC is not subjected to bending.

(B) bar AB is not subjected to bending but bar BC is subjected to bending.

(C) neither bar AB nor bar BC is subjected to bending.

(D) both bars AB and BC are subjected to bending.

YEAR 2001 TWO MARKS

MCQ 5.50 Two helical tensile springs of the same material and also having identical mean coil diameter and weight, have wire diameters d and $d/2$. The ratio of their stiffness is

(A) 1 (B) 4

(C) 64 (D) 128

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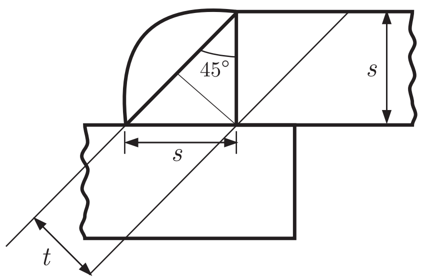
SOLUTION

SOL 5.1 Option (C) is correct.

Given : Width of fillets $s = 10\,\text{mm}$, $l = 30\,\text{mm}$, $\tau = 94\,\text{MPa}$



The shear strength of the joint for single parallel fillet weld is,

$$P = \text{Throat Area} \times \text{Allowable stress}$$
$$= t \times l \times \tau$$

From figure
$$t = s\sin 45^\circ = 0.707\,s$$
$$P = 0.707 \times s \times l \times \tau$$
$$= 0.707 \times (0.01) \times (0.03) \times (94 \times 10^6)$$
$$= 19937\,\text{N or } 19.93\,\text{kN}$$

SOL 5.2 Option (B) is correct.

Given :
$$T_1 = 400\,\text{N},\ \mu = 0.25,\ \theta = 180^\circ = 180^\circ \times \frac{\pi}{180^\circ} = \pi\,\text{rad.}$$
$$D = 0.5\,\text{m},\ r = \frac{D}{2} = 0.25\,\text{m}$$

For the band brake, the limiting ratio of the tension is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu\theta}$$
$$\frac{400}{T_2} = e^{0.25 \times \pi} = 2.19$$
$$T_2 = \frac{400}{2.19} = 182.68\,\text{N}$$

For Band-drum brake, Braking Torque is

$$T_B = (T_1 - T_2) \times r$$
$$= (400 - 182.68) \times 0.25 = 54.33\,\text{Nm} \cong 54.4\,\text{Nm}$$

SOL 5.3 Option (B) is correct.

$$F.O.S = \frac{\text{Allowable shear stress}}{\text{Design shear stress}}$$

Design shear stress for solid circular shaft

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 50 \times 10^3}{\pi d^3} \qquad \text{From } \frac{T}{J} = \frac{\tau}{r}$$

Therefore
$$F.O.S = \frac{140 \times \pi d^3}{16 \times 50 \times 10^3}$$

or,
$$2 = \frac{140 \times \pi d^3}{16 \times 50 \times 10^3}$$

$$d^3 = \frac{2 \times 16 \times 50 \times 10^3}{140 \times \pi}$$

$$d = 15.38 \text{ mm} \cong 16 \text{ mm}$$

SOL 5.4 Option (B) is correct.

Given : $W_P = 30$ kN, $W_Q = 45$ kN

Life of bearing,
$$L = \left(\frac{C}{W}\right)^k \times 10^6 \text{ revolutions}$$

C = Basic dynamic load rating = Constant

For ball bearing, $k = 3$

So,
$$L = \left(\frac{C}{W}\right)^3 \times 10^6 \text{ revolutions}$$

These are the identical bearings. So for the Life of P and Q.

$$\left(\frac{L_P}{L_Q}\right) = \left(\frac{W_Q}{W_P}\right)^3 = \left(\frac{45}{30}\right)^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$$

SOL 5.5 Option (D) is correct.

Given : $b = 80$ mm, $d = 250$ mm, $\mu = 0.25$, $\theta = 270^\circ$, $T_B = 1000$ N-m

Let, $T_1 \rightarrow$ Tension in the tight side of the band (Maximum Tension)

$T_2 \rightarrow$ Tension in the slack side of the band (Minimum Tension)

Braking torque on the drum,

$$T_B = (T_1 - T_2) r$$

$$T_1 - T_2 = \frac{T_B}{r} = \frac{1000}{0.125} = 8000 \text{ N} \qquad \text{...(i)}$$

We know that limiting ratio of the tension is given by,

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{\left(0.25 \times \frac{\pi}{180} \times 270\right)} = 3.246$$

$$T_2 = \frac{T_1}{3.246}$$

Substitute T_2 in equation (i), we get

$$T_1 - \frac{T_1}{3.246} = 8000 \quad \Rightarrow \quad 3.246 T_1 - T_1 = 25968$$

$$2.246 T_1 = 25968 \quad \Rightarrow \quad T_1 = \frac{25968}{2.246} = 11.56 \text{ kN}$$

SOL 5.6 Option (B) is correct.

Given : $d = 6\text{ mm},\ l = 12\text{ mm},\ P = 1000\text{ N}$

Each rivets have same diameter, So equal Load is carried by each rivet.

Primary or direct force on each rivet,

$$F = \frac{P}{4} = \frac{1000}{4} = 250\text{ N}$$

Shear area of each rivet is,

$$A = \frac{\pi}{4}(6 \times 10^{-3})^2 = 28.26 \times 10^{-6}\text{ mm}^2$$

Direct shear stress on each rivet,

$$\tau = \frac{F}{A} = \frac{250}{28.26 \times 10^{-6}} = 8.84 \times 10^6 \simeq 8.8\text{ MPa}$$

SOL 5.7 Option (A) is correct.

Given : $d = 50\text{ mm},\ D = 50.05\text{ mm},\ l = 20\text{ mm},\ N = 1200\text{ rpm},\ \mu = 0.03\text{ Pa s}$

Tangential velocity of shaft,

$$u = \frac{\pi dN}{60} = \frac{3.14 \times 50 \times 10^{-3} \times 1200}{60} = 3.14\text{ m/sec}$$

And Radial clearance, $y = \frac{D-d}{2} = \frac{50.05 - 50}{2} = 0.025\text{ mm}$

Shear stress from the Newton's law of viscosity,

$$\tau = \mu \times \frac{u}{y} = 0.03 \times \frac{3.14}{0.025 \times 10^{-3}} = 3768\text{ N/m}^2$$

Shear force on the shaft,

$$F = \tau \times A = 3768 \times (\pi \times d \times l)$$

$$= 3768 \times 3.14 \times 50 \times 10^{-3} \times 20 \times 10^{-3} = 11.83\text{ N}$$

Torque, $T = F \times \frac{d}{2} = 11.83 \times \frac{50}{2} \times 10^{-3} = 0.2957\text{ N-m}$

We know that power loss,

$$P = \frac{2\pi NT}{60} = \frac{2 \times 3.14 \times 1200 \times 0.2957}{60}$$

$$= 37.13\text{ W} \simeq 37\text{ W}$$

SOL 5.8 Option (A) is correct.

Given : S_u or $\sigma_u = 600\text{ MPa},\ S_y$ or $\sigma_y = 420\text{ MPa},\ S_e$ or $\sigma_e = 240\text{ MPa}$,
$d = 30\text{ mm}\ \ F_{max} = 160\text{ kN}$ (Tension), $F_{min} = -40\text{ kN}$ (Compression)

Maximum stress, $\sigma_{max} = \frac{F_{max}}{A} = \frac{160 \times 10^3}{\frac{\pi}{4}(30)^2} = 226.47\text{ MPa}$

Minimum stress, $\sigma_{min} = \frac{F_{min}}{A} = -\frac{40 \times 10^3}{\frac{\pi}{4} \times (30)^2} = -56.62\text{ MPa}$

Mean stress, $\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2} = \frac{226.47 - 56.62}{2} = 84.925\,\text{MPa}$

Variable stress, $\sigma_v = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{226.47 - (-56.62)}{2}$

$= 141.545\,\text{MPa}$

From the Soderberg's criterion,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{F.S.} = \frac{84.925}{420} + \frac{141.545}{240} = 0.202 + 0.589 = 0.791$$

So, $F.S. = \frac{1}{0.791} = 1.26$

SOL 5.9 Option (A) is correct.

Given : $m = 4\,\text{mm},\ Z = 21,\ P = 15\,\text{kW} = 15 \times 10^3\,\text{W} \ \ N = 960\,\text{rpm}$

$b = 25\,\text{mm},\ \phi = 20^\circ$

Pitch circle diameter, $D = mZ = 4 \times 21 = 84\,\text{mm}$

Tangential Force is given by,

$$F_T = \frac{T}{r} \qquad \text{...(i)}$$

Power transmitted, $P = \frac{2\pi NT}{60} \quad \Rightarrow \quad T = \frac{60P}{2\pi N}$

Then $F_T = \frac{60P}{2\pi N} \times \frac{1}{r}$ $\qquad r =$ Pitch circle radius

$$= \frac{60 \times 15 \times 10^3}{2 \times 3.14 \times 960} \times \frac{1}{42 \times 10^{-3}}$$

$$= 3554.36\,\text{N} \simeq 3552\,\text{N}$$

SOL 5.10 Option (B) is correct.

From Lewis equation

$$\sigma_b = \frac{F_T p_d}{by} = \frac{F_T}{b \times y \times m} \qquad p_d = \frac{\pi}{p_c} = \frac{\pi}{\pi m} = \frac{1}{m}$$

$$= \frac{3552}{25 \times 10^{-3} \times 0.32 \times 4 \times 10^{-3}}$$

$$\sigma_b = 111\,\text{MPa}$$

Minimum allowable (working stress)

$$\sigma_W = \sigma_b \times C_v = 111 \times 1.5 = 166.5\,\text{MPa}$$

SOL 5.11 Option (A) is correct.

Given : $d = 40\,\text{mm},\ l = 40\,\text{mm},\ \omega = 20\,\text{rad/sec}$

$Z(\mu) = 20\,\text{mPa-s} = 20 \times 10^{-3}\,\text{Pa-s},\ c(y) = 0.020\,\text{mm}$

Shear stress, $\tau = \mu \frac{u}{y}$ From the Newton's law of viscosity...(i)

$$u = r\omega = 0.020 \times 20 = 0.4 \text{ m/sec}$$

$$\tau = \frac{20 \times 10^{-3} \times 0.4}{0.020 \times 10^{-3}} = 400 \text{ N/m}^2$$

Shear force is generated due to this shear stress,

$$F = \tau A = \tau \times \pi dl \qquad A = \pi dl = \text{Area of shaft}$$

$$= 400 \times 3.14 \times 0.040 \times 0.040 = 2.0096 \text{ N}$$

Loss of torque, $T = F \times r = 2.0096 \times 0.020$

$$= 0.040192 \text{ N-m} \simeq 0.040 \text{ N-m}$$

SOL 5.12 Option (B) is correct.

Given : $d_1 = 100 \text{ mm} \Rightarrow r_1 = 50 \text{ mm}, d_2 = 40 \text{ mm} \Rightarrow r_1 = 20 \text{ mm}$

$p = 2 \text{ MPa} = 2 \times 10^6 \text{ Pa}, \mu = 0.4$

When the pressure is uniformly distributed over the entire area of the friction faces, then total frictional torque acting on the friction surface or on the clutch is given by,

$$T = 2\pi\mu p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times 3.14 \times 0.4 \times 2 \times 10^6[(50)^3 - (20)^3] \times 10^{-9}$$

$$= 195.39 \text{ N-m} \simeq 196 \text{ N-m}$$

SOL 5.13 Option (B) is correct.

Given : $m = 3 \text{ mm}, Z = 16, b = 36 \text{ mm}, \phi = 20°, P = 3 \text{ kW}$

$N = 20 \text{ rev/sec} = 20 \times 60 \text{ rpm} = 1200 \text{ rpm}, C_v = 1.5, y = 0.3$

Module, $m = \frac{D}{Z}$

$$D = m \times Z = 3 \times 16 = 48 \text{ mm}$$

Power, $P = \frac{2\pi NT}{60}$

$$T = \frac{60P}{2\pi N} = \frac{60 \times 3 \times 10^3}{2 \times 3.14 \times 1200}$$

$$= 23.88 \text{ N-m} = 23.88 \times 10^3 \text{ N-mm}$$

Tangential load, $W_T = \frac{T}{R} = \frac{2T}{D} = \frac{2 \times 23.88 \times 10^3}{48} = 995 \text{ N}$

From the lewis equation Bending stress (Beam strength of Gear teeth)

$$\sigma_b = \frac{W_T P_d}{by} = \frac{W_T}{bym} \qquad \left[P_d = \frac{\pi}{P_C} = \frac{\pi}{\pi m} = \frac{1}{m}\right]$$

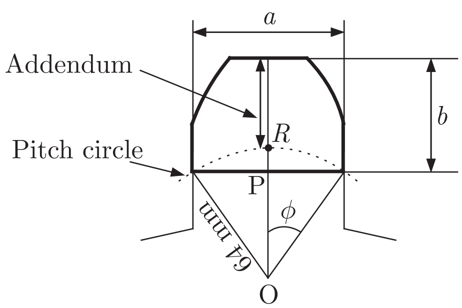
$$= \frac{995}{36 \times 10^{-3} \times 0.3 \times 3 \times 10^{-3}}$$

$$\sigma_b = \frac{995}{3.24 \times 10^{-5}} = 30.70 \times 10^6 \text{ Pa} = 30.70 \text{ MPa}$$

Permissible working stress

$$\sigma_W = \sigma_b \times C_v = 30.70 \times 1.5 = 46.06 \text{ MPa} \cong 46 \text{ MPa}$$

SOL 5.14 Option (D) is correct.



Given : $m = 4$, $Z = 32$, Tooth space = Tooth thickness = a

We know that, $m = \frac{D}{Z}$

Pitch circle diameter, $D = mZ = 4 \times 32 = 128$ mm

And for circular pitch, $P_c = \pi m = 3.14 \times 4 = 12.56$ mm

We also know that circular pitch,

$$P_c = \text{Tooth space} + \text{Tooth thickness}$$

$$= a + a = 2a$$

$$a = \frac{P_c}{2} = \frac{12.56}{2} = 6.28 \text{ mm}$$

From the figure, $b = \text{addendum} + PR$

or $\sin\phi = \frac{PQ}{OQ} = \frac{a/2}{64} = \frac{3.14}{64}$

$$\phi = \sin^{-1}(0.049) = 2.81^\circ$$

$$OP = 64\cos 2.81^\circ = 63.9 \text{ mm}$$

$$PR = OR - OP = 64 - 63.9 = 0.1 \text{ mm}$$

$$OR = \text{Pitch circle radius}$$

And $b = m + PR = 4 + 0.1 = 4.1$ mm

Therefore, $a = 6.28$ mm and $b = 4.1$ mm

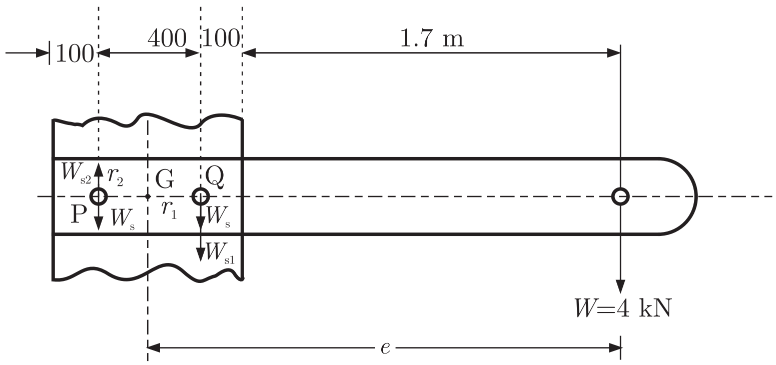
SOL 5.15 Option (A) is correct.

	Types of Gear		Description
P.	Helical	2.	Axes parallel and teeth are inclined to the axis
Q.	Spiral Bevel	4.	Axes are perpendicular and intersecting, and teeth are inclined to the axis

R.	Hypoid	1.	Axes non parallel and non-intersecting
S.	Rack and pinion	6.	Axes are parallel and one of the gear has infinite radius

So, correct pairs are P-2, Q-4, R-1, S-6

SOL 5.16 Option (A) is correct.



In this figure W_S represent the primary shear load whereas W_{S1} and W_{S2} represent the secondary shear loads.

Given : $A = 10 \times 50\ \text{mm}^2$, $n = 2$, $W = 4\ \text{kN} = 4 \times 10^3\ \text{N}$

We know that primary shear load on each bolt acting vertically downwards,

$$W_s = \frac{W}{n} = \frac{4\ \text{kN}}{2} = 2\ \text{kN}$$

Since both the bolts are at equal distances from the centre of gravity G of the two bolts, therefore the secondary shear load on each bolt is same.

For secondary shear load, taking the moment about point G,

$$W_{s1} \times r_1 + W_{s2} \times r_2 = W \times e$$

$$r_1 = r_2 \text{ and } W_{s1} = W_{s2}$$

So,
$$2r_1 W_{s1} = 4 \times 10^3 \times (1.7 + 0.2 + 0.1)$$

$$2 \times 0.2 \times W_{s1} = 4 \times 10^3 \times 2$$

$$W_{s1} = \frac{8 \times 10^3}{2 \times 0.2} = 20 \times 10^3 = 20\ \text{kN}$$

SOL 5.17 Option (B) is correct.

From the figure, resultant Force on bolt P is

$$F = W_{s2} - W_s = 20 - 2 = 18\ \text{kN}$$

Shear stress on bolt P is,

$$\tau = \frac{F}{\text{Area}} = \frac{18 \times 10^3}{\frac{\pi}{4} \times (12 \times 10^{-3})^2} = 159.23\ \text{MPa} \simeq 159\ \text{MPa}$$

SOL 5.18 Option (D) is correct.

Given : $W_1 = F$, $W_2 = 2F$, $L_1 = 8000\text{ hr}$

We know that, life of bearing is given by

$$L = \left(\frac{C}{W}\right)^k \times 10^6 \text{ revolution}$$

For ball bearing, $k = 3$, $\quad L = \left(\frac{C}{W}\right)^3 \times 10^6 \text{ revolution}$

For initial condition life is,

$$L_1 = \left(\frac{C}{F}\right)^3 \times 10^6$$

$$8000\text{ hr} = \left(\frac{C}{F}\right)^3 \times 10^6 \qquad ...(i)$$

For final load, $\quad L_2 = \left(\frac{C}{2F}\right)^3 \times 10^6 = \frac{1}{8} \times \left(\frac{C}{F}\right)^3 \times 10^6$

$$= \frac{1}{8}(8000\text{ hr}) = 1000\text{ hr} \qquad \text{From equation (i)}$$

SOL 5.19 Option (B) is correct.

Given : $d = 200\text{ mm}$, $t = 1\text{ mm}$, $\sigma_u = 800\text{ MPa}$, $\sigma_e = 400\text{ MPa}$

Circumferential stress induced in spherical pressure vessel is,

$$\sigma = \frac{p \times r}{2t} = \frac{p \times 100}{2 \times 1} = 50p\text{ MPa}$$

Given that, pressure vessel is subject to an internal pressure varying from 4 to 8 MPa.

So, $\quad \sigma_{\min} = 50 \times 4 = 200\text{ MPa}$

$$\sigma_{\max} = 50 \times 8 = 400\text{ MPa}$$

Mean stress, $\quad \sigma_m = \frac{\sigma_{\min} + \sigma_{\max}}{2} = \frac{200 + 400}{2} = 300\text{ MPa}$

Variable stress, $\quad \sigma_v = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{400 - 200}{2} = 100\text{ MPa}$

From the Goodman method,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e} = \frac{300}{800} + \frac{100}{400} = \frac{3}{8} + \frac{1}{4} = \frac{5}{8} \quad \Rightarrow \quad \text{F.S.} = \frac{8}{5} = 1.6$$

SOL 5.20 Option (B) is correct.

Given : $d = 50\text{ mm}$, $l = 50\text{ mm}$, $N = 20\text{ rps}$,

$Z = 20\text{ mPa-sec} = 20 \times 10^{-3}\text{ Pa-sec}$

Radial clearance $= 50\,\mu\text{m} = 50 \times 10^{-3}\text{ mm}$, Load $= 2\text{ kN}$

We know that,

$$p = \text{Bearing Pressure on the projected bearing area}$$

$$= \frac{\text{Load on the journal}}{l \times d}$$

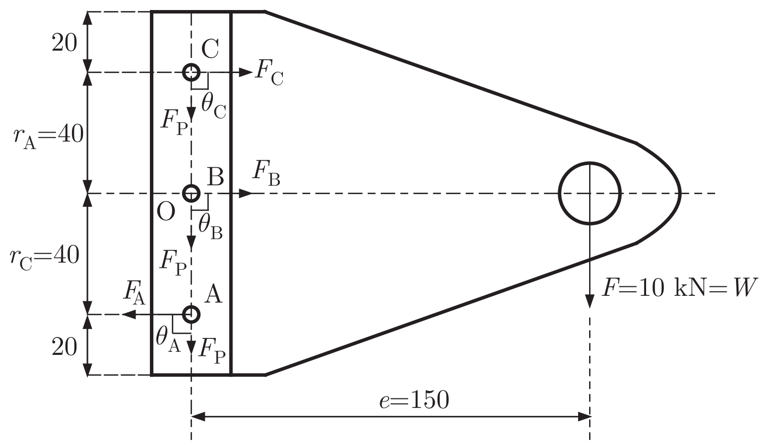
$$= \frac{2 \times 10^3}{50 \times 50} = 0.8\ \mathrm{N/mm^2} = 0.8 \times 10^6\ \mathrm{N/m^2}$$

Sommerfeld Number $= \frac{ZN}{p}\left(\frac{d}{c}\right)^2$ $\qquad c =$ diameteral clearance $= 2 \times$ radial clearance

$$S.N. = \frac{20 \times 10^{-3} \times 20}{0.8 \times 10^6} \times \left(\frac{50}{100 \times 10^{-3}}\right)^2$$

$$= \frac{20 \times 10^{-3} \times 20}{0.8 \times 10^6} \times \left(\frac{1}{2}\right)^2 \times 10^6 = 0.125$$

SOL 5.21 Option (A) is correct.



Given : Diameter of bolt $d = 10$ mm, $F = 10$ kN, No. of bolts $n = 3$

Direct or Primary shear load of each rivet

$$F_P = \frac{F}{n} = \frac{10 \times 10^3}{3}\ \mathrm{N}$$

$$F_P = 3333.33\ \mathrm{N}$$

The centre of gravity of the bolt group lies at O (due to symmetry of figure).

$$e = 150\ \mathrm{mm} \qquad \text{(eccentricity given)}$$

Turning moment produced by the load F due to eccentricity

$$= F \times e = 10 \times 10^3 \times 150$$

$$= 1500 \times 10^3\ \text{N-mm}$$

Secondary shear load on bolts from fig. $r_A = r_C = 40$ mm and $r_B = 0$

We know that $F \times e = \frac{F_A}{r_A}[(r_A)^2 + (r_B)^2 + (r_C)^2]$

$$= \frac{F_A}{r_A} \times [2(r_A)^2] \qquad (r_A = r_C \text{ and } r_B = 0)$$

$$1500 \times 10^3 = \frac{F_A}{40} \times [2(40)^2] = 80F_A$$

$$F_A = \frac{1500 \times 10^3}{80} = 18750\ \mathrm{N}$$

$$F_B = 0 \qquad (r_B = 0)$$

$$F_C = F_A \times \frac{r_C}{r_A} = 18750 \times \frac{40}{40}$$

$$= 18750\,\text{N}$$

From fig we find that angle between

$$F_A \text{ and } F_P = \theta_A = 90^\circ$$

$$F_B \text{ and } F_P = \theta_B = 90^\circ$$

$$F_C \text{ and } F_P = \theta_C = 90^\circ$$

Resultant load on bolt A,

$$R_A = \sqrt{(F_P)^2 + (F_A)^2 + 2F_P \times F_A \cos\theta_A}$$

$$= \sqrt{(3333.33)^2 + (18750)^2 + 2 \times 3333.33 \times 18750 \times \cos 90^\circ}$$

$$R_A = 19044\,\text{N}$$

Maximum shear stress at A

$$\tau_A = \frac{R_A}{\frac{\pi}{4}(d)^2} = \frac{19044}{\frac{\pi}{4}(10)^2} = 242.6\,\text{MPa}$$

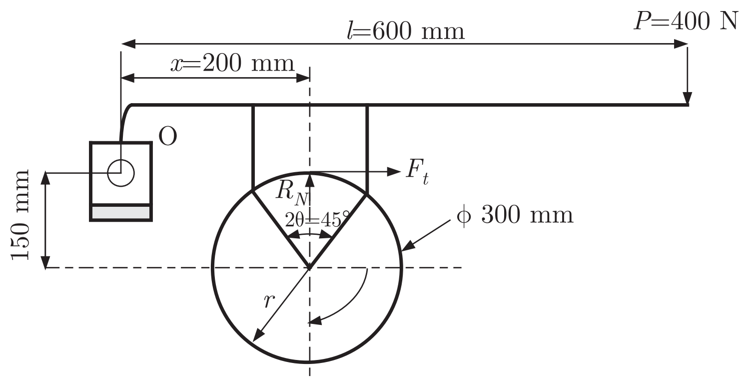
Resultant load on Bolt B,

$$R_B = F_P = 3333.33\,\text{N} \qquad (F_B = 0)$$

Maximum shear stress at B,

$$\tau_B = \frac{R_B}{\frac{\pi}{4}(d)^2} = \frac{3333.33}{\frac{\pi}{4} \times (10)^2} = 42.5\,\text{MPa}$$

SOL 5.22 Option (C) is correct.



Given :$P = 400\,\text{N}$, $r = \frac{300}{2}\,\text{mm} = 150\,\text{mm}$, $l = 600\,\text{mm}$

$x = 200\,\text{mm}$, $\mu = 0.25$ and $2\theta = 45^\circ$

Let, R_N → Normal force pressing the brake block on the wheel

F_t →Tangential braking force or the frictional force acting at the contact surface of the block & the wheel.

Here the line of action of tangential braking force F_t passes through the fulcrum O of the lever and brake wheel rotates clockwise. Then for equilibrium, Taking the moment about the fulcrum O,

$$R_N \times x = P \times l$$

$$R_N = \frac{P \times l}{x} = \frac{400 \times 0.6}{0.2} = 1200 \text{ N}$$

Tangential braking force on the wheel,

$$F_t = \mu R_N$$

Braking Torque, $T_B = F_t \times r = \mu R_N \times r$

$$= 0.25 \times 1200 \times 0.15 = 45 \text{ N-m}$$

SOL 5.23 Option (C) is correct.

Given : $d = 20$ mm, $l = 700$ mm,

$E = 200 \text{ GPa} = 200 \times 10^9 \text{ N/m}^2 = 200 \times 10^3 \text{ N/mm}^2$

Compressive or working Load $= 10$ kN

According to Euler's theory, the crippling or buckling load (W_{cr}) under various end conditions is given by the general equation,

$$W_{cr} = \frac{c\pi^2 EI}{l^2} \qquad \text{...(i)}$$

Given that one end is guided at the piston end and hinged at the other end.

So, $c = 2$

From equation (i),

$$W_{cr} = \frac{2\pi^2 EI}{l^2} = \frac{2\pi^2 E}{l^2} \times \frac{\pi}{64} d^4 \qquad I = \frac{\pi}{64} d^4$$

$$= \frac{2 \times 9.81 \times 200 \times 10^3}{(700)^2} \times \frac{3.14}{64} \times (20)^4$$

$$= 62864.08 \text{ N} = 62.864 \text{ kN}$$

We know that, factor of safety (FOS)

$$\text{FOS} = \frac{\text{Crippling Load}}{\text{Working Load}} = \frac{62.864}{10} = 6.28$$

The most appropriate option is (C).

SOL 5.24 Option (B) is correct.

Given : $Z_P = 20$, $Z_G = 40$, $N_P = 30$ rev/sec, $P = 20 \text{ kW} = 20 \times 10^3$ W, $m = 5$ mm

Module, $m = \frac{D}{Z} = \frac{D_P}{Z_P} = \frac{D_G}{Z_G}$

$$D_P = m \times Z_P = 5 \times 20 = 100 \text{ mm}$$

or, $D_G = m \times Z_G = 5 \times 40 = 200$ mm

Centre distance for the gear set,

$$L = \frac{D_P + D_G}{2} = \frac{100 + 200}{2} = 150 \text{ mm}$$

SOL 5.25 Option (C) is correct.

Given :

Length of line of action, $L = 19$ mm

Pressure angle, $\phi = 20°$

$$\text{Length of arc of contact} = \frac{\text{Length of path of contact}(L)}{\cos\phi}$$

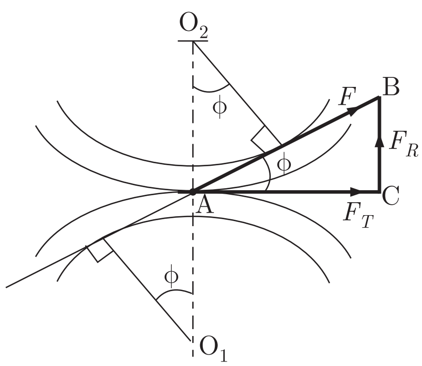
$$= \frac{19}{\cos 20°} = 20.21 \text{ mm}$$

Contact ratio or number of pairs of teeth in contact,

$$= \frac{\text{Length of arc of contact}}{\text{circular pitch}}$$

$$= \frac{20.21}{\pi m} = \frac{20.21}{3.14 \times 5} = 1.29$$

SOL 5.26 Option (C) is correct.



Let, $T \rightarrow$ Torque transmitted in N - m

We know that power transmitted is,

$$P = T\omega = T \times \frac{2\pi N}{60}$$

$$T = \frac{60P}{2\pi N} = \frac{60 \times 20 \times 10^3}{2 \times 3.14 \times 1800} = 106.157 \text{ N-m}$$

$$F_T = \frac{T}{R_P} \qquad \text{Tangential load on the pinion}$$

$$= \frac{106.157}{0.05} = 2123.14 \text{ N}$$

From the geometry, total load due to power transmitted,

$$F = \frac{F_T}{\cos\phi} = \frac{2123.14}{\cos 20°} \simeq 2258.1 \text{ N}$$

SOL 5.27 Option (A) is correct.

Given : $P = 5\,\text{kW},\ N = 2000\,\text{rpm},\ \mu = 0.25,\ r_2 = 25\,\text{mm} = 0.025\,\text{m},$
$p = 1\,\text{MPa}$

Power transmitted, $$P = \frac{2\pi NT}{60}$$

Torque, $$T = \frac{60P}{2\pi N} = \frac{60 \times 5 \times 10^3}{2 \times 3.14 \times 2000} = 23.885\ \text{N-m}$$

When pressure is uniformly distributed over the entire area of the friction faces, then total frictional torque acting on the friction surface or on the clutch,

$$T = 2\pi\mu p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$23.885 \times 3 = 2 \times 3.14 \times 0.25 \times 1 \times 10^6 \times [r_1^3 - (0.025)^3]$$

$$r_1^3 - (0.025)^3 = \frac{23.885 \times 3}{2 \times 3.14 \times 0.25 \times 10^6}$$

$$r_1^3 - 1.56 \times 10^{-5} = 45.64 \times 10^{-6} = 4.564 \times 10^{-5}$$

$$r_1^3 = (4.564 + 1.56) \times 10^{-5} = 6.124 \times 10^{-5}$$

$$r_1 = (6.124 \times 10^{-5})^{1/3} = 3.94 \times 10^{-2}\ \text{m} = 39.4\ \text{mm}$$

SOL 5.28 Option (A) is correct.

Given : $Z_P = 19,\ Z_G = 37,\ m = 5\,\text{mm}$

Also, $$m = \frac{D}{Z}$$

For pinion, pitch circle diameter is,

$$D_P = m \times Z_P = 5 \times 19 = 95\ \text{mm}$$

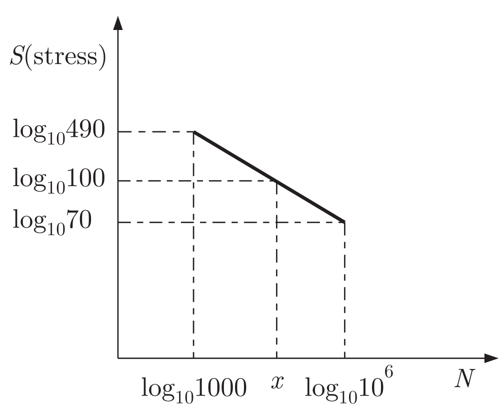
And pitch circle diameter of the gear,

$$D_G = m \times Z_G = 5 \times 37 = 185\ \text{mm}$$

Now, centre distance between the gear pair (shafts),

$$L = \frac{D_P}{2} + \frac{D_G}{2} = \frac{95 + 185}{2} = 140\ \text{mm}$$

SOL 5.29 Option (C) is correct.



We know that in S-N curve the failure occurs at 10^6 cycles (at endurance strength)

We have to make the S-N curve from the given data, on the scale of $\log_{10}$.

Now equation of line whose end point co-ordinates are

$$(\log_{10}1000, \log_{10}490) \text{ and } (\log_{10}10^6, \log_{10}70)$$

or $$(3, \log_{10}490) \text{ and } (6, \log_{10}70),$$

$$\frac{y-\log_{10}490}{x-3} = \frac{\log_{10}70-\log_{10}490}{6-3} \qquad \left(\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}\right)$$

$$\frac{y-2.69}{x-3} = \frac{1.845-2.69}{3}$$

$$y-2.69 = -0.281(x-3) \qquad \text{...(i)}$$

Given, the shaft is subject to an alternating stress of 100 MPa

So, $$y = \log_{10}100 = 2$$

Substitute this value in equation (i), we get

$$2-2.69 = -0.281(x-3)$$

$$-0.69 = -0.281x + 0.843$$

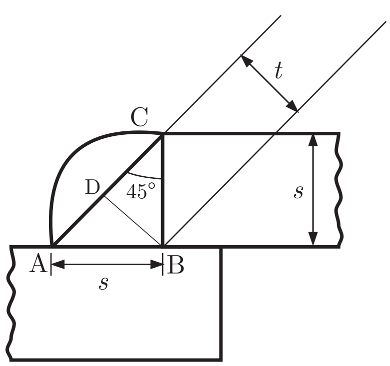
$$x = \frac{-0.843-0.69}{-0.281} = 5.455$$

And $$\log_{10}N = 5.455$$

$$N = 10^{5.455} = 285101$$

The nearest shaft life is 281914 cycles.

SOL 5.30 Option (B) is correct.



Given : $l = 60\text{ mm} = 0.06\text{ m}$, $s = 6\text{ mm} = 0.006\text{ m}$, $P = 15\text{ kN} = 15\times10^3\text{ N}$

Shear strength $= 200$ MPa

We know that, if τ is the allowable shear stress for the weld metal, then the shear strength of the joint for single parallel fillet weld,

$$P = \text{Throat Area} \times \text{Allowable shear stress}$$

$$= t \times l \times \tau$$

$$P = 0.707s \times l \times \tau \qquad t = s\sin 45^\circ = 0.707s$$

$$\tau = \frac{P}{0.707 \times s \times l} = \frac{15 \times 10^3}{0.707 \times 0.006 \times 0.06} = 58.93\,\text{MPa}$$

Factor of Safety,

$$FOS = \frac{\text{Shear strength}}{\text{Allowable shear stress}} = \frac{200\,\text{MPa}}{58.93\,\text{MPa}} = 3.39 \simeq 3.4$$

SOL 5.31 Option (A) is correct.

The coefficient of friction for a full lubricated journal bearing is a function of three variables, i.e.

$$\mu = \phi\left(\frac{ZN}{p}, \frac{d}{c}, \frac{l}{d}\right)$$

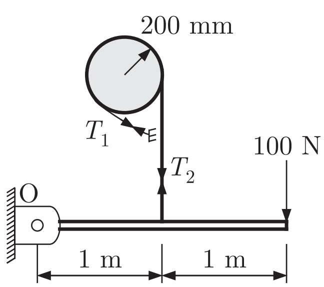
Here, $\frac{ZN}{p}$ = Bearing characteristic Number, d = Diameter of the bearing

l = Length of the bearing, c = Diameteral clearance

$$\text{Sommerfeld Number} = \frac{ZN}{p}\left(\frac{d}{c}\right)^2$$

It is a dimensionless parameter used extensively in the design of journal bearing. i.e. sommerfeld number is also function of $\left(\frac{ZN}{p}, \frac{d}{c}\right)$. Therefore option (A) is correct.

SOL 5.32 Option (B) is correct.



Given : $r = 200\text{ mm} = 0.2\text{ m}$, $\theta = 270^\circ = 270 \times \frac{\pi}{180} = \frac{3\pi}{2}$ radian, $\mu = 0.5$

At the time of braking, maximum tension is generated at the fixed end of band near the wheel.

Let, $T_2 \rightarrow$ Tension in the slack side of band

$T_1 \rightarrow$ Tension in the tight side of band at the fixed end

Taking the moment about the point O,

$$T_2 \times 1 = 100 \times 2 \qquad \Rightarrow \; T_2 = 200\text{ N}$$

For the band brake, the limiting ratio of the tension is given by the relation

$$\frac{T_1}{T_2} = e^{\mu\theta} \qquad \Rightarrow T_1 = T_2 \times e^{\mu\theta}$$

$$T_1 = 200 \times e^{0.5 \times \frac{3\pi}{2}} = 200 \times 10.54 = 2108 \text{ N}$$
$$\simeq 2110 \text{ N}$$

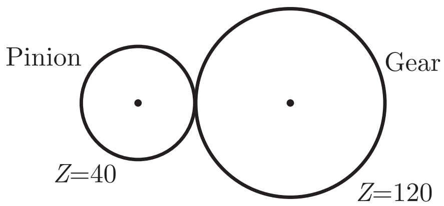
So, maximum tension that can be generated in the band during braking is equal to 2110 N

SOL 5.33 Option (B) is correct.

Maximum wheel torque or braking torque is given by,

$$T_W = (T_1 - T_2) r = (2110 - 200) \times 0.2 = 382 \text{ N-m}$$

SOL 5.34 Option (D) is correct.



Given : $Z_P = 40$ teeth, $Z_G = 120$ teeth, $N_P = 1200$ rpm, $T_P = 20$ N-m

Velocity Ratio, $$\frac{Z_P}{Z_G} = \frac{N_G}{N_P}$$

$$N_G = \frac{Z_P}{Z_G} \times N_P = \frac{40}{120} \times 1200 = 400 \text{ rpm}$$

Power transmitted is same for both pinion & Gear.

$$P = \frac{2\pi N_P T_P}{60} = \frac{2\pi N_G T_G}{60}$$

$$N_P T_P = N_G T_G$$

$$T_G = \frac{N_P T_P}{N_G} = \frac{1200}{400} \times 20 = 60 \text{ N-m}$$

So, the torque transmitted by the Gear is 60 N-m

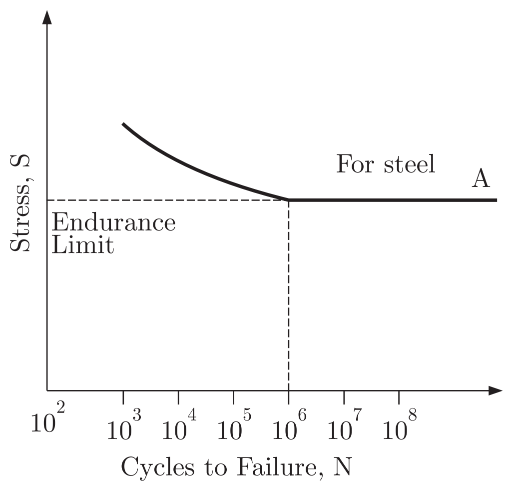
SOL 5.35 Option (A) is correct.

When the notch sensitivity factor q is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relation.

$$K_f = 1 + q(K_t - 1)$$
$$K_f - 1 = q(K_t - 1)$$
$$q = \frac{K_f - 1}{K_t - 1}$$

SOL 5.36 Option (C) is correct.

The S-N curve for the steel is shown below :



We can easily see from the S-N curve that, steel becomes asymptotic nearly at 10^6 cycles.

SOL 5.37 Option (C) is correct.

Given : $F_t = 2200\text{ N},\ p = 4\text{ mm} = 0.004\text{ m}$

Torque required for achieving the tightening force is,

$$T = F_t \times r = F_t \times \frac{\text{Pitch}}{2\pi} = 2200 \times \frac{0.004}{2 \times 3.14} = 1.4\text{ N-m}$$

SOL 5.38 Option (B) is correct.

	Type of Gears		Arrangement of shafts
P.	Bevel gears	2.	Non-parallel intersecting shafts
Q.	Worm gears	3.	Non-parallel, non-intersecting shafts
R.	Herringbone gears	4.	Parallel shafts
S.	Hypoid gears	1.	Non-parallel off-set shafts

So, correct pairs are P-2, Q-3, R-4, S-1.

SOL 5.39 Option (C) is correct.

The wire ropes are designated by the number of strands multiplied by the number of wires in each strand. Therefore,

$$6 \times 19 = \text{Number of strands} \times \text{Number of wires in each strand.}$$

SOL 5.40 Option (C) is correct.

Given : Diameter of shaft $= d$

Torque transmitted $= T$

Length of the key $= l$

We know that, width and thickness of a square key are equal.

i.e. $$w = t = \frac{d}{4}$$

Force acting on circumference of shaft

$$F = \frac{T}{r} = \frac{2T}{d} \qquad (r = d/2)$$

Shearing Area, $$A = \text{width} \times \text{length} = \frac{d}{4} \times l = \frac{dl}{4}$$

Average shear stress, $$\tau = \frac{\text{Force}}{\text{shearing Area}} = \frac{2T/d}{dl/4} = \frac{8T}{ld^2}$$

SOL 5.41 Option (D) is correct.

Let, $T_1 \rightarrow$ Tension in the tight side of the band,

$T_2 \rightarrow$ Tension in the slack side of the band

$\theta \rightarrow$ Angle of lap of the band on the drum

Given : $\frac{T_1}{T_2} = 3,\ \theta = 180^\circ = \frac{\pi}{180} \times 180 = \pi$ radian

For band brake, the limiting ratio of the tension is given by the relation,

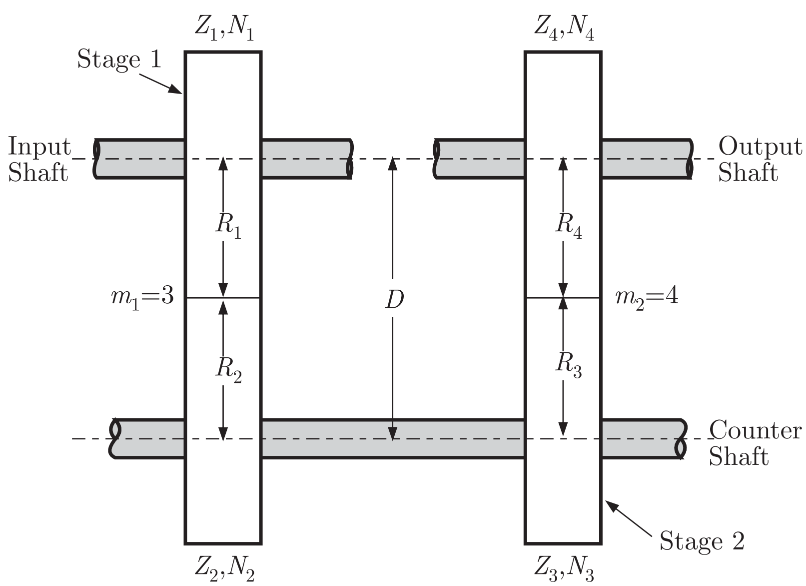
$$\frac{T_1}{T_2} = e^{\mu\theta} \text{ or } 2.3\log\left(\frac{T_1}{T_2}\right) = \mu\theta$$

$$2.3 \times \log(3) = \mu \times \pi$$

$$2.3 \times 0.4771 = \mu \times 3.14$$

$$\mu = \frac{1.09733}{3.14} = 0.349 \simeq 0.35$$

SOL 5.42 Option (A) is correct.



Let N_1, N_2, N_3 and N_4 are the speeds of pinion 1, gear 2, pinion 3 and gear 4 respectively.

Given : $Z_1 = 16$ teeth , $Z_3 = 15$ teeth and $Z_4 = ?$, $Z_2 = ?$

Velocity ratio $\frac{N_1}{N_4} = \frac{Z_2/Z_1}{Z_3/Z_4}$ $N \propto 1/Z$

$$= \frac{Z_2}{Z_1} \times \frac{Z_4}{Z_3} = 12 \quad ...(i)$$

But for stage 1, $\frac{N_1}{N_2} = \frac{Z_2}{Z_1} = 4$ ...(ii)

So, $4 \times \frac{Z_4}{Z_3} = 12$ from eq. (i)

$$\frac{Z_4}{Z_3} = 3, \quad \Rightarrow \quad Z_4 = 3 \times 15 = 45 \text{ teeth}$$

From equation (ii), $Z_2 = 4 \times Z_1 = 4 \times 16 = 64$ teeth

SOL 5.43 Option (B) is correct.

Let centre distance in the second stage is D.

$$D = R_4 + R_3 = \frac{D_4 + D_3}{2}$$

But, $\frac{D_4}{Z_4} = \frac{D_3}{Z_3} = 4$ $m = D/Z$ module

$$D_4 = 4 \times Z_4 = 4 \times 45 = 180$$

Or, $D_3 = 4 \times Z_3 = 4 \times 15 = 60$

So, $D = \frac{180 + 60}{2} = 120$ mm

SOL 5.44 Option (C) is correct.

In standard full height involute teeth gear mechanism the arc of approach is not be less than the circular pitch, therefore

Maximum length of arc of approach = Circular pitch
...(i)

where Maximum length of the arc of approach

$$= \frac{\text{Max. length of the path of approach}}{\cos\phi}$$

$$= \frac{r\sin\phi}{\cos\phi} = r\tan\phi$$

Circular pitch, $P_C = \pi m = \frac{2\pi r}{Z}$ $m = \frac{2r}{Z}$

Hence, from equation (i), we get

$$r\tan\phi = \frac{2\pi r}{Z}$$

$$Z = \frac{2\pi}{\tan\phi} = \frac{2\pi}{\tan 20^\circ} = 17.25 \simeq 18 \text{ teeth}$$

SOL 5.45 Option (D) is correct.

(A) **Flange coupling :-** It is used to connect two shaft having perfect coaxial alignment and no misalignment is allowed between them.

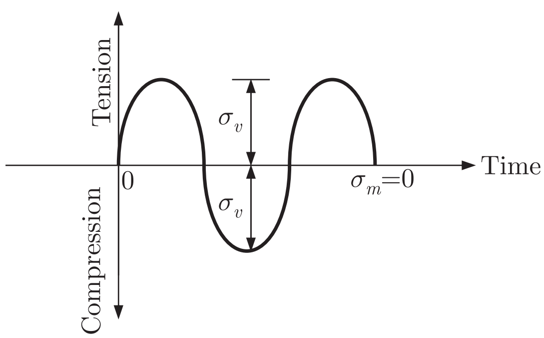
(B) **Oldham's coupling :-** It is used to join two shafts which have lateral misalignment.

(C) **Flexible bush coupling :-** It is used to join the abutting ends of shafts when they are not in exact alignment.

(D) **Hook's joint :-** It is used to connect two shafts with large angular misalignment.

SOL 5.46 Option (D) is correct.

When the shaft rotates, the bending stress at the upper fibre varies from maximum compressive to maximum tensile while the bending stress at the lower fibres varies from maximum tensile to maximum compressive. The specimen subjected to a completely reversed stress cycle. This is shown in the figure.



When shaft is subjected to repeated stress, then it will be designed for fatigue loading.

SOL 5.47 Option (B) is correct.

For a worm gear the velocity ratio ranges between 10 : 1 to 100 : 1. So, Large speed reductions (greater than 20) in one stage of a gear train are possible through worm gearing.

SOL 5.48 Option (A) is correct.

For Helical spring, deflection is given by,

$$\delta = \frac{64PR^3n}{Gd^4} = \frac{8PD^3n}{Gd^4}$$

where, P = Compressive load

d = Wire diameter

R = Coil diameter

$$G = \text{Modulus of rigidity}$$

From the given conditions

$$\delta \propto \frac{1}{d^4}$$

Given $d_1 = 1\,\text{cm}$ and $d_2 = 2\,\text{cm}$

$$\frac{\delta_2}{\delta_1} = \left(\frac{d_1}{d_2}\right)^4$$

$$\frac{\delta_2}{\delta_1} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$\delta_2 = \frac{\delta_1}{16}$$

So, deflection will decrease by a factor of 16.

SOL 5.49 Option (C) is correct.

Bars AB and BC have negligible mass. The support load P acting at the free end of bars AB and BC. Due to this load P, In bar AB compressive stress and in bar BC tensile stress are induced.

However, none of these bars will be subjected to bending because there is no couple acting on the bars.

SOL 5.50 Option (C) is correct.

Let L_1 & L_2 are lengths of the springs and n_1 & n_2 are the number of coils in both the springs.

Given : $$W_1 = W_2$$

$$m_1 g = m_2 g$$

$$\rho \nu_1 g = \rho \nu_2 g \qquad m = \rho \nu$$

$$A_1 \times L_1 \times \rho g = A_2 \times L_2 \times \rho g$$

$$\frac{\pi}{4} d_1^2 \times \pi D_1 n_1 = \frac{\pi}{4} d_2^2 \times \pi D_2 n_2 \qquad L = \pi D n$$

$$d_1^2 \times n_1 = d_2^2 \times n_2 \qquad D_1 = D_2$$

Given : $$d_1 = d \ \& \ d_2 = \frac{d}{2}$$

$$d^2 \times n_1 = \frac{d^2}{4} \times n_2$$

or, $$n_1 = \frac{n_2}{4}$$

The deflection of helical spring is given by,

$$\delta = \frac{8PD^3 n}{Gd^4}$$

Spring stiffness, $$k = \frac{P}{\delta} = \frac{Gd^4}{8D^3 n}$$

From the given conditions, we get

$$k \propto \frac{d^4}{n}$$

So,

$$\frac{k_1}{k_2} = \left(\frac{d_1}{d_2}\right)^4 \times \left(\frac{n_2}{n_1}\right)$$

$$\frac{k_1}{k_2} = \left(\frac{d}{d/2}\right)^4 \times \frac{n_2}{n_2/4}$$

$$\frac{k_1}{k_2} = 16 \times 4 = 64$$

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CHAPTER 6

FLUID MECHANICS

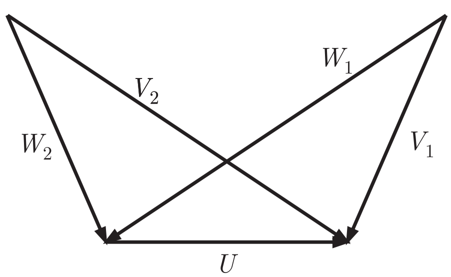
YEAR 2012 ONE MARK

MCQ 6.1 Oil flows through a $200\,\text{mm}$ diameter horizontal cast iron pipe (friction factor, $f = 0.0225$) of length $500\,\text{m}$. The volumetric flow rate is $0.2\,\text{m}^3/\text{s}$. The head loss (in m) due to friction is (assume $g = 9.81\,\text{m/s}^2$)

(A) 116.18 (B) 0.116

(C) 18.22 (D) 232.36

MCQ 6.2 The velocity triangles at the inlet and exit of the rotor of a turbomachine are shown. V denotes the absolute velocity of the fluid, W denotes the relative velocity of the fluid and U denotes the blade velocity. Subscripts 1 and 2 refer to inlet and outlet respectively. If $V_2 = W_1$ and $V_1 = W_2$, then the degree of reaction is



(A) 0 (B) 1

(C) 0.5 (D) 0.25

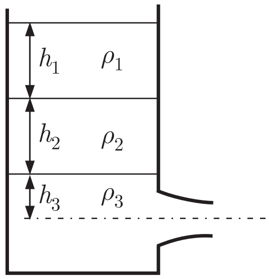
YEAR 2012 TWO MARKS

MCQ 6.3 An incompressible fluid flows over a flat plate with zero pressure gradient. The boundary layer thickness is $1\,\text{mm}$ at a location where the Reynolds number is 1000. If the velocity of the fluid alone is increased by a factor of 4, then the boundary layer thickness at the same location, in mm will be

(A) 4 (B) 2

(C) 0.5 (D) 0.25

MCQ 6.4 A large tank with a nozzle attached contains three immiscible, inviscide fluids as shown. Assuming that the change in h_1, h_2 and h_3 are negligible, the instantaneous discharge velocity is



(A) $\sqrt{2gh_3\left(1+\frac{\rho_1}{\rho_3}\frac{h_1}{h_3}+\frac{\rho_2}{\rho_3}\frac{h_2}{h_3}\right)}$

(B) $\sqrt{2g(h_1+h_2+h_3)}$

(C) $\sqrt{2g\left(\frac{\rho_1 h_1+\rho_2 h_2+\rho_3 h_3}{\rho_1+\rho_2+\rho_3}\right)}$

(D) $\sqrt{2g\frac{\rho_1 h_2 h_3+\rho_2 h_3 h_1+\rho_3 h_1 h_2}{\rho_1 h_1+\rho_2 h_2+\rho_3 h_3}}$

YEAR 2011 ONE MARK

MCQ 6.5 A streamline and an equipotential line in a flow field

(A) are parallel to each other

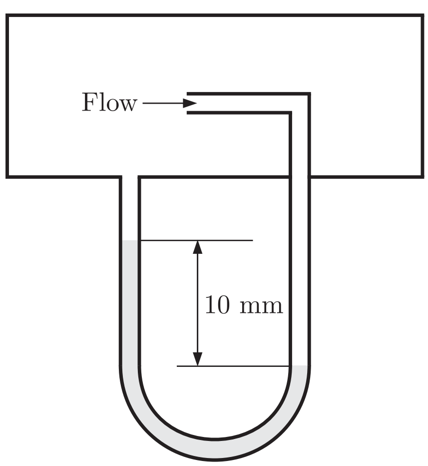
(B) are perpendicular to each other

(C) intersect at an acute angle

(D) are identical

YEAR 2011 TWO MARKS

MCQ 6.6 Figure shows the schematic for the measurement of velocity of air (density $= 1.2\ \text{kg/m}^3$) through a constant area duct using a pitot tube and a water tube manometer. The differential head of water (density $= 1000\ \text{kg/m}^3$) in the two columns of the manometer is $10\ \text{mm}$. Take acceleration due to gravity as $9.8\ \text{m/s}^2$. The velocity of air in m/s is



(A) 6.4

(B) 9.0

(C) 12.8

(D) 25.6

MCQ 6.7 A pump handing a liquid raises its pressure from 1 bar to 30 bar. Take the density of the liquid as $990\ kg/m^3$. The isentropic specific work done by the pump in kJ/kg is

(A) 0.10 (B) 0.30

(C) 2.50 (D) 2.93

YEAR 2010 ONE MARK

MCQ 6.8 For the stability of a floating body, under the influence of gravity alone, which of the following is TRUE ?

(A) Metacenter should be below centre of gravity.

(B) Metacenter should be above centre of gravity.

(C) Metacenter and centre of gravity must lie on the same horizontal line.

(D) Metacenter and centre of gravity must lie on the same vertical line.

MCQ 6.9 The maximum velocity of a one-dimensional incompressible fully developed viscous flow, between two fixed parallel plates, is $6\ ms^{-1}$. The mean velocity (in ms^{-1}) of the flow is

(A) 2 (B) 3

(C) 4 (D) 5

MCQ 6.10 A phenomenon is modeled using n dimensional variables with k primary dimensions. The number of non-dimensional variables is

(A) k (B) n

(C) $n-k$ (D) $n+k$

MCQ 6.11 A hydraulic turbine develops $1000\ kW$ power for a head of $40\ m$. If the head is reduced to $20\ m$, the power developed (in kW) is

(A) 177 (B) 354

(C) 500 (D) 707

YEAR 2010 TWO MARKS

MCQ 6.12 Velocity vector of a flow field is given as $V = 2xyi - x^2zj$. The vorticity vector at $(1,1,1)$ is

(A) $4i - j$ (B) $4i - k$

(C) $i - 4j$ (D) $i - 4k$

MCQ 6.13 A smooth pipe of diameter $200\ mm$ carries water. The pressure in the pipe at section S_1 (elevation : 10 m) is 50 kPa. At section S_2 (elevation : 12 m) the pressure is 20 kPa and velocity is $2\ ms^{-1}$. Density of water is $1000\ kgm^{-3}$

and acceleration due to gravity is $9.8\,ms^{-2}$. Which of the following is TRUE

(A) flow is from S_1 to S_2 and head loss is 0.53 m

(B) flow is from S_2 to S_1 and head loss is 0.53 m

(C) flow is from S_1 to S_2 and head loss is 1.06 m

(D) flow is from S_2 to S_1 and head loss is 1.06 m

MCQ 6.14 Match the following

P.	Compressible flow	U.	Reynolds number
Q.	Free surface flow	V.	Nusselt number
R.	Boundary layer flow	W.	Weber number
S.	Pipe flow	X.	Froude number
T.	Heat convection	Y.	Mach number
		Z.	Skin friction coefficient

(A) P-U; Q-X; R-V; S-Z; T-W

(B) P-W; Q-X; R-Z; S-U; T-V

(C) P-Y; Q-W; R-Z; S-U; T-X

(D) P-Y; Q-W; R-Z; S-U; T-V

YEAR 2009 TWO MARKS

MCQ 6.15 Consider steady, incompressible and irrotational flow through a reducer in a horizontal pipe where the diameter is reduced from 20 cm to 10 cm. The pressure in the 20 cm pipe just upstream of the reducer is 150 kPa. The fluid has a vapour pressure of 50 kPa and a specific weight of $5\,kN/m^3$. Neglecting frictional effects, the maximum discharge (in m^3/s) that can pass through the reducer without causing cavitation is

(A) 0.05 (B) 0.16

(C) 0.27 (D) 0.38

MCQ 6.16 You are asked to evaluate assorted fluid flows for their suitability in a given laboratory application. The following three flow choices, expressed in terms of the two dimensional velocity fields in the xy-plane, are made available.

P : $u = 2y, v = -3x$

Q : $u = 3xy, v = 0$

R : $u = -2x, v = 2y$

Which flow(s) should be recommended when the application requires the flow to be incompressible and irrotational ?

(A) P and R (B) Q

(C) Q and R (D) R

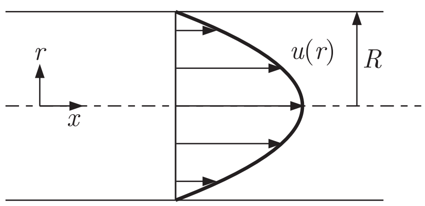
MCQ 6.17 Water at $25^\circ C$ is flowing through a $1.0\,km$ long. G.I. pipe of $200\,mm$ diameter at the rate of $0.07\,m^3/s$. If value of Darcy friction factor for this pipe is 0.02 and density of water is $1000\,kg/m^3$, the pumping power (in kW) required to maintain the flow is

(A) 1.8
(B) 17.4
(C) 20.5
(D) 41.0

MCQ 6.18 The velocity profile of a fully developed laminar flow in a straight circular pipe, as shown in the figure, is given by the expression

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

Where $\frac{dp}{dx}$ is a constant. The average velocity of fluid in the pipe is



(A) $-\frac{R^2}{8\mu}\left(\frac{dp}{dx}\right)$
(B) $-\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)$
(C) $-\frac{R^2}{2\mu}\left(\frac{dp}{dx}\right)$
(D) $-\frac{R^2}{\mu}\left(\frac{dp}{dx}\right)$

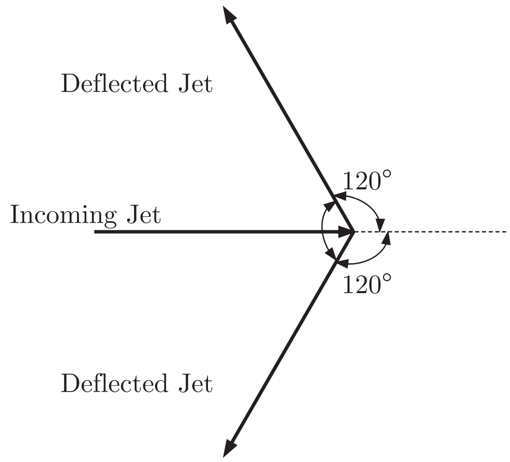
YEAR 2008 ONE MARK

MCQ 6.19 For the continuity equation given by $\nabla \cdot V = 0$ to be valid, where V is the velocity vector, which one of the following is a necessary condition ?

(A) steady flow
(B) irrotational flow
(C) inviscid flow
(D) incompressible flow

YEAR 2008 TWO MARKS

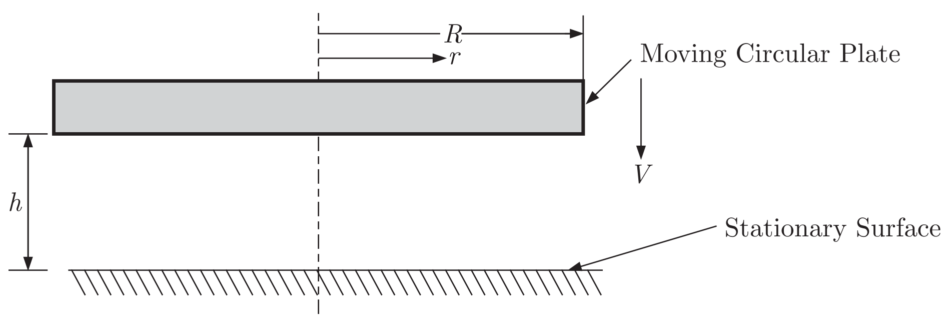
MCQ 6.20 Water, having a density of $1000\,kg/m^3$, issues from a nozzle with a velocity of $10\,m/s$ and the jet strikes a bucket mounted on a Pelton wheel. The wheel rotates at $10\,rad/s$. The mean diameter of the wheel is $1\,m$. The jet is split into two equal streams by the bucket, such that each stream is deflected by 120° as shown in the figure. Friction in the bucket may be neglected. Magnitude of the torque exerted by the water on the wheel, per unit mass flow rate of the incoming jet, is



(A) 0 (N-m)/(kg/s)
(B) 1.25 (N-m)/(kg/s)
(C) 2.5 (N-m)/(kg/s)
(D) 3.75 (N-m)/(kg/s)

• Common Data For Q. 21 and Q.22

The gap between a moving circular plate and a stationary surface is being continuously reduced, as the circular plate comes down at a uniform speed V towards the stationary bottom surface, as shown in the figure. In the process, the fluid contained between the two plates flows out radially. The fluid is assumed to be incompressible and inviscid.



MCQ 6.21 The radial velocity V_r at any radius r, when the gap width is h, is

(A) $V_r = \frac{Vr}{2h}$
(B) $V_r = \frac{Vr}{h}$
(C) $V_r = \frac{2Vh}{r}$
(D) $V_r = \frac{Vh}{r}$

MCQ 6.22 The radial component of the fluid acceleration at $r = R$ is

(A) $\frac{3V^2R}{4h^2}$
(B) $\frac{V^2R}{4h^2}$
(C) $\frac{V^2R}{2h^2}$
(D) $\frac{V^2h}{2R^2}$

YEAR 2007 ONE MARK

MCQ 6.23 Consider an incompressible laminar boundary layer flow over a flat plate of length L, aligned with the direction of an incoming uniform free stream. If F is the ratio of the drag force on the front half of the plate to the drag force on the rear half, then

(A) $F < 1/2$ (B) $F = 1/2$

(C) $F = 1$ (D) $F > 1$

MCQ 6.24 In a steady flow through a nozzle, the flow velocity on the nozzle axis is given by $v = u_0(1 + 3x/L)$, where x is the distance along the axis of the nozzle from its inlet plane and L is the length of the nozzle. The time required for a fluid particle on the axis to travel from the inlet to the exit plane of the nozzle is

(A) $\frac{L}{u_0}$ (B) $\frac{L}{3u_0}\ln 4$

(C) $\frac{L}{4u_0}$ (D) $\frac{L}{2.5u_0}$

MCQ 6.25 Consider steady laminar incompressible anti-symmetric fully developed viscous flow through a straight circular pipe of constant cross-sectional area at a Reynolds number of 5. The ratio of inertia force to viscous force on a fluid particle is

(A) 5 (B) 1/5

(C) 0 (D) ∞

YEAR 2007 TWO MARKS

MCQ 6.26 The inlet angle of runner blades of a Francis turbine is 90°. The blades are so shaped that the tangential component of velocity at blade outlet is zero. The flow velocity remains constant throughout the blade passage and is equal to half of the blade velocity at runner inlet. The blade efficiency of the runner is

(A) 25% (B) 50%

(C) 80% (D) 89%

MCQ 6.27 A model of a hydraulic turbine is tested at a head of $1/4^{\text{th}}$ of that under which the full scale turbine works. The diameter of the model is half of that of the full scale turbine. If N is the RPM of the full scale turbine, the RPM of the model will be

(A) $N/4$ (B) $N/2$

(C) N (D) $2N$

MCQ 6.28 Which combination of the following statements about steady incompressible forced vortex flow is correct ?

P: Shear stress is zero at all points in the flow.

Q: Vorticity is zero at all points in the flow.

R: Velocity is directly proportional to the radius from the center of the vortex.

S: Total mechanical energy per unit mass is constant in the entire flow field.

(A) P and Q (B) R and S

(C) P and R (D) P and S

MCQ 6.29 Match List-I with List-II and select the correct answer using the codes given below the lists :

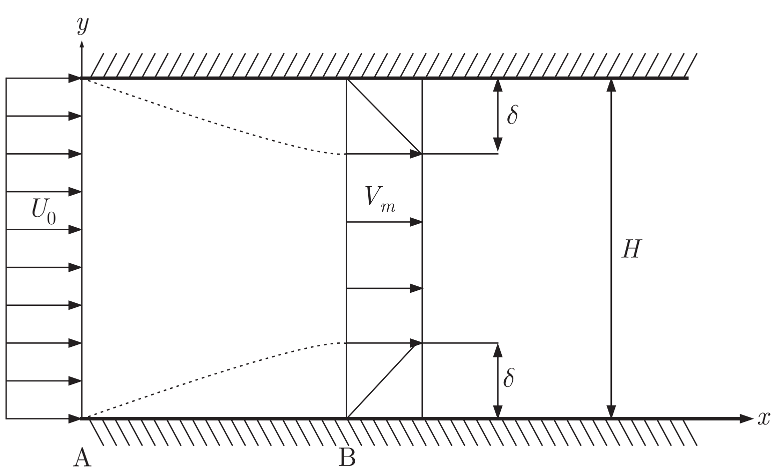
	List-I		List-II
P.	Centrifugal compressor	1.	Axial flow
Q.	Centrifugal pump	2.	Surging
R.	Pelton wheel	3.	Priming
S.	Kaplan turbine	4.	Pure impulse

Codes :

	P	Q	R	S
(A)	2	3	4	1
(B)	2	3	1	4
(C)	3	4	1	2
(D)	1	2	3	4

• **Common Data For Q.30 and Q.31 :**

Consider a steady incompressible flow through a channel as shown below.



The velocity profile is uniform with a value of U_0 at the inlet section A. The velocity profile at section B downstream is

$$u = \begin{cases} V_m \dfrac{y}{\delta}, & 0 \le y \le \delta \\ V_m, & \delta \le y \le H-\delta \\ V_m \dfrac{H-y}{\delta}, & H-\delta \le y \le H \end{cases}$$

MCQ 6.30 The ratio V_m/U_0 is

(A) $\dfrac{1}{1-2(\delta/H)}$

(B) 1

(C) $\dfrac{1}{1-(\delta/H)}$

(D) $\dfrac{1}{1+(\delta/H)}$

MCQ 6.31 The ratio $\dfrac{p_A - p_B}{\frac{1}{2}\rho U_0^2}$ (where p_A and p_B are the pressures at section A and B) respectively, and ρ is the density of the fluid) is

(A) $\dfrac{1}{[1-(\delta/H)]^2} - 1$

(B) $\dfrac{1}{[1-(\delta/H)]^2}$

(C) $\dfrac{1}{[1-(2\delta/H)]^2} - 1$

(D) $\dfrac{1}{1+(\delta/H)}$

YEAR 2006 ONE MARK

MCQ 6.32 For a Newtonian fluid

(A) Shear stress is proportional to shear strain

(B) Rate of shear stress is proportional to shear strain

(C) Shear stress is proportional to rate of shear strain

(D) Rate of shear stress is proportional to rate of shear strain

MCQ 6.33 In a two-dimensional velocity field with velocities u and v along the x and y directions respectively, the convective acceleration along the x-direction is given by

(A) $u\dfrac{\partial v}{\partial x} + v\dfrac{\partial u}{\partial y}$

(B) $u\dfrac{\partial u}{\partial x} + v\dfrac{\partial v}{\partial y}$

(C) $u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y}$

(D) $v\dfrac{\partial u}{\partial x} + u\dfrac{\partial u}{\partial y}$

MCQ 6.34 In a Pelton wheel, the bucket peripheral speed is $10\,\text{m/s}$, the water jet velocity is $25\,\text{m/s}$ and volumetric flow rate of the jet is $0.1\,\text{m}^3/\text{s}$. If the jet deflection angle is 120° and the flow is ideal, the power developed is

(A) 7.5 kW

(B) 15.0 kW

(C) 22.5 kW

(D) 37.5 kW

YEAR 2006 TWO MARKS

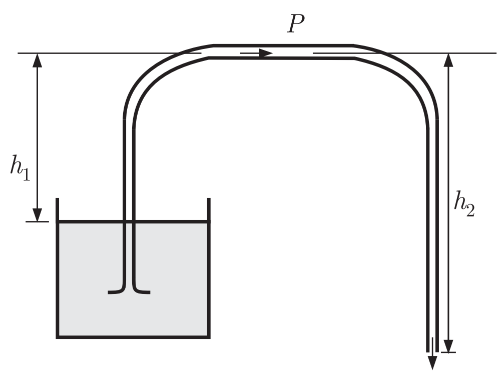
MCQ 6.35 A two-dimensional flow field has velocities along the x and y directions given by $u = x^2 t$ and $v = -2xyt$ respectively, where t is time. The equation of stream line is

(A) $x^2 y =$ constant
(B) $xy^2 =$ constant
(C) $xy =$ constant
(D) not possible to determine

MCQ 6.36 The velocity profile in fully developed laminar flow in a pipe of diameter D is given by $u = u_0(1 - 4r^2/D^2)$, where r is the radial distance from the center. If the viscosity of the fluid is μ, the pressure drop across a length L of the pipe is

(A) $\frac{\mu u_0 L}{D^2}$
(B) $\frac{4\mu u_0 L}{D^2}$
(C) $\frac{8\mu u_0 L}{D^2}$
(D) $\frac{16\mu u_0 L}{D^2}$

MCQ 6.37 A siphon draws water from a reservoir and discharge it out at atmospheric pressure. Assuming ideal fluid and the reservoir is large, the velocity at point P in the siphon tube is

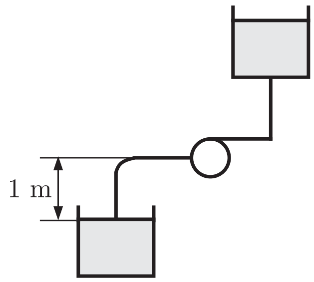


(A) $\sqrt{2gh_1}$
(B) $\sqrt{2gh_2}$
(C) $\sqrt{2g(h_2 - h_1)}$
(D) $\sqrt{2g(h_2 + h_1)}$

MCQ 6.38 A large hydraulic turbine is to generate 300 kW at 1000 rpm under a head of 40 m. For initial testing, a 1 : 4 scale model of the turbine operates under a head of 10 m. The power generated by the model (in kW) will be

(A) 2.34
(B) 4.68
(C) 9.38
(D) 18.75

MCQ 6.39 A horizontal-shaft centrifugal pump lifts water at 65°C. The suction nozzle is one meter below pump center line. The pressure at this point equals 200 kPa gauge and velocity is 3 m/s. Steam tables show saturation pressure at 65°C is 25 kPa, and specific volume of the saturated liquid is 0.001020 m^3/kg. The pump Net Positive Suction Head (NPSH) in meters is

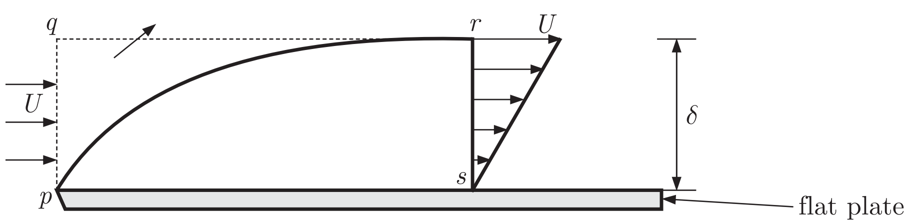


(A) 24 (B) 26

(C) 28 (D) 30

• Common Data For Q.40 and Q.41

A smooth flat plate with a sharp leading edge is placed along a gas stream flowing at $U = 10\,\text{m/s}$. The thickness of the boundary layer at section r-s is $10\,\text{mm}$, the breadth of the plate is $1\,\text{m}$ (into the paper) and the density of the gas $\rho = 1.0\,\text{kg/m}^3$. Assume that the boundary layer is thin, two-dimensional, and follows a linear velocity distribution, $u = U(y/\delta)$, at the section r-s, where y is the height from plate.



MCQ 6.40 The mass flow rate (in kg/s) across the section $q - r$ is

(A) zero (B) 0.05

(C) 0.10 (D) 0.15

MCQ 6.41 The integrated drag force (in N) on the plate, between p-s, is

(A) 0.67 (B) 0.33

(C) 0.17 (D) zero

YEAR 2005 ONE MARK

MCQ 6.42 The velocity components in the x and y directions of a two dimensional potential flow are u and v, respectively. Then $\partial u/\partial x$ is equal to

(A) $\frac{\partial v}{\partial x}$ (B) $-\frac{\partial v}{\partial x}$

(C) $\frac{\partial v}{\partial y}$ (D) $-\frac{\partial v}{\partial y}$

YEAR 2005 TWO MARKS

MCQ 6.43 A venturimeter of 20 mm throat diameter is used to measure the velocity of water in a horizontal pipe of 40 mm diameter. If the pressure difference between the pipe and throat sections is found to be 30 kPa then, neglecting frictional losses, the flow velocity is

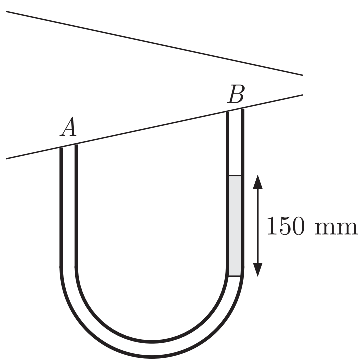
(A) 0.2 m/s

(B) 1.0 m/s

(C) 1.4 m/s

(D) 2.0 m/s

MCQ 6.44 A U-tube manometer with a small quantity of mercury is used to measure the static pressure difference between two locations A and B in a conical section through which an incompressible fluid flows. At a particular flow rate, the mercury column appears as shown in the figure. The density of mercury is 13600 kg/m^3 and $g = 9.81$ m/s^2. Which of the following is correct ?



(A) Flow direction is A to B and $p_A - p_B = 20$ kPa

(B) Flow direction is B to A and $p_A - p_B = 1.4$ kPa

(C) Flow direction is A to B and $p_B - p_A = 20$ kPa

(D) Flow direction is B to A and $p_B - p_A = 1.4$ kPa

MCQ 6.45 A leaf is caught in a whirlpool. At a given instant, the leaf is at a distance of 120 m from the centre of the whirlpool. The whirlpool can be described by the following velocity distribution:

$$V_r = -\left(\frac{60 \times 10^3}{2\pi r}\right) \text{ m/s and } V_\theta = \frac{300 \times 10^3}{2\pi r} \text{ m/s}$$

Where r (in metres) is the distance from the centre of the whirlpool. What will be the distance of the leaf from the centre when it has moved through half a revolution ?

(A) 48 m (B) 64 m

(C) 120 m (D) 142 m

YEAR 2004 ONE MARK

MCQ 6.46 An incompressible fluid (kinematic viscosity, 7.4×10^{-7} m^2/s, specific gravity, 0.88) is held between two parallel plates. If the top plate is moved with a velocity of 0.5 m/s while the bottom one is held stationary, the fluid attains a linear velocity profile in the gap of 0.5 mm between these plates; the shear stress in Pascals on the surfaces of top plate is

(A) 0.651×10^{-3}
(B) 0.651
(C) 6.51
(D) 0.651×10^{3}

MCQ 6.47 A fluid flow is represented by the velocity field $V = axi + ayj$, where a is a constant. The equation of stream line passing through a point (1, 2) is

(A) $x - 2y = 0$
(B) $2x + y = 0$
(C) $2x - y = 0$
(D) $x + 2y = 0$

YEAR 2004 TWO MARKS

MCQ 6.48 The following data about the flow of liquid was observed in a continuous chemical process plant :

Flow rate (litres / sec)	7.5 to 7.7	7.7 to 7.9	7.9 to 8.1	8.1 to 8.3	8.3 to 8.5	8.5 to 8.7
Frequency	1	5	35	17	12	10

Mean flow rate of the liquid is

(A) 8.00 litres/sec
(B) 8.06 litres/sec
(C) 8.16 litres/sec
(D) 8.26 litres/sec

MCQ 6.49 For a fluid flow through a divergent pipe of length L having inlet and outlet radii of R_1 and R_2 respectively and a constant flow rate of Q, assuming the velocity to be axial and uniform at any cross-section, the acceleration at the exit is

(A) $\frac{2Q(R_1 - R_2)}{\pi L R_2^3}$
(B) $\frac{2Q^2(R_1 - R_2)}{\pi L R_2^3}$
(C) $\frac{2Q^2(R_1 - R_2)}{\pi^2 L R_2^5}$
(D) $\frac{2Q^2(R_2 - R_1)}{\pi^2 L R_2^5}$

MCQ 6.50 A closed cylinder having a radius R and height H is filled with oil of density ρ. If the cylinder is rotated about its axis at an angular velocity of ω, then thrust at the bottom of the cylinder is

(A) $\pi R^2 \rho g H$
(B) $\pi R^2 \frac{\rho \omega^2 R^2}{4}$
(C) $\pi R^2 (\rho \omega^2 R^2 + \rho g H)$
(D) $\pi R^2 \left(\frac{\rho \omega^2 R^2}{4} + \rho g H\right)$

MCQ 6.51 For air flow over a flat plate, velocity (U) and boundary layer thickness (δ) can be expressed respectively, as

$$\frac{U}{U_\infty} = \frac{3y}{2\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3; \ \delta = \frac{4.64x}{\sqrt{\text{Re}_x}}$$

If the free stream velocity is 2 m/s, and air has kinematic viscosity of 1.5×10^{-5} m^2/s and density of 1.23 kg/m^3, the wall shear stress at $x = 1$ m, is

(A) 2.36×10^2 N/m^2

(B) 43.6×10^{-3} N/m^2

(C) 4.36×10^{-3} N/m^2

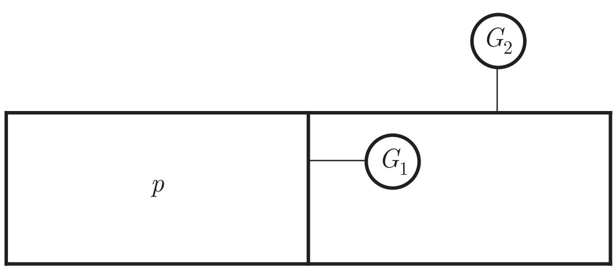
(D) 2.18×10^{-3} N/m^2

MCQ 6.52 A centrifugal pump is required to pump water to an open water tank situated 4 km away from the location of the pump through a pipe of diameter 0.2 m having Darcy's friction factor of 0.01. The average speed of water in the pipe is 2 m/s. If it is to maintain a constant head of 5 m in the tank, neglecting other minor losses, then absolute discharge pressure at the pump exit is

(A) 0.449 bar (B) 5.503 bar

(C) 44.911 bar (D) 55.203 bar

MCQ 6.53 The pressure gauges G_1 and G_2 installed on the system show pressure of $p_{G1} = 5.00$ bar and $p_{G2} = 1.00$ bar. The value of unknown pressure p is



(A) 1.01 bar (B) 2.01 bar

(C) 5.00 bar (D) 7.01 bar

MCQ 6.54 At a hydro electric power plant site, available head and flow rate are 24.5 m and 10.1 m^3/s respectively. If the turbine to be installed is required to run at 4.0 revolution per second (rps) with an overall efficiency of 90%, the suitable type of turbine for this site is

(A) Francis (B) Kaplan

(C) Pelton (D) Propeller

MCQ 6.55 Match List-I with List-II and select the correct answer using the codes given below the lists :

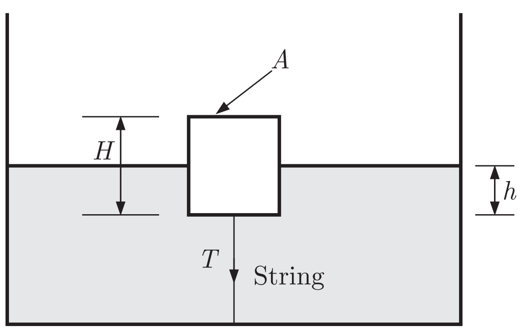
	List-I		List-II
P.	Reciprocating pump	1.	Plant with power output below 100 kW
Q.	Axial flow pump	2.	Plant with power output between 100 kW to 1 MW
R.	Microhydel plant	3.	Positive displacement
S.	Backward curved vanes	4.	Draft tube
		5.	High flow rate, low pressure ratio
		6.	Centrifugal pump impeller

Codes :

	P	Q	R	S
(A)	3	5	6	2
(B)	3	5	2	6
(C)	3	5	1	6
(D)	4	5	1	6

YEAR 2003 ONE MARK

MCQ 6.56 A cylindrical body of cross-sectional area A, height H and density ρ_s, is immersed to a depth h in a liquid of density ρ, and tied to the bottom with a string. The tension in the string is



(A) $\rho g h A$

(B) $(\rho_s - \rho) g h A$

(C) $(\rho - \rho_s) g h A$

(D) $(\rho h - \rho_s H) g A$

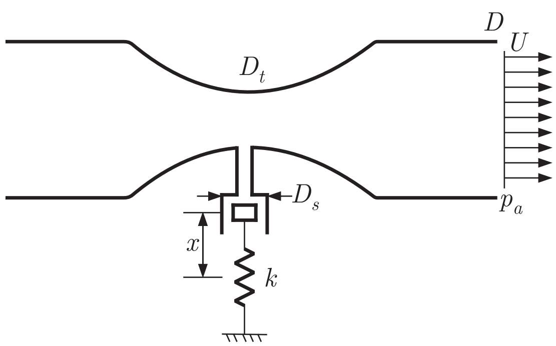
YEAR 2003 TWO MARKS

MCQ 6.57 A water container is kept on a weighing balance. Water from a tap is falling vertically into the container with a volume flow rate of Q; the velocity of the water when it hits the water surface is U. At a particular instant of time the total mass of the container and water is m. The force registered by the weighing balance at this instant of time is

(A) $mg + \rho QU$ (B) $mg + 2\rho QU$

(C) $mg + \rho QU^2/2$ (D) $\rho QU^2/2$

MCQ 6.58 Air flows through a venturi and into atmosphere. Air density is ρ; atmospheric pressure is p_a; throat diameter is D_t; exit diameter is D and exit velocity is U. The throat is connected to a cylinder containing a frictionless piston attached to a spring. The spring constant is k. The bottom surface of the piston is exposed to atmosphere. Due to the flow, the piston moves by distance x. Assuming incompressible frictionless flow, x is



(A) $(\rho U^2/2k)\pi D_s^2$ (B) $(\rho U^2/8k)\left(\frac{D^2}{D_t^2} - 1\right)\pi D_s^2$

(C) $(\rho U^2/2k)\left(\frac{D^2}{D_t^2} - 1\right)\pi D_s^2$ (D) $(\rho U^2/8k)\left(\frac{D^4}{D_t^4} - 1\right)\pi D_s^2$

MCQ 6.59 A centrifugal pump running at 500 rpm and at its maximum efficiency is delivering a head of 30 m at a flow rate of 60 litres per minute. If the rpm is changed to 1000, then the head H in metres and flow rate Q in litres per minute at maximum efficiency are estimated to be

(A) $H = 60, Q = 120$ (B) $H = 120, Q = 120$

(C) $H = 60, Q = 480$ (D) $H = 120, Q = 30$

MCQ 6.60 Match List-I with the List-II and select the correct answer using the codes given below the lists :

	List-I		List-II
P	Curtis	1.	Reaction steam turbine
Q	Rateau	2.	Gas turbine
R	Kaplan	3.	Velocity compounding
S	Francis	4.	Pressure compounding
		5.	Impulse water turbine
		6.	Axial turbine

7. Mixed flow turbine
8. Centrifugal pump

Codes :

	P	Q	R	S
(A)	2	1	1	6
(B)	3	1	5	7
(C)	1	3	1	5
(D)	3	4	7	6

MCQ 6.61 Assuming ideal flow, the force F in newtons required on the plunger to push out the water is

(A) 0 (B) 0.04

(C) 0.13 (D) 1.15

MCQ 6.62 Neglect losses in the cylinder and assume fully developed laminar viscous flow throughout the needle; the Darcy friction factor is 64/Re. Where Re is the Reynolds number. Given that the viscosity of water is 1.0×10^{-3} kg/s-m, the force F in newtons required on the plunger is

(A) 0.13 (B) 0.16

(C) 0.3 (D) 4.4

YEAR 2002 ONE MARK

MCQ 6.63 If there are m physical quantities and n fundamental dimensions in a particular process, the number of non-dimentional parameters is

(A) $m + n$ (B) $m \times n$

(C) $m - n$ (D) m/n

MCQ 6.64 If x is the distance measured from the leading edge of a flat plate, the laminar boundary layer thickness varies as

(A) $\frac{1}{x}$ (B) $x^{4/5}$

(C) x^2 (D) $x^{1/2}$

MCQ 6.65 Flow separation in flow past a solid object is caused by

(A) a reduction of pressure to vapour pressure

(B) a negative pressure gradient

(C) a positive pressure gradient

(D) the boundary layer thickness reducing to zero

MCQ 6.66 The value of Biot number is very small (less than 0.01) when
(A) the convective resistance of the fluid is negligible
(B) the conductive resistance of the fluid is negligible
(C) the conductive resistance of the solid is negligible
(D) None of the above

YEAR 2002 TWO MARKS

MCQ 6.67 The properties of mercury at $300\,K$ are; density $= 13529\,kg/m^3$, specific heat at constant pressure $= 0.1393\,kJ/kg\text{-}K$, dynamic viscosity $= 0.1523 \times 10^{-2}\,N\text{-}s/m^2$ and thermal conductivity $= 8.540\,W/m\text{-}K$. The Prandtl number of the mercury at $300\,K$ is
(A) 0.0248
(B) 2.48
(C) 24.8
(D) 248

YEAR 2001 ONE MARK

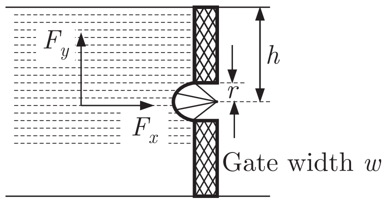
MCQ 6.68 The SI unit of kinematic viscosity (υ) is
(A) m^2/s
(B) $kg/m\text{-}s$
(C) m/s^2
(D) m^3/s^2

MCQ 6.69 A static fluid can have
(A) non-zero normal and shear stress
(B) negative normal stress and zero shear stress
(C) positive normal stress and zero shear stress
(D) zero normal stress and non-zero shear stress

MCQ 6.70 Lumped heat transfer analysis of a solid object suddenly exposed to a fluid medium at a different temperature is valid when
(A) Biot number < 0.1
(B) Biot number > 0.1
(C) Fourier number < 0.1
(D) Fourier number > 0.1

YEAR 2001 TWO MARKS

MCQ 6.71 The horizontal and vertical hydrostatic forces F_x and F_y on the semi-circular gate, having a width w into the plane of figure, are



(A) $F_x = \rho ghrw$ and $F_y = 0$

(B) $F_x = 2\rho ghrw$ and $F_y = 0$

(C) $F_x = \rho ghrw$ and $F_y = \rho gwr^2/2$

(D) $F_x = 2\rho ghrw$ and $F_y = \pi\rho gwr^2/2$

MCQ 6.72 The two-dimensional flow with velocity $v = (x + 2y + 2)\,i + (4 - y)\,j$ is

(A) compressible and irrotational

(B) compressible and not irrotational

(C) incompressible and irrotational

(D) incompressible and not irrotational

MCQ 6.73 Water (Prandtl number $= 6$) flows over a flat plate which is heated over the entire length. Which one of the following relationships between the hydrodynamic boundary layer thickness (δ) and the thermal boundary layer thickness (δ_t) is true?

(A) $\delta_t > \delta$

(B) $\delta_t < \delta$

(C) $\delta_t = \delta$

(D) cannot be predicted

****

SOLUTION

SOL 6.1 Option (A) is correct.

From Darcy Weischback equation head loss

$$h = f \times \frac{L}{D} \times \frac{V^2}{2g} \qquad ...(1)$$

Given that $h = 500\text{ m},\ D = \frac{200}{1000} = 0.2\text{ m},\ f = 0.0225$

Since volumetric flow rate

$$\dot{\nu} = \text{Area} \times \text{velocity of flow}\,(V)$$

$$V = \frac{\dot{\nu}}{\text{Area}} = \frac{0.2}{\frac{\pi}{4} \times (0.2)^2} = 6.37\text{ m/s}$$

Hence,
$$h = 0.0225 \times \frac{500}{0.2} \times \frac{(6.37)^2}{2 \times 9.81}$$

$$h = 116.33\text{ m} \simeq 116.18\text{ m}$$

SOL 6.2 Option (C) is correct.

Degree of reaction

$$R = 1 - \frac{(V_1^2 - V_2^2)}{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (W_2^2 - W_1^2)}$$

where V_1 and V_2 are absolute velocities

W_1 and W_2 are relative velocities

U_1 and $U_2 = U$ for given figure

Given $W_2 = V_1,\ W_1 = V_2$

Hence
$$R = 1 - \frac{(V_1^2 - V_2^2)}{(V_1^2 - V_2^2) + (U^2 - U^2) + (V_1^2 - V_2^2)}$$

$$= 1 - \frac{(V_1^2 - V_2^2)}{2(V_1^2 - V_2^2)} = 1 - \frac{1}{2} = 0.5$$

SOL 6.3 Option (C) is correct.

For flat plate with zero pressure gradient and Re = 1000 (laminar flow).

Boundary layer thickness

$$\delta(x) = \frac{4.91x}{\sqrt{\text{Re}_x}} = \frac{4.91x}{\sqrt{\frac{Vx}{\upsilon}}} = \frac{4.91x^{1/2}}{\sqrt{\frac{V}{\upsilon}}}$$

$\Rightarrow$
$$\delta \propto \frac{x^{1/2}}{V^{1/2}}$$
For a same location ($x = 1$)

$$\delta \propto (V)^{-1/2}$$

where V = velocity of fluid

$$\frac{\delta_1}{\delta_2} = \left(\frac{V_1}{V_2}\right)^{-1/2}$$

$$\delta_2 = \left(\frac{V_1}{V_2}\right)^{1/2} \times \delta_1 = \left(\frac{V_1}{4V_1}\right)^{1/2} \times 1 \quad V_2 = 4V_1 \text{ (Given)}$$

$$= \left(\frac{1}{4}\right)^{1/2} \times 1 = \frac{1}{2} = 0.5$$

SOL 6.4 Option (A) is correct.

Takes point (1) at top and point (2) at bottom

By Bernoulli equation between (1) and (2)

$$p_1 + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3 + \frac{V_1^2(p_1 + p_2 + p_3)}{2g} = p_{atm.} + \frac{V_2^2}{2g}$$

At Reference level (2) $z_2 = 0$ and $V_1 = 0$ at point (1)

Therefore

$$\Rightarrow \quad p_1 + \rho_1 g h_1 + \rho_1 g h_2 + \rho_3 g h_3 = p_{atm.} + \frac{V_2^2}{2g} \qquad ...(1)$$

Since p_1 = atmospheric pressure (because tank is open)

Hence $p_1 = p_{\text{atm.}}$

Therefore

$$V_2 = \sqrt{2g \times [\rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3]}$$

By Rearranging

$$V_2 = \sqrt{2g \times \left[\frac{\rho_1 g h_1}{\rho_3 g} + \frac{\rho_2 g h_2}{\rho_3 g} + h_3\right]}$$

$$= \sqrt{2g \times \left[\frac{\rho_1 h_1}{\rho_3} + \frac{\rho_2 h_2}{\rho_3} + h_3\right]} = \sqrt{2gh_3 \times \left[1 + \frac{\rho_1 h_1}{\rho_3 h_3} + \frac{\rho_2 h_2}{\rho_3 h_3}\right]}$$

SOL 6.5 Option (B) is correct.

For Equipotential line, $\frac{dy}{dx} = -\frac{u}{v}$ = Slope of equipotential line ...(i)

For stream function,

$$\frac{dy}{dx} = \frac{v}{u} = \text{Slope of stream line} \qquad ...(ii)$$

It is clear from equation (i) and (ii) that the product of slope of equipotential line and slope of the stream line at the point of intersection is equal to -1.

$$-\frac{u}{v} \times \frac{v}{u} = -1$$

And, when $m_1 m_2 = -1$, Then lines are perpendicular, therefore the stream line and an equipotential line in a flow field are perpendicular to each other.

SOL 6.6 Option (C) is correct.

Given : $p_a = 1.2\ \text{kg/m}^3,\ \rho_w = 1000\ \text{kg/m}^3,\ x = 10 \times 10^{-3}\ \text{m},\ g = 9.8\ \text{m/sec}^2$

If the difference of pressure head 'h' is measured by knowing the difference of the level of the manometer liquid say x. Then

$$h = x\left[\frac{S.G_w}{S.G_a} - 1\right] = x\left[\frac{\rho_w}{\rho_a} - 1\right]$$

$$= 10 \times 10^{-3}\left[\frac{1000}{1.2} - 1\right] = 8.32\ \text{m}$$

Where $S.G = \dfrac{\text{Weight density of liquid}}{\text{Weight density of water}}$

$S.G \propto$ Density of Liquid

Velocity of air $V = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 8.32} = 12.8\ \text{m/sec}$

SOL 6.7 Option (D) is correct.

Given : $p_1 = 1\ \text{bar},\ p_2 = 30\ \text{bar},\ \rho = 990\ \text{kg/m}^3$

Isentropic work down by the pump is given by,

$$W = \nu dp = \frac{m}{\rho}dp \qquad \nu = \frac{m}{\rho}$$

$$\frac{W}{m} = \frac{1}{\rho}dp = \frac{1}{990} \times (30-1) \times 10^5\ \text{pascal}$$

$$= 2929.29\ \text{J/kg} = 2.93\ \text{kJ/kg}$$

SOL 6.8 Option (B) is correct.

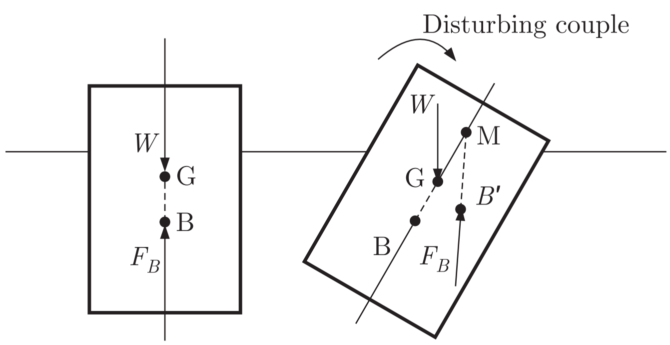


Fig. (I) Fig. (II)

As shown in figure above. If point B' is sufficiently far from B, these two forces (Gravity force and Buoyant force) create a restoring moment and return the body to the original position.

A measure of stability for floating bodies is the metacentric height GM, which is the distance between the centre of gravity G and the metacenter M (the intersection point of the lines of action of the buoyant force through the body before and after rotation.)

A floating body is stable if point M is above the point G, and thus GM is positive, and unstable if point M is below point G, and thus GM is negative. Stable equilibrium occurs when M is above G.

SOL 6.9 Option (C) is correct.

In case of two parallel plates, when flow is fully developed, the ratio of V_{max} and V_{avg} is a constant.

$$\frac{V_{max}}{V_{avg}} = \frac{3}{2} \qquad V_{max} = 6 \text{ m/sec}$$

$$V_{avg} = \frac{2}{3} \times V_{max} = \frac{2}{3} \times 6 = 4 \text{ m/sec}$$

SOL 6.10 Option (C) is correct.

From Buckingham's π-theorem

It states "If there are n variable (Independent and dependent variables) in a physical phenomenon and if these variables contain m fundamental dimensions (M, L, T), then variables are arranged into $(n-m)$ dimensionless terms.

Here n = dimensional variables

k = Primary dimensions (M, L, T)

So, non dimensional variables, $\Rightarrow n - k$

SOL 6.11 Option (B) is correct.

Given : $P_1 = 10^3 \text{ kW}$, $H_1 = 40 \text{ m}$, $H_2 = 40 - 20 = 20 \text{ m}$

If a turbine is working under different heads, the behavior of turbine can be easily known from the values of unit quantities i.e. from the unit power.

So
$$P_u = \frac{P}{H^{3/2}}$$

$$\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

$$P_2 = \left(\frac{H_2}{H_1}\right)^{3/2} \times P_1 = \left(\frac{20}{40}\right)^{3/2} \times 1000 = 353.6 \approx 354 \text{ kW}$$

SOL 6.12 Option (D) is correct.

Given : $V = 2xyi - x^2zj \qquad P(1,1,1)$

The vorticity vector is defined as,

$$\text{Vorticity Vector} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

Substitute, $u = 2xy$, $v = -x^2z$, $w = 0$

So,
$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & -x^2z & 0 \end{vmatrix}$$

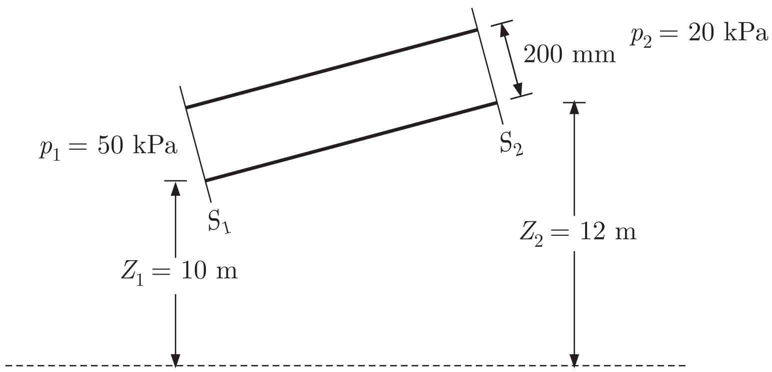
$$= i\left[-\frac{\partial}{\partial z}(-x^2z)\right] - j\left[-\frac{\partial}{\partial z}(2xy)\right] + k\left[\frac{\partial}{\partial x}(-x^2z) - \frac{\partial}{\partial y}(2xy)\right]$$

$$= x^2 i - 0 + k[-2xz - 2x]$$

Vorticity vector at $P(1,1,1)$, $= i + k[-2-2] = i - 4k$

SOL 6.13 Option (C) is correct.

Given : $p_1 = 50\text{ kPa}$, $Z_1 = 10\text{ m}$, $V_2 = 2\text{ m/sec}$, $p_2 = 20\text{ kPa}$, $Z_2 = 12\text{ m}$, $\rho = 1000\text{ kg/m}^3$, $g = 9.8\text{ m/sec}^2$



Applying continuity equation at section S_1 and S_2,

$$A_1V_1 = A_2V_2$$

$$V_1 = V_2 \qquad D_1 = D_2 \text{ so } A_1 = A_2 \text{... (i)}$$

Applying Bernoulli's equation at section S_1 and S_2 with head loss h_L,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$\frac{p_1}{\rho g} + z_1 = \frac{p_2}{\rho g} + z_2 + h_L \qquad \text{From equation (i)}$$

$$h_L = \left(\frac{p_1 - p_2}{\rho g}\right) + (z_1 - z_2) = \frac{(50-20)\times 10^3}{(1000 \times 9.8)} + (10 - 12)$$

$$= 3.058 - 2 = 1.06\text{ m}$$

Head at section (S_1) is given by,

$$H_1 = \frac{p_1}{\rho g} + Z_1 = \frac{50 \times 10^3}{10^3 \times 9.8} + 10 = 15.09\text{ m}$$

Head at section S_2,

$$H_2 = \frac{p_2}{\rho g} + Z_2 = \frac{20 \times 10^3}{10^3 \times 9.8} + 12 = 14.04\text{ m}$$

From H_1 and H_2 we get $H_1 > H_2$. So, flow is from S_1 to S_2

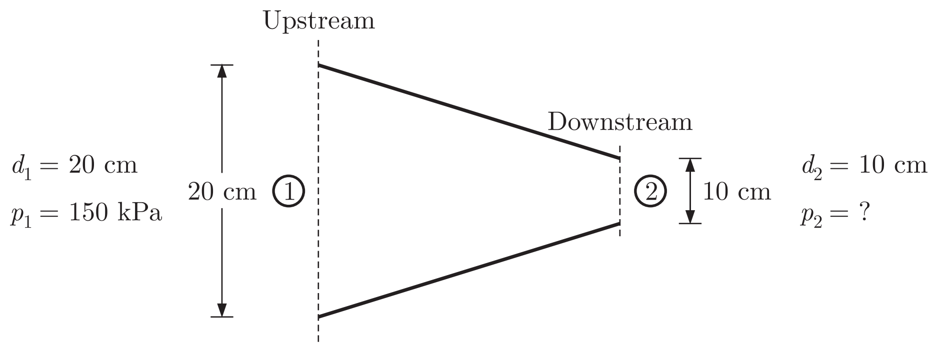
SOL 6.14 Option (D) is correct.

Here type of flow is related to the dimensionless numbers (Non-dimensional numbers). So

P.	Compressible flow	Y.	Mach number
Q.	Free surface flow	W.	Weber number
R.	Boundary layer	Z.	Skin friction coefficient
S.	Pipe flow	U.	Reynolds number
T.	Heat convection	V.	Nusselt number

So, correct pairs are P-Y, Q-W, R-Z, S-U, T-V

SOL 6.15 Option (B) is correct.



Reducer

Given : $p_V = 50\,\text{kPa}$, $w = 5\,\text{kN/m}^3 = \rho g$

Consider steady, incompressible and irrotational flow and neglecting frictional effect. First of all applying continuity equation at section (1) and (2).

$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4}(d_1)^2 \times V_1 = \frac{\pi}{4}(d_2)^2 \times V_2$$

Substitute the values of d_1 and d_2, we get

$$\frac{\pi}{4}(20)^2 \times V_1 = \frac{\pi}{4}(10)^2 \times V_2$$

$$400 V_1 = 100 V_2 \qquad \Rightarrow V_2 = 4V_1 \qquad \text{...(i)}$$

Cavitation is the phenomenon of formation of vapor bubbles of a flowing liquid in a region where the pressure of liquid falls below the vapor pressure $[p_L < p_V]$

So, we can say that maximum pressure in downstream of reducer should be equal or greater than the vapor pressure. For maximum discharge

$$p_V = p_2 = 50\,\text{kPa}$$

Applying Bernoulli's equation at point (1) and (2)

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Here $z_1 = z_2$ for horizontal pipe and $w = \rho g = 5\,\text{kN/m}^2$

$$\frac{150}{5} + \frac{V_1^2}{2g} = \frac{50}{5} + \frac{(4V_1)^2}{2g} \qquad \text{From equation (i) } V_2 = 4V_1$$

$$\frac{150}{5} - \frac{50}{5} = \frac{16V_1^2}{2g} - \frac{V_1^2}{2g}$$

$$20 = \frac{15V_1^2}{2g}$$

$$V_1^2 = \frac{40 \times 9.81}{15} = 5.114\,\text{m/sec}$$

And $\quad V_2 = 4V_1 = 4 \times 5.114 = 20.46\,\text{m/sec}$

Maximum discharge,

$$Q_{\max} = A_2 V_2 = \frac{\pi}{4}(d_2)^2 V_2 = \frac{\pi}{4}(10 \times 10^{-2})^2 \times 20.46$$

$$= \frac{\pi}{4} \times 10^{-2} \times 20.46 = 0.16\,\text{m}^3/\text{sec}$$

SOL 6.16 Option (D) is correct.

Given :

P : $\quad u = 2y, V = -3x$

Q : $\quad u = 3xy, V = 0$

R : $\quad u = -2x, V = 2y$

For incompressible fluid,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \text{...(i)}$$

For irrotational flow $\zeta_z = 0$,

$$\zeta_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \qquad \text{...(ii)}$$

From equation (i) and (ii), check P, Q and R

For P : $\quad u = 2y, \quad \frac{\partial u}{\partial x} = 0, \ \frac{\partial u}{\partial y} = 2$

$\quad v = -3x, \quad \frac{\partial v}{\partial y} = 0, \ \frac{\partial v}{\partial x} = -3$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \Rightarrow 0 + 0 = 0 \qquad \text{(Flow is incompressible)}$$

Or, $\quad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$

$$-3 - 2 = 0 \quad \Rightarrow -5 \neq 0 \qquad \text{(Rotational flow)}$$

For Q : $\quad u = 3xy \quad \frac{\partial u}{\partial x} = 3y, \ \frac{\partial u}{\partial y} = 3x$

$$v = 0 \qquad \frac{\partial v}{\partial y} = 0,\ \frac{\partial v}{\partial x} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \Rightarrow 3y \neq 0 \qquad \text{(Compressible flow)}$$

Or, $$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$0 - 3x = 0 \qquad \Rightarrow -3x \neq 0 \qquad \text{(Rotational flow)}$$

For R : $$u = -2x \qquad \frac{\partial u}{\partial x} = -2,\ \frac{\partial u}{\partial y} = 0$$

$$v = 2y \qquad \frac{\partial v}{\partial y} = 2,\ \frac{\partial v}{\partial x} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$-2 + 2 = 0 \qquad \Rightarrow 0 = 0 \qquad \text{(Incompressible flow)}$$

Or, $$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$0 - 0 = 0 \qquad \Rightarrow 0 = 0 \qquad \text{(Irrotational flow)}$$

So, we can easily see that R is incompressible and irrotational flow.

SOL 6.17 Option (A) is correct.

Given : $L = 1\,\text{km} = 1000\,\text{m},\ D = 200\,\text{mm} = 0.2\,\text{m},\ \ Q = 0.07\,\text{M}^3/\text{sec}$

$f = 0.02,\ \rho = 1000\,\text{kg/m}^3$

Head loss is given by,

$$h_f = \frac{fLV^2}{D \times 2g} = \frac{fL}{D \times 2g}\left(\frac{4Q}{\pi D^2}\right)^2 = \frac{16fLQ^2}{\pi^2 D^5 \times 2g} = \frac{8fLQ^2}{\pi^2 D^5 g} \qquad Q = \frac{\pi D^2}{4} \times V$$

$$= \frac{8 \times 0.02 \times 1000 \times (0.07)^2}{(3.14)^2 \times (0.2)^5 \times (9.81)}$$

$$= \frac{0.784}{0.30} = 2.61\,\text{m}$$ of water Pumping power required,

$$P = \rho g Q \times h_f = 1000 \times 9.81 \times 0.07 \times 2.61$$

$$= 1752.287 = 1.752\,\text{kW} \approx 1.8\,\text{kW}$$

SOL 6.18 Option (A) is correct.

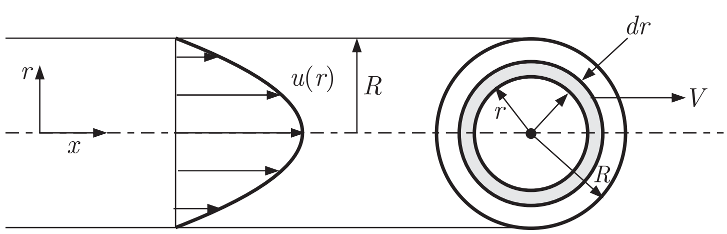


Fig. (I) Fig. (II)

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

Therefore, the velocity profile in fully developed laminar flow in a pipe is parabolic with a maximum at the center line and minimum at the pipe wall. The average velocity is determined from its definition,

$$V_{avg} = \int_0^R u(r)\,r dr = -\frac{2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) r dr$$

$$= -\frac{1}{2\mu}\left(\frac{dp}{dx}\right)\int_0^R \left(r - \frac{r^3}{R^2}\right) dr$$

$$= -\frac{1}{2\mu}\left(\frac{dp}{dx}\right)\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R = -\frac{1}{2\mu}\left(\frac{dp}{dx}\right)\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right]$$

$$= -\frac{1}{2\mu}\left(\frac{dp}{dx}\right) \times \frac{R^2}{4} = -\frac{R^2}{8\mu}\left(\frac{dp}{dx}\right)$$

Alternate Method :

Now we consider a small element (ring) of pipe with thickness dr and radius r.

We find the flow rate through this elementary ring.

$$dQ = (2\pi r) \times dr \times u(r) \qquad \text{Put the value of } u(r)$$

$$dQ = (2\pi r) \times dr \times \left(-\frac{R^2}{4\mu}\right)\left(\frac{dp}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

Now for total discharge integrate both the rides within limit.

$$Q \Rightarrow 0 \text{ to } Q \text{ and } R \Rightarrow 0 \text{ to } R$$

So

$$\int_0^Q dQ = -2\pi\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\int_0^R r\left(1 - \frac{r^2}{R^2}\right) dr$$

$$[Q]_0^Q = -2\pi\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R$$

Now put the limits, we have

$$Q = -2\pi\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right] = -2\pi\frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)\left[\frac{R^2}{2} - \frac{R^2}{4}\right]$$

$$= -2\pi\left(\frac{R^2}{4\mu}\right)\left(\frac{dp}{dx}\right)\left[\frac{R^2}{4}\right] = -\frac{\pi R^4}{8\mu}\left(\frac{dp}{dx}\right)$$

Now

$$Q = \text{Area} \times \text{Average velocity} = A \times V_{avg.}$$

$$V_{avg.} = \frac{Q}{A} = \frac{-\pi R^4}{8\mu}\left(\frac{dp}{dx}\right) \times \frac{1}{\pi R^2} = -\frac{R^2}{8\mu}\left(\frac{dp}{dx}\right)$$

SOL 6.19 Option (D) is correct.

The continuity equation in three dimension is given by,

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

For incompressible flow ρ = Constant

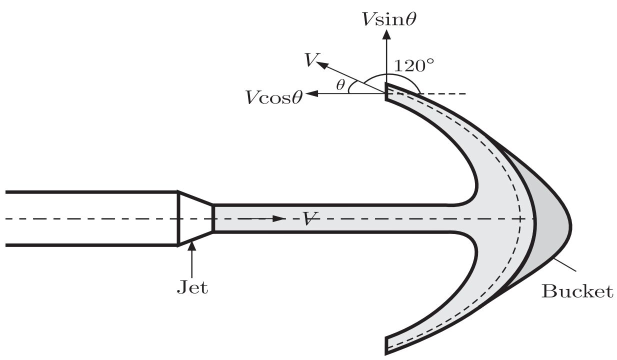
$$\rho\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right]=0$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$\nabla \cdot V = 0$$

So, the above equation represents the incompressible flow.

SOL 6.20 None of these is correct.



Given : $\rho = 1000\,\text{kg/m}^3$, $V = 10\,\text{m/sec}$, $\theta = 180 - 120 = 60^\circ$, $R = 0.5\,\text{m}$

Initial velocity in the direction of jet $= V$

Final velocity in the direction of the jet $= -V\cos\theta$.

Force exerted on the bucket

$$F_x = \rho AV[V-(-V\cos\theta)] = \rho AV[1+\cos\theta]V$$

$$= Q(1+\cos\theta)V \qquad \text{Mass flow rate } Q = \rho AV$$

Torque, $T_x = F_x \times R = QV(1+\cos\theta)R$

Torque per unit mass flow rate

$$\frac{T_x}{Q} = V(1+\cos\theta)R = 10(1+\cos 60^\circ)\times 0.5$$

$$= 7.5\,\text{N-m/kg/sec}$$

And $F_y = \rho AV(0 - V\sin\theta) = -QV\sin\theta$

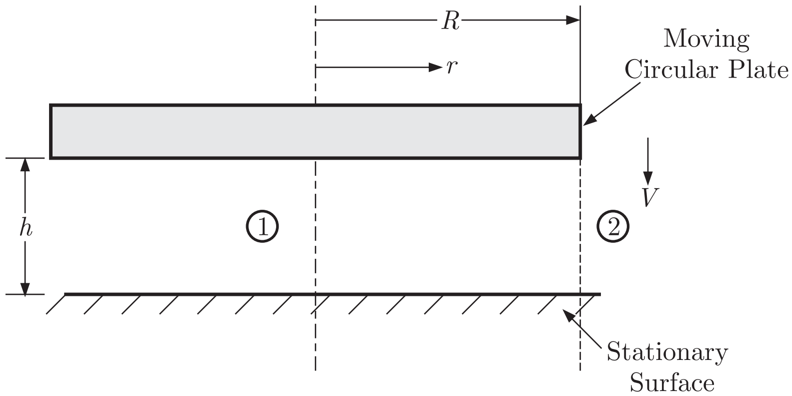
Torque in y-direction

$$T_y = F_y \times R = 0 \qquad R = 0$$

Total Torque will be

$$T = \sqrt{T_x^2 + T_y^2} = T_x = 7.5\,\text{N}-\text{m/kg/sec}$$

SOL 6.21 Option (A) is correct.



Here Gap between moving and stationary plates are continuously reduced, so we can say that

Volume of fluid moving out radially

= Volume of fluid displaced by moving plate within radius r

Volume displaced by the moving plate

= Velocity of moving plate × Area $= V \times \pi r^2$ ...(i)

Volume of fluid which flows out at radius r

$= V_r \times 2\pi r \times h$ ...(ii)

Equating equation (i) and (ii),

$$V \times \pi r^2 = V_r \times 2\pi r h$$

$$Vr = 2V_r h \quad \Rightarrow \quad V_r = \frac{Vr}{2h}$$

Alternate Method :

Apply continuity equation at point (i) and (ii),

$$A_1 V_1 = A_2 V_2$$

$$V \times \pi r^2 = V_r \times 2\pi r h$$

$$V_r = \frac{Vr}{2h}$$

SOL 6.22 Option (B) is correct.

From previous part of question,

$$V_r = \frac{Vr}{2h}$$

Acceleration at radius r is given by

$$a_r = V_r \times \frac{dV_r}{dr} = V_r \times \frac{d}{dr}\left[\frac{Vr}{2h}\right] = V_r \times \frac{V}{2h} \quad \text{...(i)}$$

At $r = R$ $\quad a_r = \frac{VR}{2h} \times \frac{V}{2h} = \frac{V^2 R}{4h^2}$

SOL 6.23 Option (D) is correct.

$$F_D = C_D \times \frac{\rho A V^2}{2} = \frac{1.33}{\sqrt{\text{Re}_L}} \times \frac{\rho A V^2}{2} \qquad C_D = \frac{1.33}{\sqrt{\text{Re}_L}}$$

$$= \frac{1.33}{\sqrt{\frac{\rho VL}{\mu}}} \times \frac{1}{2}\rho \times bLV^2 \qquad \mathrm{Re}_L = \frac{\rho VL}{\mu}$$

$$= \frac{1.33}{\sqrt{\frac{\rho V}{\mu}}} \times \frac{1}{2}\rho b V^2 \sqrt{L} \qquad ...(i)$$

So from equation (i)

$$F_D \propto \sqrt{L} \qquad ...(ii)$$

Drag force on front half of plate

$$F_{D/2} = \sqrt{\frac{L}{2}} = \frac{F_D}{\sqrt{2}} \qquad \text{From Equation (ii)}$$

Drag on rear half,

$$F'_{D/2} = F_D - F_{D/2} = \left(1 - \frac{1}{\sqrt{2}}\right)F_D$$

Now ratio of $F_{D/2}$ and $F'_{D/2}$ is

$$F = \frac{F_{D/2}}{F'_{D/2}} = \frac{\frac{F_D}{\sqrt{2}}}{\left(1 - \frac{1}{\sqrt{2}}\right)F_D} = \frac{1}{\sqrt{2} - 1} > 1$$

SOL 6.24 Option (B) is correct.

Given :

$$v = u_0\left(1 + \frac{3x}{L}\right)$$

$$\frac{dx}{dt} = u_0\left(1 + \frac{3x}{L}\right) = \frac{u_0}{L}(L + 3x)$$

$$dt = \frac{L}{u_0} \times \frac{1}{(L + 3x)}dx$$

On integrating both the sides within limits $t \Rightarrow 0$ to t and $x \Rightarrow 0$ to L, we get

$$\int_0^t dt = \frac{L}{u_0}\int_0^L \frac{1}{(L + 3x)}dx$$

$$[t]_0^t = \frac{L}{3u_0}[\ln(L + 3x)]_0^L$$

$$t = \frac{L}{3u_0}[\ln 4L - \ln L] = \frac{L}{3u_0}\ln 4$$

SOL 6.25 Option (A) is correct.

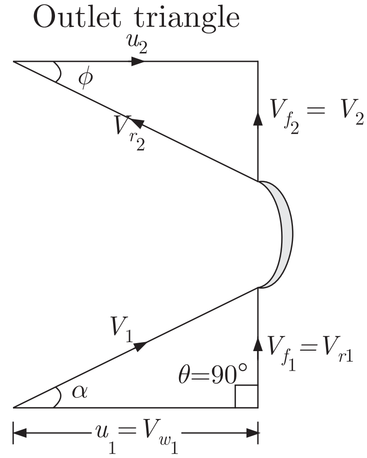
Reynolds Number,

$$\mathrm{Re} = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho A V^2}{\mu \times \frac{V}{L} \times A}$$

$$= \frac{\rho VL}{\mu} = 5 = \frac{I.F.}{V.F.}$$

SOL 6.26 Option (C) is correct.

Given figure shows the velocity triangle for the pelton wheel.



Velocity triangle for Francis turbine

Given :

Flow velocity at Inlet V_{f_1} = flow velocity at outlet V_{f_2}

$$V_{f_1} = V_{f_2} = \frac{u_1}{2} \text{ (blade velocity)}$$

$$V_2 = V_{f_2}$$

$$u_1 = V_{w_1} \qquad \theta = 90°$$

From Inlet triangle, $$V_1^2 = (V_{f_1})^2 + (V_{w_1})^2 = \left(\frac{u_1}{2}\right)^2 + (u_1)^2 = \frac{5}{4}u_1^2$$

$$\text{Blade efficiency} = \frac{V_1^2 - V_2^2}{V_1^2} \times 100 = \frac{\frac{5}{4}u_1^2 - \frac{u_1^2}{4}}{\frac{5}{4}u_1^2} \times 100$$

$$= \frac{u_1^2}{\frac{5}{4}u_1^2} \times 100 = 80\%$$

SOL 6.27 Option (C) is correct.

$$u = \frac{\pi DN}{60} = \sqrt{2gH}$$

From this equation, $$\sqrt{H} \propto DN$$

$$\frac{\sqrt{H}}{DN} = \text{Constant}$$

So using this relation for the given model or prototype,

$$\left(\frac{\sqrt{H}}{DN}\right)_p = \left(\frac{\sqrt{H}}{DN}\right)_m$$

$$\frac{N_p}{N_m} = \sqrt{\frac{H_p}{H_m}} \times \frac{D_m}{D_p} \qquad \text{...(i)}$$

Given : $H_m = \frac{1}{4}H_p,\ D_m = \frac{1}{2}D_p,\ N_p = N$

$$\frac{N}{N_m} = \sqrt{\frac{H_p}{\frac{1}{4}H_p}} \times \frac{\frac{1}{2}D_p}{D_p} = \sqrt{4} \times \frac{1}{2} = 1$$

So, $N_m = N$

SOL 6.28 Option (B) is correct.

For forced Vortex flow the relation is given by,

$$V = r\omega \qquad \text{...(i)}$$

From equation (i) it is shown easily that velocity is directly proportional to the radius from the centre of the vortex (Radius of fluid particle from the axis of rotation)

And also for forced vortex flow,

$$\frac{1}{2}\rho\omega^2(r_2^2 - r_1^2) - \rho g(z_2 - z_1) = 0$$

$$\Delta K.E. - \Delta P.E. = 0 \qquad \Rightarrow \quad \Delta K.E = \Delta P.E.$$

Now total mechanical energy per unit mass is constant in the entire flow field.

SOL 6.29 Option (A) is correct.

	List-I		List-II
P.	Centrifugal compressor	2.	Surging
Q.	Centrifugal pump	3.	Priming
R.	Pelton wheel	4.	Pure Impulse
S.	Kaplan Turbine	1.	Axial Flow

So, correct pairs are P-2, Q-3, R-4, S-1

SOL 6.30 Option (C) is correct.

Let width of the channel $= b$

From mass conservation

Flow rate at section A = flow rate at B

or Velocity $A \times$ Area of A = Velocity at $B \times$ Area of B

$$U_0 \times (H \times b) = \text{Velocity for } (0 \le y \le \delta) \times dy \times b$$
$$+\text{velocity for } (\delta \le y \le H - \delta) \times dy \times b$$
$$+\text{velocity for } (H - \delta \le y \le H) \times dy \times b$$

or
$$U_0 \times H = V_m \int_0^{\delta} \frac{y}{\delta} dy + V_m \int_{\delta}^{H-\delta} dy + V_m \int_{H-\delta}^{H} \frac{H-y}{\delta} dy$$

or $$U_0 \times H = V_m \frac{\delta}{2} + V_m (H - 2\delta) + \frac{V_m \delta}{2}$$

$$U_0 \times H = V_m \delta + V_m (H - 2\delta) = V_m (\delta + H - 2\delta)$$

or $$\frac{V_m}{U_0} = \frac{H}{\delta + H - 2\delta}$$

$$= \frac{H}{H - \delta} = \frac{1}{1 - \frac{\delta}{H}}$$

SOL 6.31 Option (A) is correct.

Applying Bernoulli's Equation at the section A and B.

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B$$

Here, $z_A = z_B = 0$

So, $$\frac{p_A - p_B}{\rho g} = \frac{V_B^2 - V_A^2}{2g}$$

$$\frac{p_A - p_B}{\rho} = \frac{V_B^2 - V_A^2}{2} = \frac{V_m^2 - U_0^2}{2} \qquad V_B = V_m \text{ and } V_A = U_0$$

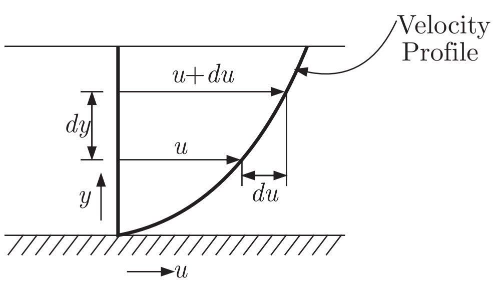
$$= \frac{U_0^2\left[\frac{V_m^2}{U_0^2} - 1\right]}{2}$$

$$\frac{p_A - p_B}{\frac{1}{2}\rho U_0^2} = \frac{V_m^2}{U_0^2} - 1 = \left(\frac{V_m}{U_0}\right)^2 - 1$$

Substitute, $$\frac{V_m}{U_0} = \frac{1}{1 - \frac{\delta}{H}}$$ From previous part of question

$$\frac{p_A - p_B}{\frac{1}{2}\rho U_0^2} = \frac{1}{[1 - \delta/H]^2} - 1$$

SOL 6.32 Option (C) is correct.



Velocity variation near a body

From the Newton's law of Viscosity, the shear stress (τ) is directly proportional to the rate of shear strain (du/dy).

So, $$\tau \propto \frac{du}{dy} = \mu \frac{du}{dy}$$

Where $\mu =$ Constant of proportionality and it is known as coefficient of Viscosity.

SOL 6.33 Option (C) is correct.

Convective Acceleration is defined as the rate of change of velocity due to the change of position of fluid particles in a fluid flow.

In Cartesian coordinates, the components of the acceleration vector along the x-direction is given by.

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

In above equation term $\partial u/\partial t$ is known as local acceleration and terms other then this, called convective acceleration.

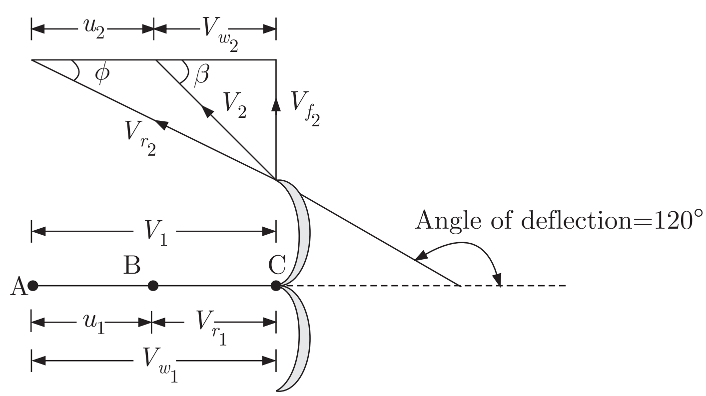
Hence for given flow.

Convective acceleration along x-direction.

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \qquad [w = 0]$$

SOL 6.34 Option (C) is correct.

The velocity triangle for the pelton wheel is given below.



Given : $u = u_1 = u_2 = 10 \text{ m/sec}, \; V_1 = 25 \text{ m/sec}, \; Q = 0.1 \text{ m}^3/\text{sec}$

Jet deflection angle $= 120^\circ$ C

$$\phi = 180^\circ - 120^\circ = 60^\circ$$

$$P = \frac{\rho Q[V_{w_1} + V_{w_2}] \times u}{1000} \text{ kW} \qquad ...(i)$$

From velocity triangle,

$$V_{w_1} = V_1 = 25 \text{ m/sec}$$

$$V_{w_2} = V_{r_2}\cos\phi - u_2 \qquad V_{r_2} = V_{r_1} = V_1 - u_1$$

$$= 15\cos 60^\circ - 10 \qquad = 25 - 10 = 15 \text{ m/sec}$$

$$= \frac{15}{2} - 10 = -2.5 \text{ m/sec}$$

Now put there values in equation (i)

$$P = \frac{1000 \times 0.1[25 - 2.5] \times 10}{1000}\text{ kW} = 22.5\text{ kW}$$

SOL 6.35 Option (D) is correct.

Given : $u = x^2 t$, $v = -2xyt$

The velocity component in terms of stream function are

$$\frac{\partial \psi}{\partial x} = v = -2xyt \quad ...(i)$$

$$\frac{\partial \psi}{\partial y} = -u = -x^2 t \quad ...(ii)$$

Integrating equation (i), w.r.t 'x', we get

$$\psi = \int (-2xyt)\,dx$$

$$= -x^2 yt + K \quad ...(iii)$$

Where, K is a constant of integration which is independent of 'x' but can be a function of 'y'

Differentiate equation (iii) w.r.t y, we get

$$\frac{\partial \psi}{\partial y} = -x^2 t + \frac{\partial K}{\partial y}$$

But from equation (ii),

$$\frac{\partial \psi}{\partial y} = -x^2 t$$

Comparing the value of $\frac{\partial \psi}{\partial y}$, we get

$$-x^2 t + \frac{\partial K}{\partial y} = -x^2 t$$

$$\frac{\partial K}{\partial y} = 0$$

$$K = \text{Constant}(K_1)$$

From equation (iii)

$$\psi = -x^2 yt + K_1$$

The line for which stream function ψ is zero called as stream line.

So, $-x^2 yt + K_1 = 0$

$$K_1 = x^2 yt$$

If 't' is constant then equation of stream line is,

$$x^2 y = \frac{K_1}{t} = K_2$$

But in the question, there is no condition for t is constant. Hence, it is not possible to determine equation of stream line.

SOL 6.36 Option (D) is correct.

Given : $u = u_o\left(1 - \frac{4r^2}{D^2}\right) = u_o\left(1 - \frac{r^2}{R^2}\right)$

Drop of pressure for a given length (L)of a pipe is given by,

$$\Delta p = p_1 - p_2 = \frac{32\mu\bar{u}L}{D^2} \quad ..(i)$$

(From the Hagen poiseuille formula)

Where $\bar{u}$ = average velocity

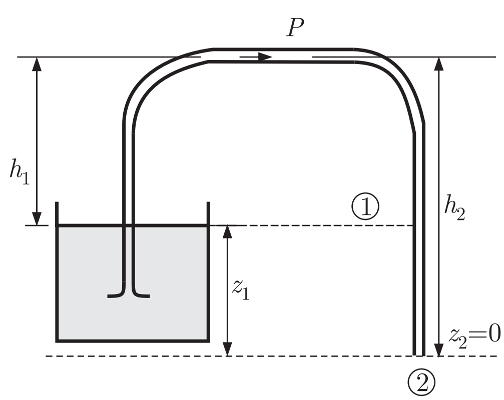
And
$$\bar{u} = \frac{2}{R^2}\int_0^R u(r)\,rdr = \frac{2}{R^2}\int_0^R u_o\left(1 - \frac{r^2}{R^2}\right)rdr$$
$$= \frac{2u_o}{R^2}\int_0^R\left(r - \frac{r^3}{R^2}\right)dr = \frac{2u_o}{R^2}\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R$$
$$= \frac{2u_o}{R^2}\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right]_0^R = \frac{2u_o}{R^2}\left[\frac{R^2}{4}\right]$$
$$\overline{u} = \frac{u_o}{2}$$

Substitute the value of $\overline{u}$ in equation(1)

So, $$\Delta p = \frac{32\mu L}{D^2} \times \frac{u_o}{2} = \frac{16\mu u_o L}{D^2}$$

Note : The average velocity in fully developed laminar pipe flow is one-half of the maximum velocity.

SOL 6.37 Option (C) is correct.



In a steady and ideal flow of incompressible fluid, the total energy at any point of the fluid is constant. So applying the Bernoulli's Equation at section (1) and (2)

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$V_1 = 0$ = Initial velocity at point (1)

$Z_2 = 0$ = At the bottom surface

$$p_1 = p_2 = p_{atm}$$

And $$z_1 = h_2 - h_1$$

So, $$h_2 - h_1 = \frac{V_2^2}{2g}$$

$$V_2^2 = 2g(h_2 - h_1)$$

$$V_2 = \sqrt{2g(h_2 - h_1)}$$

So, velocity of fluid is same inside the tube

$$V_p = V_2 = \sqrt{2g(h_2 - h_1)}$$

SOL 6.38 Option (A) is correct.

Given : $P_1 = 300\text{ kW},\ N_1 = 1000\text{ rpm},\ H_1 = 40\text{ m}$

$$\frac{d_2}{d_1} = \frac{1}{4},\ H_2 = 10\text{ m}$$

Specific power for similar turbine is same. So from the relation, we have

$$\frac{P}{d^2H^{3/2}} = \text{Constant}$$

For both the cases,

$$\frac{P_1}{d_1^2H_1^{3/2}} = \frac{P_2}{d_2^2H_2^{3/2}}$$

$$P_2 = \left(\frac{d_2}{d_1}\right)^2\left(\frac{H_2}{H_1}\right)^{3/2} \times P_1 = \left(\frac{1}{4}\right)^2\left(\frac{10}{40}\right)^{3/2} \times 300 = 2.34$$

SOL 6.39 Option (A) is correct.

Net positive suction head, (NPSH) = Pressure head + static head

Pressure difference, $\Delta p = 200 - (-25) = 225\text{ kPa}$

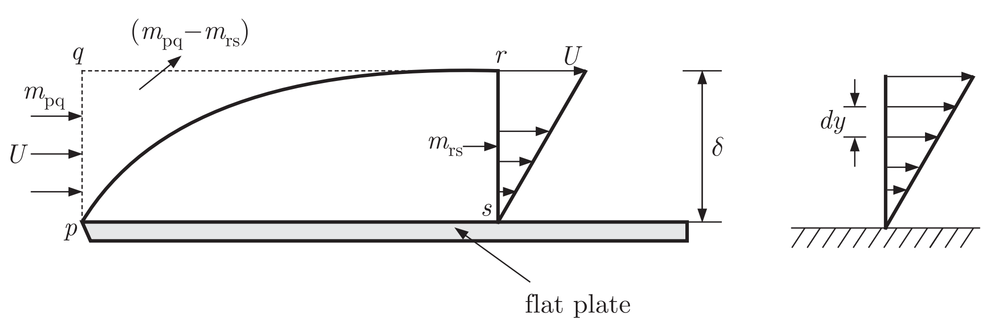
(Negative sign shows that the pressure acts on liquid in opposite direction)

$$\Delta p = 225 \times 10^3\text{ Pa} = 2.25\text{ bar} = \frac{2.25 \times 10.33}{1.013}\text{ m} = 22.95\text{ m of water}$$

Static head $= 1\text{ m}$ (Given)

Now, NPSH $= 22.95 + 1 = 23.95 \simeq 24\text{ m of water}$

SOL 6.40 Option (B) is correct.



Given : $U = 10\ \text{m/sec},\ \delta = 10\ \text{mm} = 10^{-2}\ \text{meter},\ \rho = 1.0\ \text{kg/m}^3$,

$B = 1\ \text{m}$ and $u = U\left(\frac{y}{\delta}\right)$

From the figure we easily find that mass entering from the side qp

= Mass leaving from the side qr + Mass Leaving from the side rs

$$m_{pq} = (m_{pq} - m_{rs}) + m_{rs}$$

So, firstly Mass flow rate entering from the side pq is

$$\dot{m}_{pq} = \rho \times \text{Volume} = \rho \times (A \times U)$$
$$= 1 \times (B \times \delta) \times U$$

Substitute the values, we get

$$\dot{m}_{pq} = 1 \times (1 \times 10^{-2}) \times 10 = 0.1\ \text{kg/sec}$$

For mass flow through section $r-s$, we have to take small element of dy thickness.

Then Mass flow rate through this element,

$$d\dot{m} = \rho \times \text{Volume} = \rho \times (A \times u)$$
$$= \rho \times u \times B \times (dy) = \rho BU\left(\frac{y}{\delta}\right)dy$$

For total Mass leaving from rs, integrating both sides within the limits,

$$dm \Rightarrow 0 \text{ to } m$$
$$y \Rightarrow 0 \text{ to } \delta$$
$$\int_0^m d\dot{m} = \int_0^\delta y\left(\frac{\rho UB}{\delta}\right)dy$$
$$[\dot{m}]_0^m = \frac{\rho UB}{\delta}\left[\frac{y^2}{2}\right]_0^\delta$$
$$\dot{m} = \frac{\rho UB}{\delta} \times \frac{\delta^2}{2} = \frac{1}{2}\rho UB\delta$$

So, $$\dot{m}_{rs} = \frac{1}{2} \times 10^{-2} \times 10 \times 1 \times 1 = 5 \times 10^{-2} = 0.05\ \text{kg/sec}$$

Mass leaving from qr

$$\dot{m}_{qr} = \dot{m}_{pq} - \dot{m}_{rs} = 0.1 - 0.05 = 0.05\ \text{kg/sec}$$

SOL 6.41 Option (D) is correct.

Von Karman momentum Integral equation for boundary layer flows is,

$$\frac{\tau_o}{\rho U^2} = \frac{\partial \theta}{\partial x}$$

and $$\theta = \text{momentum thickness}$$
$$= \int_0^\delta \frac{u}{U}\left[1 - \frac{u}{U}\right]dy$$

So, $$\frac{\tau_o}{\rho U^2} = \frac{\partial}{\partial x}\left[\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy\right] \qquad \frac{u}{U} = \frac{y}{\delta}$$

$$= \frac{\partial}{\partial x}\left[\int_0^{\delta} \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy\right] = \frac{\partial}{\partial x}\left[\int_0^{\delta}\left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) dy\right]$$

Integrating this equation, we get

$$= \frac{\partial}{\partial x}\left[\left(\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right)_0^{\delta}\right] = \frac{\partial}{\partial x}\left[\left(\frac{\delta^2}{2\delta} - \frac{\delta^3}{3\delta^2}\right)\right] = \frac{\partial}{\partial x}\left[\frac{\delta}{6}\right] = 0$$

$$\tau_o = 0$$

And drag force on the plate of length L is,

$$F_D = \int_0^L \tau_o \times b \times dx = 0$$

SOL 6.42 Option (D) is correct.

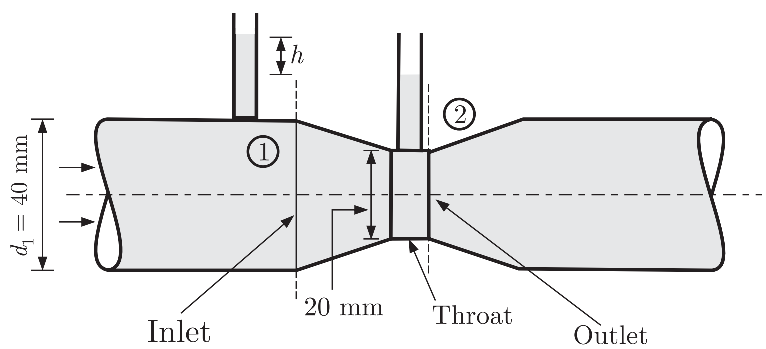
We know that potential flow (ideal flow) satisfy the continuity equation.

The continuity equation for two dimensional flow for incompressible fluid is given by,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$$

SOL 6.43 Option (D) is correct.



Given : $d_2 = 20\text{ mm} = 0.020\text{ m}$, $d_1 = 40\text{ mm} = 0.040\text{ m}$

$$\Delta p = p_1 - p_2 = 30\text{ kPa}$$

Applying continuity equation at section (1) and (2),

$$A_1 V_1 = A_2 V_2$$

$$V_1 = \left(\frac{A_2}{A_1}\right) V_2 = \frac{\frac{\pi}{4} d_2^2}{\frac{\pi}{4} d_1^2} \times V_2$$

$$= \frac{d_2^2}{d_1^2} \times V_2 = \left(\frac{20}{40}\right)^2 V_2 = \frac{V_2}{4}$$

$$V_2 = 4V_1 \qquad ..(i)$$

Now applying Bernoulli's equation at section (1) and (2),

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \qquad \text{For horizontal pipe } z_1 = z_2$$

$$\frac{p_1 - p_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$\frac{\Delta p}{\rho} = \frac{V_2^2 - V_1^2}{2}$$

$$\frac{30 \times 10^3}{1000} = \frac{(4V_1)^2 - V_1^2}{2} \qquad \text{From equation (i)}$$

$$30 = \frac{16V_1^2 - V_1^2}{2} = \frac{15V_1^2}{2}$$

$$V_1^2 = \frac{30 \times 2}{15} = 4 \qquad \Rightarrow V_1 = 2 \text{ m/sec}$$

SOL 6.44 Option (A) is correct.

It is a U-tube differential Manometer.

In this manometer A and B at different level and the liquid contain in manometer has the same specific gravity (only mercury is fill in the manometer)

Given : $\rho_{mercury} = 13600 \text{ kg/m}^3$, $g = 9.81 \text{ m/sec}^2$, $\Delta h = 150 \text{ mm} = 0.150$ meter

Static pressure difference for U-tube differential manometer is given by,

$$p_A - p_B = \rho g (h_A - h_B) = \rho g \Delta h$$

$$= 13600 \times 9.81 \times 0.150$$

$$= 20.01 \times 10^3 \text{ Pa} = 20.01 \text{ kPa} \approx 20 \text{ kPa}$$

Hence $p_A - p_B$ is positive and $p_A > p_B$, Flow from A to B.

SOL 6.45 Option (B) is correct.

Given : $$V_r = -\left(\frac{60 \times 10^3}{2\pi r}\right) \text{ m/sec} \qquad \text{...(i)}$$

And $$V_\theta = \frac{300 \times 10^3}{2\pi r} \text{ m/sec} \qquad \text{...(ii)}$$

Dividing equation (i) by equation (ii), we get

$$\frac{V_r}{V_\theta} = -\frac{60 \times 10^3}{2\pi r} \times \frac{2\pi r}{300 \times 10^3} = -\frac{1}{5}$$

$$V_r = -\frac{V_\theta}{5} \qquad \text{...(iii)}$$

In this equation (iii)

$$V_r = \text{Radial Velocity} = \frac{dr}{dt}$$

$$V_\theta = \text{Angular Velocity} = r\omega = r\frac{d\theta}{dt}$$

So, $$\frac{dr}{dt} = -\frac{1}{5} r \frac{d\theta}{dt}$$

$$\frac{dr}{r} = -\frac{1}{5}d\theta$$

On integrating both the sides and put limits, between $r \Rightarrow 120$ to r and $\theta \Rightarrow 0$ to π (for half revolution).

$$\int_{120}^{r}\frac{dr}{r} = -\frac{1}{5}\int_{0}^{\pi}d\theta$$

$$[\ln r]_{120}^{r} = -\frac{1}{5}[\theta]_{0}^{\pi}$$

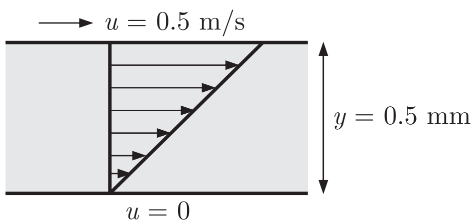
$$\ln r - \ln 120 = -\frac{1}{5}[\pi - 0] = -\frac{\pi}{5}$$

$$\ln\frac{r}{120} = -\frac{\pi}{5}$$

$$\frac{r}{120} = e^{-\pi/5} = 0.533$$

$$r = 0.533 \times 120 = 64 \text{ meter}$$

SOL 6.46 Option (B) is correct.



Given : $\upsilon = 7.4 \times 10^{-7}\ \text{m}^2/\text{sec},\ S = 0.88,\ y = 0.5\ \text{mm} = 0.5 \times 10^{-3}$ meter

$$\text{Density of liquid} = S \times \text{density of water}$$
$$= 0.88 \times 1000 = 880\ \text{kg/m}^3$$

Kinematic Viscosity $\upsilon = \frac{\mu}{\rho} = \frac{\text{Dynamic viscosity}}{\text{Density of liquid}}$

$$\mu = \upsilon \times \rho = 7.4 \times 10^{-7} \times 880$$
$$= 6.512 \times 10^{-4}\ \text{Pa-s}$$

From the Newton's law of viscosity,

$$\tau = \mu \times \frac{u}{y} = 6.512 \times 10^{-4} \times \frac{0.5}{0.5 \times 10^{-3}} = 0.6512\ \text{N/m}^2$$
$$= 0.651\ \text{Pa}$$

SOL 6.47 Option (C) is correct.

Given : $V = axi + ayj$ ...(i)

The equation of stream line is,

$$\frac{dx}{u_x} = \frac{dy}{u_y} = \frac{dz}{u_z} \quad \text{...(ii)}$$

From equation (i), $u_x = ax$, $u_y = ay$ and $u_z = 0$
Substitute there values in equation (ii), we get

$$\frac{dx}{ax} = \frac{dy}{ay}$$

$$\frac{dx}{x} = \frac{dy}{y}$$

Integrating both sides, we get

$$\int \frac{dx}{x} = \int \frac{dy}{y}$$

$$\log x = \log y + \log c = \log yc \Rightarrow x = yc \quad \text{...(iii)}$$

At point (1, 2), $1 = 2c \Rightarrow c = \frac{1}{2}$

From equation (iii), $x = \frac{y}{2} \Rightarrow 2x - y = 0$

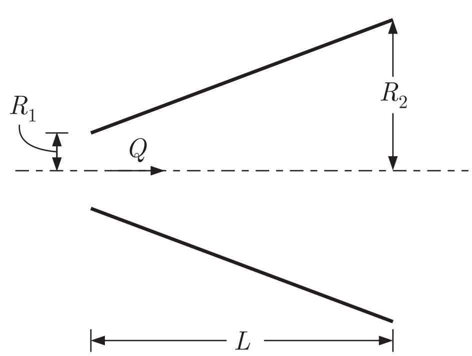
SOL 6.48 Option (C) is correct.

In this question we have to make the table for calculate mean flow rate :

Flow rate litres/ sec.	Mean flow rate $x = \left(\frac{x_i + x_f}{2}\right)$	Frequency f	fx
7.5 to 7.7	7.6	1	7.6
7.7 to 7.9	7.8	5	39
7.9 to 8.1	8.0	35	280
8.1 to 8.3	8.2	17	139.4
8.3 to 8.5	8.4	12	100.8
8.5 to 8.7	8.6	10	86
		$\Sigma f = 80$	$\Sigma fx = 652.8$

Mean flow rate, $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{652.8}{80} = 8.16 \text{ litres/ sec}$

SOL 6.49 Option (C) is correct.



Flow rate, $Q = AV$

Inlet velocity, $V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi}{4}(2R_1)^2} = \frac{Q}{\pi R_1^2}$ $A_1 = \frac{\pi}{4} d_1^2$

Outlet Velocity, $V_2 = \frac{Q}{A_2} = \frac{Q}{\pi R_2^2}$

Therefore, resultant velocity will be,

$$dV = V_2 - V_1 = \frac{Q}{\pi}\left[\frac{1}{R_2^2} - \frac{1}{R_1^2}\right]$$

Acceleration at the exit section,

$$a = \frac{dV}{dt} = V\frac{dV}{dx}$$

In this case $dV = V_2 - V_1$

$V = V_2$

And $dx = L$

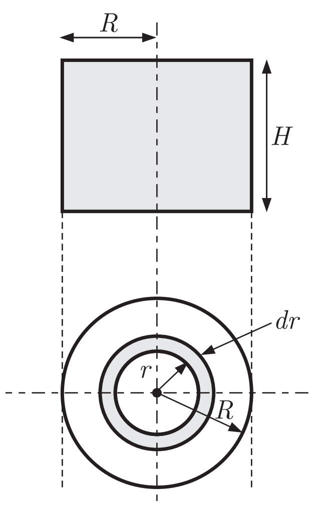
So, $$a = \frac{Q}{\pi R_2^2} \times \frac{Q}{\pi L}\left[\frac{1}{R_2^2} - \frac{1}{R_1^2}\right] = \frac{Q^2}{\pi^2 R_2^2 L}\left[\frac{R_1^2 - R_2^2}{R_1^2 R_2^2}\right]$$

$$= \frac{Q^2}{\pi^2 R_2^2 L}\left[\frac{(R_1 + R_2)(R_1 - R_2)}{R_1^2 R_2^2}\right]$$

Considering limiting case $R_1 \to R_2$

Then, $$a = \frac{Q^2}{\pi^2 R_2^2 L}\left[\frac{(R_1 - R_2)2R_2}{R_2^2 R_2^2}\right] = \frac{2Q^2}{\pi^2 R_2^5 L}[R_1 - R_2] = \frac{2Q^2(R_1 - R_2)}{\pi^2 R_2^5 L}$$

SOL 6.50 Option (D) is correct.



Total thrust at the bottom of cylinder = Weight of water in cylinder

+ Pressure force on the cylinder

For rotating motion,

$$\frac{\partial p}{\partial r} = \frac{\rho V^2}{r} = \frac{\rho r^2 \omega^2}{r} = \rho \omega^2 r$$ $p =$ Pressure, $V = r\omega$

And $\partial p = \rho\omega^2 r dr$

Integrating both the sides within limits p between 0 to p and r between 0 to r,

$$\int_0^p \partial p = \int_0^r \rho\omega^2 r dr$$

$$[p]_0^p = \rho\omega^2\left[\frac{r^2}{2}\right]_0^r$$

For calculating the total pressure on the cylinder,

$$p = \rho\omega^2 \times \left[\frac{r^2}{2} - 0\right] = \frac{\rho\omega^2 r^2}{2}$$

Dividing whole area of cylinder in the infinite small rings with thickness dr,

Force on elementary ring

Pressure intensity $\times$ Area of ring $= \frac{\rho\omega^2 r^2}{2} \times 2\pi r dr$

Total force, $F = \int_0^R \frac{\rho\omega^2 r^2}{2} \times 2\pi r dr = \pi\rho\omega^2 \int_0^R r^3 dr$

$$= \pi\rho\omega^2\left[\frac{r^4}{4}\right]_0^R = \pi\rho\omega^2 \frac{R^4}{4}$$

Weight of water $= mg = \rho\nu g$ $\qquad m = \rho\nu$

$= \rho\pi R^2 \times Hg = \rho g H\pi R^2$ $\qquad A = \pi R^2$

So, Net force $= \rho g H\pi R^2 + \rho\omega^2 \frac{\pi R^4}{4} = \pi R^2\left[\frac{\rho\omega^2 R^2}{4} + \rho g H\right]$

SOL 6.51 Option (C) is correct.

Given relation is,

$$\frac{U}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \text{ and } \delta = \frac{4.64x}{\sqrt{\text{Re}_x}} \qquad \text{...(i)}$$

$U_\infty = U = 2\text{ m/sec},\ v = 1.5 \times 10^{-5}\text{ m}^2/\text{s},\ \rho = 1.23\text{ kg/m}^3,\ L = x = 1$

Kinematic viscosity,

$$\upsilon = \frac{\mu}{\rho}$$

$$\mu = \upsilon \times \rho = 1.5 \times 10^{-5} \times 1.23$$

$$= 1.845 \times 10^{-5}\text{ kg/m sec}$$

Reynolds Number is given as,

$$\text{Re}_x = \frac{\rho U x}{\mu} = \frac{1.23 \times 2 \times 1}{1.845 \times 10^{-5}} = 1.33 \times 10^5$$

$$\delta = \frac{4.64 \times 1}{\sqrt{1.33 \times 10^5}} = 0.0127$$

And $\frac{U}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

$$\frac{dU}{dy} = U_\infty \frac{d}{dy}\left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right] = U_\infty\left[\frac{3}{2} \times \frac{1}{\delta} - \frac{3}{2}\frac{y^2}{\delta^3}\right]$$

where U_∞ = Free stream velocity $= U$

$$\left(\frac{dU}{dy}\right)_{y=0} = U_\infty\left[\frac{3}{2\delta}\right] = \frac{3U_\infty}{2\delta}$$

We know that shear stress by the Newton's law of viscosity,

$$\tau_0 = \mu\left(\frac{dU}{dy}\right)_{y=0} = 1.845 \times 10^{-5} \times \frac{3U_\infty}{2\delta}$$

Substitute the values of U_∞ and δ, we get

$$= 1.845 \times 10^{-5} \times \frac{3 \times 2}{2 \times 0.0127}$$

$$= 435.82 \times 10^{-5}\,\text{N/m}^2 = 4.36 \times 10^{-3}\,\text{N/m}^2$$

SOL 6.52 Option (B) is correct.

Given : $L = 4\,\text{km} = 4 \times 1000 = 4000\,\text{m},\ d = 0.2\,\text{m}\ \ f = 0.01,\ V = 2\,\text{m/sec},$ $H = 5\,\text{meter}$

Head loss due to friction in the pipe,

$$h_f = \frac{fLV^2}{2gd} = \frac{0.01 \times 4000 \times (2)^2}{2 \times 9.81 \times 0.2} = 40.77\ \text{m of water}$$

Now total pressure (absolute discharge pressure) to be supplied by the pump at exit = Pressure loss by pipe + Head pressure of tank + Atmospheric pressure head

Total pressure, $p = \rho g h_f + \rho g H + \rho g h_{atm}$

$$p = \rho g[h_f + H + h_{atm}] \quad \text{Pressure head, } \frac{p}{\rho g} = H \Rightarrow p = H\rho g$$

$$= 1000 \times 9.81[40.77 + 5 + 10.3]$$

$$= 5.5 \times 10^5\,\text{N/m}^2 = 5.5\,\text{bar} \qquad \text{For water } h_{atm} = 10.3\,\text{m}$$

SOL 6.53 Option (D) is correct.

Given : $p_{G_1} = 5.00\,\text{bar},\ p_{G_2} = 1.00\,\text{bar},\ p_{atm} = 1.01\,\text{bar}$

Absolute pressure of G_2 = Atmospheric pressure + Gauge pressure

$$= 1.01 + 1.00 = 2.01\,\text{bar}$$

Absolute pressure of $G_1 = p_{G_1} + p_{abs(G_2)} = 5.00 + 2.01 = 7.01\,\text{bar}$

SOL 6.54 Option (A) is correct.

Given : $H = 24.5\,\text{m},\ Q = 10.1\,\text{m}^3/\text{sec},\ \eta_0 = 90\%,$

$N = 4\,\text{rps} = 4 \times 60 = 240\,\text{rpm}$

$$\eta_0 = \frac{\text{Shaft Power in kW}}{\text{Water Power in kW}} = \frac{P}{\left(\dfrac{\rho \times g \times Q \times H}{1000}\right)}$$

$$P = \frac{\eta_0 \times \rho \times g \times Q \times H}{1000}$$

$$= \frac{0.90 \times 1000 \times 9.81 \times 10.1 \times 24.5}{1000}$$

$$= 2184.74\,\text{kW} \qquad \rho_{water} = 1000\,\text{kg/m}^3$$

For turbine Specific speed,

$$N_S = \frac{N\sqrt{P}}{H^{5/4}} = \frac{240\sqrt{2184.74}}{(24.5)^{5/4}} = 205.80\,\text{rpm}$$

Hence, $51 < N_S < 255$ for francis turbine.

SOL 6.55 Option (B) is correct.

	List-I		List-II
P.	Reciprocating pump	3.	Positive Displacement
Q.	Axial flow pump	5.	High Flow rate, low pressure ratio
R.	Microhydel plant	2.	Plant with power output between 100 kW to 1 MW
S.	Backward curved vanes	6.	Centrifugal pump impeller

So, correct pairs are P-3, Q-5, R-2, S-6

SOL 6.56 Option (D) is correct.

Given :

Cross section area of body $= A$

Height of body $= H$

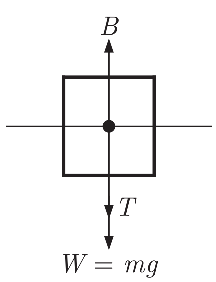
Density of body $= \rho_s$

Density of liquid $= \rho$

Tension in the string $= T$

We have to make the FBD of the block.

B = Buoyancy force



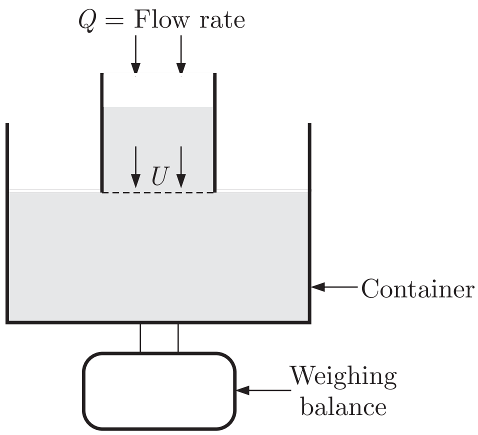
From the principal of buoyancy,

Downward force = Buoyancy force $\qquad m = \rho\nu$

$$T + mg = \rho h A g$$

$$T + \rho_s \nu g = \rho h A g \qquad \nu = A \times H$$
$$T + \rho_s AHg = \rho hAg$$
$$T = \rho hAg - \rho_s AHg = Ag(\rho h - \rho_s H)$$

SOL 6.57 Option (A) is correct.



Given : Flow rate $= Q$

Velocity of water when it strikes the water surface $= U$

Total Mass (container + water) $= m$

Force on weighing balance due to water strike = Change in momentum

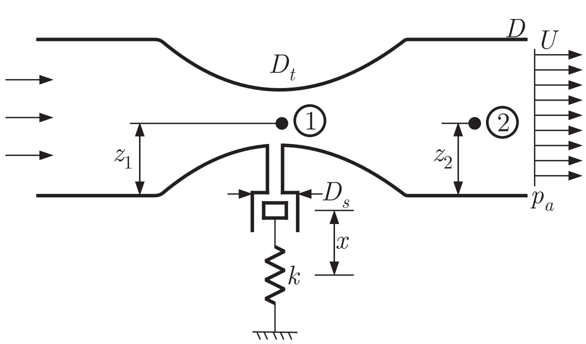
$$\Delta P = \text{Initial Momentum} - \text{Final momentum}$$
$$= \rho QU - \rho Q(0) = \rho QU \qquad \text{Final velocity is zero}$$

Weighing balance also experience the weight of the container and water.

So, Weight of container and water $= mg$

Therefore, total force on weighing Balance $= \rho QU + mg$

SOL 6.58 Option (D) is correct.



First of all we have to take two section (1) and (2)

Applying Bernoulli's equation at section (1) and (2).

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{p_1}{\rho}+\frac{V_1^2}{2} = \frac{p_2}{\rho}+\frac{V_2^2}{2} \qquad z_1 = z_2$$

$$p_1 - p_2 = \frac{\rho}{2}(V_2^2 - V_1^2) \qquad \text{...(i)}$$

Apply continuity equation, we get

$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4}D_t^2 V_1 = \frac{\pi}{4}D^2 U \quad V_2 = U\text{. Let at point (1) velocity} = V_1$$

$$V_1 = \left(\frac{D}{D_t}\right)^2 \times U \qquad \text{...(ii)}$$

Substitute the value of V_1 from equation (ii) into the equation (i),

$$p_1 - p_2 = \frac{\rho}{2}\left[U^2 - \left(\frac{D}{D_t}\right)^4 U^2\right] = \frac{\rho}{2}U^2\left[1-\left(\frac{D}{D_t}\right)^4\right] \qquad \text{...(iii)}$$

From the figure, we have

$$\text{Spring force} = \text{Pressure force due to air}$$

$$-kx = A_s(p_1 - p_2) = \frac{\pi}{4}D_s^2 \times (p_1 - p_2)$$

$$= \frac{\pi}{4}D_s^2 \times \frac{\rho}{2}U^2\left[1-\left(\frac{D}{D_t}\right)^4\right] \quad \text{From equation (iii)}$$

$$kx = \frac{\pi}{8}D_s^2 \rho U^2\left[\left(\frac{D}{D_t}\right)^4 - 1\right]$$

$$x = \frac{\rho U^2}{8k}\left[\left(\frac{D}{D_t}\right)^4 - 1\right]\pi D_s^2$$

SOL 6.59 Option (B) is correct.

Given : $N_1 = 500\,\text{rpm}$, $H_1 = 30\,\text{meter}$, $N_2 = 1000\,\text{rpm}$, $Q_1 = 60$ litres per minute

From the general relation,

$$U = \frac{\pi DN}{60} = \sqrt{2gH}$$

$$DN \propto \sqrt{H} \qquad \Rightarrow \quad N \propto \frac{\sqrt{H}}{D}$$

Centrifugal pump is used for both the cases. So $D_1 = D_2$

$$N \propto \sqrt{H}$$

$$\frac{H_1}{H_2} = \frac{N_1^2}{N_2^2}$$

$$H_2 = \frac{N_2^2}{N_1^2} \times H_1 = \frac{(1000)^2}{(500)^2} \times 30 = 120\,\text{m}$$

The specific speed will be constant for centrifugal pump and relation is,

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}} = \text{Constant}$$

So, $$\frac{N_1\sqrt{Q_1}}{H_1^{3/4}} = \frac{N_2\sqrt{Q_2}}{H_2^{3/4}}$$ For both cases

$$\sqrt{Q_2} = \frac{N_1}{N_2} \times \left(\frac{H_2}{H_1}\right)^{3/4} \times \sqrt{Q_1} = \frac{500}{1000} \times \left(\frac{120}{30}\right)^{3/4} \times \sqrt{60}$$

$$= \frac{1}{2} \times (2)^{3/2} \times \sqrt{60}$$

Squaring both sides

$$Q_2 = \frac{1}{4} \times 8 \times 60 = 120 \text{ litre/min}$$

Alternate :

From unit quantities unit speed

$$N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}} \quad \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$\sqrt{H_2} = \frac{N_2\sqrt{H_1}}{N_1}$$

or $$H_2 = \frac{N_2^2 \times H_1}{N_1^2} = \frac{(1000)^2 \times 30}{(500)^2} = 120 \text{ m}$$

Unit discharge $$Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

$$\frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

or $$Q_2 = \frac{Q_1\sqrt{H_2}}{\sqrt{H_1}} = \frac{60 \times \sqrt{120}}{\sqrt{30}} = 120 \text{ litre/min}$$

SOL 6.60 None of these is correct.

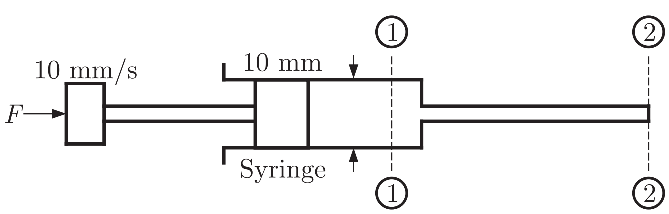
	List-I		List-II
P.	Curtis	3.	Velocity compounding
Q.	Rateau	4.	Pressure compounding
R.	Kaplan	6.	Axial flow turbine
S.	Francis	7.	Mixed flow turbine

So, correct pairs are P-3, Q-4, R-6, S-7.

SOL 6.61 Option (B) is correct.

Given : $L = 100 \text{ mm}, d = 1 \text{ mm}, D = 10 \text{ mm}, V_1 = 10 \text{ mm/sec}$

We have to take the two sections of the system (1) and (2).



Apply continuity equation on section (1) and (2),

$$A_1V_1 = A_2V_2 \qquad Q = AV,\ Q = \text{flow rate}$$

$$V_2 = \left(\frac{A_1}{A_2}\right) \times V_1 = \frac{\pi/4(0.01)^2}{\pi/4(0.001)^2} \times 0.010 = 1\ \text{m/sec}$$

Again applying the Bernoulli's equation at section (1) and (2),

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

The syringe and the plunger is situated on the same plane so $z_1 = z_2$,

Take $\quad p_2 = 0$ = Atmospheric pressure (Outside the needle)

$$\frac{p_1}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$p_1 = \frac{\rho}{2}(V_2^2 - V_1^2) = \frac{1000}{2}[(1)^2 - (0.01)^2] = 499.95\ \text{N/m}^2$$

Force required on plunger,

$$F = p_1 \times A_1 = 499.95 \times \frac{\pi}{4}(0.01)^2 = 0.04\ \text{N}$$

SOL 6.62 Option (C) is correct.

Given : $f = \frac{64}{\text{Re}}$, $\mu = 1 \times 10^{-3}$ kg/s-m

$$\text{Re} = \frac{\rho Vd}{\mu} = \frac{\rho V_2 d_2}{\mu} = \frac{1000 \times 1 \times 0.001}{1 \times 10^{-3}} = 1000 \qquad \text{For Needle}$$

And $\quad f = \frac{64}{\text{Re}} = \frac{64}{1000} = 0.064$

From the help of f we have to find Head loss in needle,

$$h_f = \frac{fLV_2^2}{2gd_2} = \frac{0.064 \times 0.1 \times (1)^2}{2 \times 9.81 \times 0.001} = 0.3265\ \text{m of water}$$

Applying Bernoulli's equation at section (1) and (2) with the head loss in account.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

$$z_1 = z_2 \qquad \text{At the same plane}$$

And $\quad p_2 = 0 \qquad$ Atmospheric pressure

$$\frac{p_1}{\rho g} = \left(\frac{V_2^2 - V_1^2}{2g}\right) + h_f$$

$$p_1 = \frac{\rho}{2}(V_2^2 - V_1^2) + \rho g h_f$$

$$= \frac{1000}{2}[(1)^2 - (0.01)^2] + 1000 \times 9.81 \times 0.3265$$

$$= 499.95 + 3202.965 = 3702.915\ \text{N/m}^2$$

Force required on plunger,

$$F = p_1 \times A_1 = 3702.915 \times \frac{\pi}{4} \times (0.01)^2 = 0.3\,\text{N}$$

SOL 6.63 Option (C) is correct.

From Buckingham's π-theorem,

"If there are m variables (Indepenent and dependent variables) in a physical phenomenon and if these variables contain n fundamental dimensions (M, L, T) then variables are arranged into $(m-n)$ dimensionless terms.

So, non dimensional variables, $\Rightarrow m - n$.

SOL 6.64 Option (D) is correct.

The laminar boundary layer generation along a flat plate for this flow, is

$$\frac{\delta}{L} \sim \frac{1}{\sqrt{\text{Re}_L}}$$

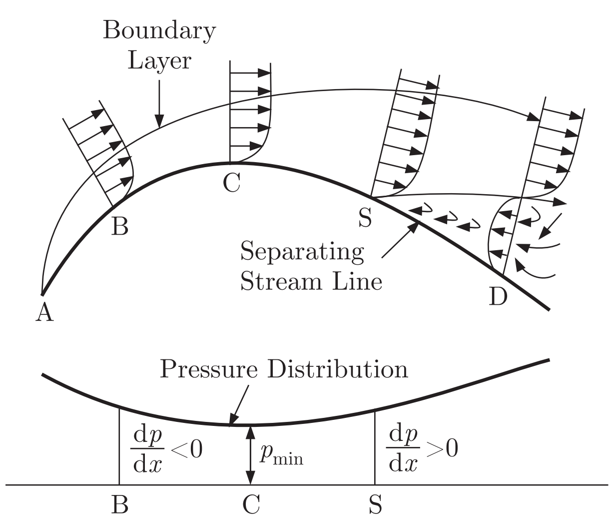
If we substitute x for L and for a laminar boundary layer on a flat plate, where $V(x) = V =$ constant, then δ grows like the square root of x.

$$\frac{\delta}{x} \sim \frac{1}{\sqrt{\frac{Vx}{\upsilon}}}$$

$$\frac{\delta}{\sqrt{x}} \sim \frac{1}{\sqrt{\frac{V}{\upsilon}}} \Rightarrow \delta \propto \sqrt{x}$$

SOL 6.65 Option (C) is correct.

The pressure is minimum at point C. Along the region CSD of the curved surface, the area of flow increases and hence velocity of flow along the direction of Fluid decreases.



Due to decrease of velocity, the pressure increases in the direction of flow and pressure gradient dp/dx is positive or $\frac{dp}{dx} > 0$

SOL 6.66 Option (C) is correct.

Biot Number $Bi = \frac{hl}{k}$

where h = Convective heat transfer coefficient

k = thermal conductivity

l = linear dimension

Biot Number gives an indication of the ratio of internal (conduction) resistance to the surface (convection) resistance.

A small value of Bi implies that the fluid has a small conduction resistance i.e.

Conduction resistance << Convection resistance

SOL 6.67 Option (A) is correct.

Given : $c_p = 0.1393 \text{ k-J/kg-K}$

$\mu = 0.1523 \times 10^{-2} \text{ N-s/m}^2$

$k = 8.540 \text{ W/m-K}$

Prandtl Number $\Pr = \frac{\mu c_p}{k}$

$$\Pr = \frac{0.1523 \times 10^{-2} \times 0.1393 \times 10^3}{8.540}$$

$$\Pr = 0.0248$$

SOL 6.68 Option (A) is correct.

The SI unit of kinematic viscosity is m^2/sec.

$$\upsilon = \frac{\mu}{\rho} = \frac{\tau\frac{dy}{du}}{\rho} \qquad \tau = \mu\frac{du}{dy}$$

$$\upsilon = \frac{\frac{F}{A} \times \frac{dy}{du}}{\rho}$$

Substitute the units of all the parameters

$$\upsilon = \frac{\frac{\text{Newton}}{\text{m}^2} \times \frac{\text{m}}{\text{m/sec}}}{\text{kg/m}^3} \qquad N = \frac{\text{kgm}}{\text{sec}^3}$$

$$\upsilon = \frac{\frac{\text{kgm}}{\text{sec}^2\text{m}^2}\frac{\text{m sec}}{\text{m}}}{\text{kg/m}^3}$$

$$v = \frac{\frac{\text{kg}}{\text{sec m}}}{\text{kg/m}^3} = \frac{\text{m}^2}{\text{sec}}$$

SOL 6.69 Option (C) is correct.

Fluid static deals with problems associated with fluids at rest. In static fluid, there is no relative motion between adjacent fluid layers and thus there are no shear (tangential) stresses in the fluid trying to deform it.

The only stress in static fluid is the normal stress, which is the pressure and the variation of pressure is due only to the weight of the fluid and it is always positive.

Therefore, the topic of fluid statics has significance only in gravity field.

SOL 6.70 Option (A) is correct.

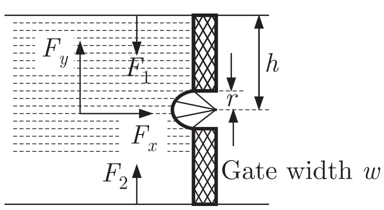
Biot number gives an indication of internal (conduction) resistance to the surface (convection) resistance.

$$Bi = \frac{hl}{k}$$

If the value of Biot number is less than 0.1, then lumped that transfer analysis is valid.

i.e. Biot Number < 0.1.

SOL 6.71 Option (D) is correct.



Here $\quad F_1 =$ weight of water column above the top surface.

$F_2 =$ weight of water column above the bottom surface.

At the depth h, pressure is given by,

$$p = \rho g h$$

then horizontal force,

$$F_x = A \times p = (2r \times w) \times \rho g h$$

where $\quad A =$ Normal area, when viewed in the direction of F_x

$$F_x = 2\rho g h r w$$

$F_y = F_2 - F_1 =$ weight of water contained in volume of semi circular gate.

$$F_y = mg = \left(\frac{\pi}{2} r^2 \times w\right)\rho g \qquad m = \rho v \text{ and } v = A \times w$$

$$F_y = \frac{\pi \rho g w r^2}{2}$$

SOL 6.72 Option (D) is correct.

Given : $v = (x + 2y + 2)\,i + (4 - y)\,j$

where $u = x + 2y + 2, \;\; v = 4 - y$

$$\frac{\partial u}{\partial x} = 1\;;\; \frac{\partial u}{\partial y} = 2 \qquad \frac{\partial v}{\partial y} = -1;\; \frac{\partial v}{\partial x} = 0$$

We know, for Incompressive flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$1 - 1 = 0$$

So, flow is incompressible.

And for irrotational flow, $\zeta_z = 0$

$$\zeta_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

$$\Rightarrow \qquad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$0 - 2 = 0$$

$$-2 \neq 0$$

So, flow is not irrotational.

SOL 6.73 Option (B) is correct.

The non-dimensional Prandtl Number for thermal boundary layer is given by,

$$\frac{\delta}{\delta_t} = (\mathrm{Pr})^{1/3}$$

where δ = hydrodynamic boundary layer thickness

δ_t = thermal boundary layer thickness

Given, $\mathrm{Pr} = 6$ $\quad \frac{\delta}{\delta_t} = (6)^{1/3} = 1.82$

$$\delta = 1.82\delta_t$$

$$\delta = 1.82\delta_t$$

So, $\quad \delta > \delta_t$ or $\delta_t < \delta$

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CHAPTER 7

HEAT TRANSFER

YEAR 2012 ONE MARK

MCQ 7.1 For an opaque surface, the absorptivity (α), transmissivity (τ) and reflectivity (ρ) are related by the equation :

(A) $\alpha + \rho = \tau$

(B) $\rho + \alpha + \tau = 0$

(C) $\alpha + \rho = 1$

(D) $\alpha + \rho = 0$

MCQ 7.2 Which one of the following configurations has the highest fin effectiveness ?

(A) Thin, closely spaced fins

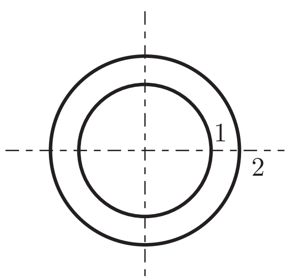
(B) Thin, widely spaced fins

(C) Thick, widely spaced fins

(D) Thick, closely spaced fins

YEAR 2012 TWO MARKS

MCQ 7.3 Consider two infinitely long thin concentric tubes of circular cross section as shown in the figure. If D_1 and D_2 are the diameters of the inner and outer tubes respectively, then the view factor F_{22} is give by



(A) $\left(\frac{D_2}{D_1}\right) - 1$

(B) zero

(C) $\left(\frac{D_1}{D_2}\right)$

(D) $1 - \left(\frac{D_1}{D_2}\right)$

MCQ 7.4 Water ($c_p = 4.18\,\text{kJ/kgK}$) at 80°C enters a counter flow heat exchanger with a mass flow rate of $0.5\,\text{kg/s}$. Air ($c_p = 1\,\text{kJ/kgK}$) enters at 30°C with a mass flow rate of $2.09\,\text{kg/s}$. If the effectiveness of the heat exchanger is 0.8, the LMTD (in $^\circ$C) is

(A) 40 (B) 20

(C) 10 (D) 5

YEAR 2011 ONE MARK

MCQ 7.5 In a condenser of a power plant, the steam condenses at a temperatures of 60° C. The cooling water enters at 30° C and leaves at 45° C. The logarithmic mean temperature difference (LMTD) of the condenser is

(A) 16.2° C (B) 21.6° C

(C) 30° C (D) 37.5° C

MCQ 7.6 A pipe of 25 mm outer diameter carries steam. The heat transfer coefficient between the cylinder and surroundings is $5\,W/m^2\,K$. It is proposed to reduce the heat loss from the pipe by adding insulation having a thermal conductivity of $0.05\,W/m\,K$. Which one of the following statements is TRUE ?

(A) The outer radius of the pipe is equal to the critical radius.

(B) The outer radius of the pipe is less than the critical radius.

(C) Adding the insulation will reduce the heat loss.

(D) Adding the insulation will increases the heat loss.

YEAR 2011 TWO MARKS

MCQ 7.7 A spherical steel ball of 12 mm diameter is initially at 1000 K. It is slowly cooled in surrounding of 300 K. The heat transfer coefficient between the steel ball and the surrounding is $5\,W/m^2\,K$. The thermal conductivity of steel is $20\,W/mK$. The temperature difference between the centre and the surface of the steel ball is

(A) large because conduction resistance is far higher than the convective resistance.

(B) large because conduction resistance is far less than the convective resistance.

(C) small because conduction resistance is far higher than the convective resistance.

(D) small because conduction resistance is far less than the convective resistance.

MCQ 7.8 The ratios of the laminar hydrodynamic boundary layer thickness to thermal boundary layer thickness of flows of two fluids P and Q on a flat plate are 1/2 and 2 respectively. The Reynolds number based on the plate length for both the flows is 10^4. The Prandtl and Nusselt numbers for P are 1/8 and 35 respectively. The Prandtl and Nusselt numbers for Q are respectively

(A) 8 and 140 (B) 8 and 70

(C) 4 and 40 (D) 4 and 35

YEAR 2010 TWO MARKS

MCQ 7.9 A fin has 5 mm diameter and 100 mm length. The thermal conductivity of fin material is $400\ \text{Wm}^{-1}\ \text{K}^{-1}$. One end of the fin is maintained at 130° C and its remaining surface is exposed to ambient air at 30° C. If the convective heat transfer coefficient is $40\ \text{Wm}^{-2}\ \text{K}^{-1}$, the heat loss (in W) from the fin is

(A) 0.08 (B) 5.0

(C) 7.0 (D) 7.8

YEAR 2009 ONE MARK

MCQ 7.10 A coolant fluid at 30°C flows over a heated flat plate maintained at constant temperature of 100°C. The boundary layer temperature distribution at a given location on the plate may be approximated as $T = 30 + 70\exp(-y)$ where y (in m) is the distance normal to the plate and T is in $^\circ$C. If thermal conductivity of the fluid is $1.0\ \text{W/mK}$, the local convective heat transfer coefficient (in $\text{W/m}^2\ \text{K}$) at that location will be

(A) 0.2 (B) 1

(C) 5 (D) 10

YEAR 2009 TWO MARKS

MCQ 7.11 In a parallel flow heat exchanger operating under steady state, the heat capacity rates (product of specific heat at constant pressure and mass flow rate) of the hot and cold fluid are equal. The hot fluid, flowing at $1\ \text{kg/s}$ with $c_p = 4\ \text{kJ/kg K}$, enters the heat exchanger at 102°C while the cold fluid has an inlet temperature of 15°C. The overall heat transfer coefficient for the heat exchanger is estimated to be $1\ \text{kW/m}^2\ \text{K}$ and the corresponding heat transfer surface area is $5\ \text{m}^2$. Neglect heat transfer between the heat exchanger and the ambient. The heat exchanger is characterized by the following relations:

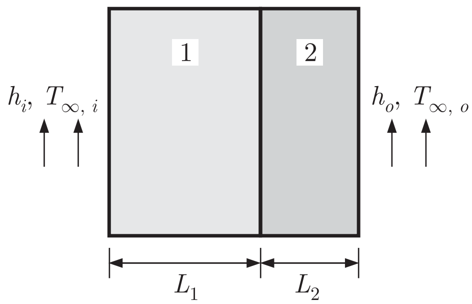
$$2\varepsilon = -\exp(-2\,\text{NTU})$$

The exit temperature (in $^\circ$C) for the cold fluid is

(A) 45 (B) 55

(C) 65 (D) 75

MCQ 7.12 Consider steady-state conduction across the thickness in a plane composite wall (as shown in the figure) exposed to convection conditions on both sides.



Given : $h_i = 20\ \text{W/m}^2\text{K}$, $h_o = 50\ \text{W/m}^2\text{K}$;, $T_{\infty,i} = 20^\circ\text{C}$; $T_{\infty,o} = -2^\circ\text{C}$, $k_1 = 20\ \text{W/mK}$; $k_2 = 50\ \text{W/mK}$; $L_1 = 0.30\ \text{m}$ and $L_2 = 0.15\ \text{m}$.

Assuming negligible contact resistance between the wall surfaces, the interface temperature, T (in $^\circ$C), of the two walls will be

(A) -0.50 (B) 2.75

(C) 3.75 (D) 4.50

• Common Data For Q.13 and Q.14

Radiative heat transfer is intended between the inner surfaces of two very large isothermal parallel metal plates. While the upper plate (designated as plate 1) is a black surface and is the warmer one being maintained at 727°C, the lower plate (plate 2) is a diffuse and gray surface with an emissivity of 0.7 and is kept at 227°C.

Assume that the surfaces are sufficiently large to form a two-surface enclosure and steady-state conditions to exits. Stefan-Boltzmann constant is given as $5.67 \times 10^{-8}\ \text{W/m}^2\ \text{K}^4$

MCQ 7.13 The irradiation (in kW/m^2) for the plate (plate 1) is

(A) 2.5 (B) 3.6

(C) 17.0 (D) 19.5

MCQ 7.14 If plate 1 is also diffuse and gray surface with an emissivity value of 0.8, the net radiation heat exchange (in kW/m^2) between plate 1 and plate 2 is

(A) 17.0 (B) 19.5

(C) 23.0 (D) 31.7

YEAR 2008 ONE MARK

MCQ 7.15 For flow of fluid over a heated plate, the following fluid properties are known

$$\text{Viscosity} = 0.001\,\text{Pa-s};$$

Specific heat at constant pressure $= 1\,\text{kJ/kg.K}$;

Thermal conductivity $= 1\text{W/m - K}$

The hydrodynamic boundary layer thickness at a specified location on the plate is 1 mm. The thermal boundary layer thickness at the same location is

(A) 0.001 mm (B) 0.01 mm

(C) 1 mm (D) 1000 mm

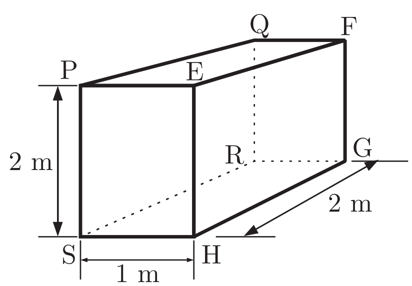
YEAR 2008 TWO MARKS

MCQ 7.16 The logarithmic mean temperature difference (LMTD) of a counter flow heat exchanger is 20°C. The cold fluid enters at 20°C and the hot fluid enters at 100°C. Mass flow rate of the cold fluid is twice that of the hot fluid. Specific heat at constant pressure of the hot fluid is twice that of the cold fluid. The exit temperature of the cold fluid

(A) is 40°C (B) is 60°C

(C) is 80°C (D) cannot be determined

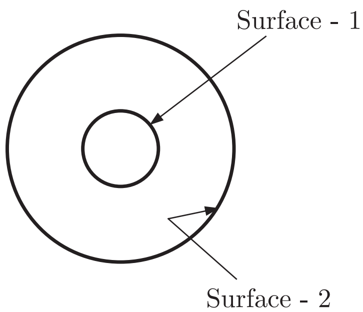
MCQ 7.17 For the three-dimensional object shown in the figure below, five faces are insulated. The sixth face (PQRS), which is not insulated, interacts thermally with the ambient, with a convective heat transfer coefficient of $10\,\text{W/m}^2\,\text{K}$. The ambient temperature is 30°C. Heat is uniformly generated inside the object at the rate of $100\,\text{W/m}^3$. Assuming the face PQRS to be at uniform temperature, its steady state temperature is



(A) 10°C (B) 20°C

(C) 30°C (D) 40°C

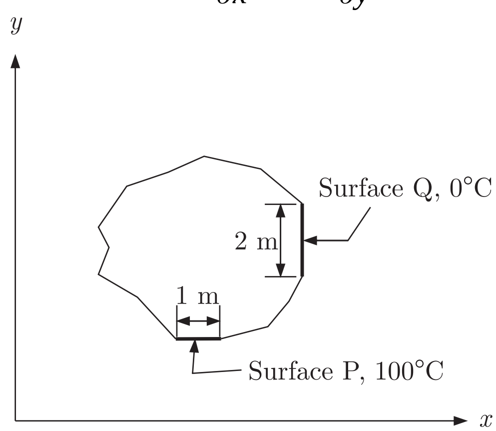
MCQ 7.18 A hollow enclosure is formed between two infinitely long concentric cylinders of radii $1\,\text{m}$ and $2\,\text{m}$, respectively. Radiative heat exchange takes place between the inner surface of the larger cylinder (surface-2) and the outer surface of the smaller cylinder (surface-1). The radiating surfaces are diffuse and the medium in the enclosure is non-participating. The fraction of the thermal radiation leaving the larger surface and striking itself is



(A) 0.25 (B) 0.5

(C) 0.75 (D) 1

MCQ 7.19 Steady two-dimensional heat conduction takes place in the body shown in the figure below. The normal temperature gradients over surfaces P and Q can be considered to be uniform. The temperature gradient $\partial T / \partial x$ at surface Q is equal to $10\,K/m$. Surfaces P and Q are maintained at constant temperature as shown in the figure, while the remaining part of the boundary is insulated. The body has a constant thermal conductivity of $0.1\,W/mK$. The values of $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial y}$ at surface P are



(A) $\frac{\partial T}{\partial x} = 20\,K/m, \quad \frac{\partial T}{\partial y} = 0\,K/m$

(B) $\frac{\partial T}{\partial x} = 0\,K/m, \quad \frac{\partial T}{\partial y} = 10\,K/m$

(C) $\frac{\partial T}{\partial x} = 10\,K/m, \quad \frac{\partial T}{\partial y} = 10\,K/m$

(D) $\frac{\partial T}{\partial x} = 0\,K/m, \quad \frac{\partial T}{\partial y} = 20\,K/m$

YEAR 2007 TWO MARKS

MCQ 7.20 The temperature distribution within the thermal boundary layer over a heated isothermal flat plate is given by

$$\frac{T - T_w}{T_\infty - T_w} = \frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3,$$

where T_w and T_∞ are the temperature of plate and free stream respectively, and y is the normal distance measured from the plate. The local Nusselt number based on the thermal boundary layer thickness δ_t is given by

(A) 1.33 (B) 1.50

(C) 2.0 (D) 4.64

MCQ 7.21 In a counter flow heat exchanger, hot fluid enters at 60° C and cold fluid leaves at 30° C. Mass flow rate of the fluid is 1 kg/s and that of the cold fluid is 2 kg/s. Specific heat of the hot fluid is 10 kJ/kgK and that of the cold fluid is 5 kJ/kgK. The Log Mean Temperature Difference (LMTD) for the heat exchanger in $^\circ$C is

(A) 15 (B) 30

(C) 35 (D) 45

MCQ 7.22 The average heat transfer co-efficient on a thin hot vertical plate suspended in still air can be determined from observations of the change in plate temperature with time as it cools. Assume the plate temperature to be uniform at any instant of time and radiation heat exchange with the surroundings negligible. The ambient temperature is 25° C, the plat has a total surface area of $0.1\ m^2$ and a mass of 4 kg. The specific heat of the plate material is 2.5 kJ/kgK. The convective heat transfer co-efficient in W/m^2 K, at the instant when the plate temperature is 225° C and the change in plate temperature with time $dT/dt = -0.02$ K/s, is

(A) 200 (B) 20

(C) 15 (D) 10

• Common Data For Q.23 and Q.24

Consider steady one-dimensional heat flow in a plate of 20 mm thickness with a uniform heat generation of $80\ MW/m^3$. The left and right faces are kept at constant temperatures of 160° C and 120° C respectively. The plate has a constant thermal conductivity of 200 W/mK.

MCQ 7.23 The location of maximum temperature within the plate from its left face is

(A) 15 mm (B) 10 mm

(C) 5 mm (D) 0 mm

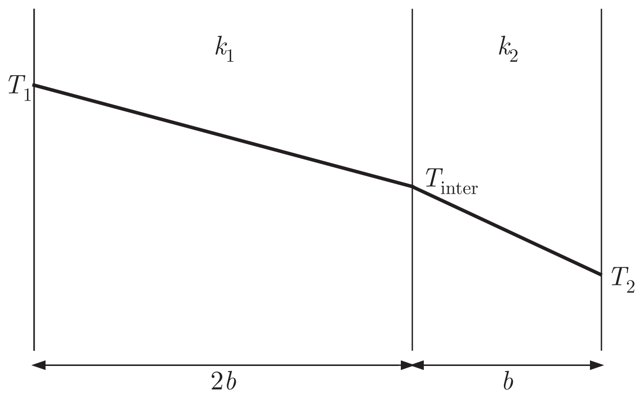
MCQ 7.24 The maximum temperature within the plate in $^\circ$C is

(A) 160 (B) 165

(C) 200 (D) 250

YEAR 2006 ONE MARK

MCQ 7.25 In a composite slab, the temperature at the interface (T_{inter}) between two material is equal to the average of the temperature at the two ends. Assuming steady one-dimensional heat conduction, which of the following statements is true about the respective thermal conductivities ?



(A) $2k_1 = k_2$ (B) $k_1 = k_2$

(C) $2k_1 = 3k_2$ (D) $k_1 = 2k_2$

YEAR 2006 TWO MARKS

MCQ 7.26 A 100 W electric bulb was switched on in a $2.5 \text{ m} \times 3 \text{ m} \times 3 \text{ m}$ size thermally insulated room having a temperature of 20°C. The room temperature at the end of 24 hours will be

(A) 321°C (B) 341°C

(C) 450°C (D) 470°C

MCQ 7.27 A thin layer of water in a field is formed after a farmer has watered it. The ambient air conditions are : temperature 20°C and relative humidity 5%. An extract of steam tables is given below.

Temp($^\circ$C)	-15	-10	-5	0.01	5	10	15	20
Saturation Pressure (kPa)	0.10	0.26	0.40	0.61	0.87	1.23	1.71	2.34

Neglecting the heat transfer between the water and the ground, the water temperature in the field after phase equilibrium is reached equals

(A) 10.3°C (B) -10.3°C

(C) -14.5°C (D) 14.5°C

MCQ 7.28 With an increase in the thickness of insulation around a circular pipe, heat loss to surrounding due to

(A) convection increase, while that the due to conduction decreases

(B) convection decrease, while that due to conduction increases

(C) convection and conduction decreases

(D) convection and conduction increases

YEAR 2005 ONE MARK

MCQ 7.29 In a case of one dimensional heat conduction in a medium with constant properties, T is the temperature at position x, at time t. Then $\frac{\partial T}{\partial t}$ is proportional to

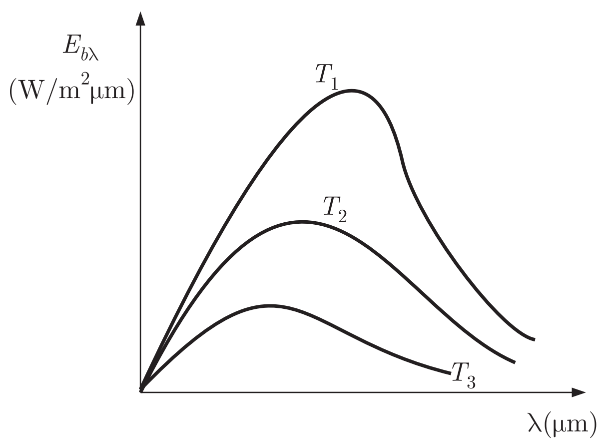
(A) $\frac{T}{x}$

(B) $\frac{\partial T}{\partial x}$

(C) $\frac{\partial^2 T}{\partial x \partial t}$

(D) $\frac{\partial^2 T}{\partial x^2}$

MCQ 7.30 The following figure was generated from experimental data relating spectral black body emissive power to wavelength at three temperature T_1, T_2 and $T_3 (T_1 > T_2 > T_3)$.



The conclusion is that the measurements are

(A) correct because the maxima in $E_{b\lambda}$ show the correct trend

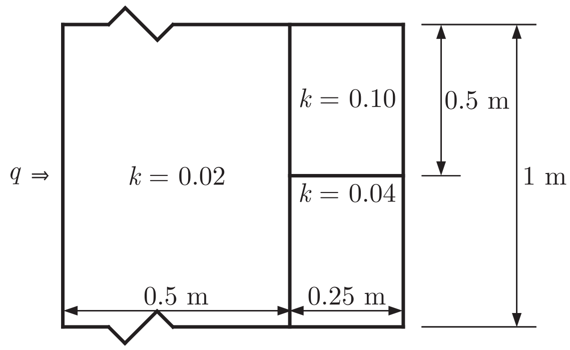
(B) correct because Planck's law is satisfied

(C) wrong because the Stefan Boltzmann law is not satisfied

(D) wrong because Wien's displacement law is not satisfied

YEAR 2005 TWO MARKS

MCQ 7.31 Heat flows through a composite slab, as shown below. The depth of the slab is 1 m. The k values are in W/mK. The overall thermal resistance in K/W is



(A) 17.2 (B) 21.9

(C) 28.6 (D) 39.2

MCQ 7.32 A small copper ball of 5 mm diameter at 500 K is dropped into an oil bath whose temperature is 300 K. The thermal conductivity of copper is 400 W/mK, its density 9000 kg/m^3 and its specific heat 385 J/kgK. If the heat transfer coefficient is 250 W/m^{2}K and lumped analysis is assumed to be valid, the rate of fall of the temperature of the ball at the beginning of cooling will be, in K/s,

(A) 8.7 (B) 13.9

(C) 17.3 (D) 27.7

MCQ 7.33 A solid cylinder (surface 2) is located at the centre of a hollow sphere (surface 1). The diameter of the sphere is 1 m, while the cylinder has a diameter and length of 0.5 m each. The radiation configuration factor F_{11} is

(A) 0.375 (B) 0.625

(C) 0.75 (D) 1

MCQ 7.34 Hot oil is cooled from 80 to 50°C in an oil cooler which uses air as the coolant. The air temperature rises from 30 to 40°C. The designer uses a LMTD value of 26°C. The type of heat exchange is

(A) parallel flow (B) double pipe

(C) counter flow (D) cross flow

• Common Data For Q.35 and Q.36

An uninsulated air conditioning duct of rectangular cross section 1 m $\times$ 0.5 m, carrying air at 20°C with a velocity of 10 m/s, is exposed to an ambient of 30°C. Neglect the effect of duct construction material. For air in the range of 20 - 30°C, data are as follows; thermal conductivity $=$ 0.025 W/mK; viscosity $=$ 18 μPas, Prandtl number $=$ 0.73; density $=$ 1.2 kg/m^3. The laminar flow Nusselt number is 3.4 for constant wall temperature conditions and for turbulent flow, $Nu = 0.023\,Re^{0.8}\,Pr^{0.33}$

MCQ 7.35 The Reynolds number for the flow is

(A) 444
(B) 890
(C) 4.44×10^5
(D) 5.33×10^5

MCQ 7.36 The heat transfer per meter length of the duct, in watts is

(A) 3.8
(B) 5.3
(C) 89
(D) 769

YEAR 2004 ONE MARK

MCQ 7.37 One dimensional unsteady state heat transfer equation for a sphere with heat generation at the rate of 'q' can be written as

(A) $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{q}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$
(B) $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{q}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$
(C) $\frac{\partial^2 T}{\partial r^2}+\frac{q}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$
(D) $\frac{\partial^2}{\partial r^2}(rT)+\frac{q}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

YEAR 2004 TWO MARKS

MCQ 7.38 A stainless steel tube $(k_s = 19 \text{ W/m K})$ of 2 cm ID and 5 cm OD is insulated with 3 cm thick asbestos $(k_a = 0.2 \text{ W/m K})$. If the temperature difference between the innermost and outermost surfaces is 600° C, the heat transfer rate per unit length is

(A) 0.94 W/m
(B) 9.44 W/m
(C) 944.72 W/m
(D) 9447.21 W/m

MCQ 7.39 A spherical thermocouple junction of diameter 0.706 mm is to be used for the measurement of temperature of a gas stream. The convective heat transfer co-efficient on the bead surface is 400 W/m^{2}K. Thermo-physical properties of thermocouple material are $k = 20$ W/mK, $c = 400$ J/kg K and $\rho = 8500$ kg/m^3. If the thermocouple initially at 30°C is placed in a hot stream of 300°C, then time taken by the bead to reach 298°C, is

(A) 2.35 s
(B) 4.9 s
(C) 14.7 s
(D) 29.4 s

MCQ 7.40 In a condenser, water enters at 30°C and flows at the rate 1500 kg/hr. The condensing steam is at a temperature of 120°C and cooling water leaves the condenser at 80°C. Specific heat of water is 4.187 kJ/kgK. If the overall heat transfer coefficient is 2000 W/m^{2}K, then heat transfer area is

(A) 0.707 m^2
(B) 7.07 m^2
(C) 70.7 m^2
(D) 141.4 m^2

YEAR 2003 ONE MARK

MCQ 7.41 A plate having $10\,cm^2$ area each side is hanging in the middle of a room of $100\,m^2$ total surface area. The plate temperature and emissivity are respectively 800 K and 0.6. The temperature and emissivity values for the surfaces of the room are 300 K and 0.3 respectively. Boltzmann's constant $\sigma = 5.67 \times 10^{-8}\,W/m^2K^4$. The total heat loss from the two surfaces of the plate is

(A) 13.66 W

(B) 27.32 W

(C) 27.87 W

(D) 13.66 MW

YEAR 2003 TWO MARKS

MCQ 7.42 In a counter flow heat exchanger, for the hot fluid the heat capacity $= 2\,kJ/kgK$, mass flow rate $= 5\,kg/s$, inlet temperature $= 150^\circ C$, outlet temperature $= 100^\circ C$. For the cold fluid, heat capacity $= 4\,kJ/kgK$, mass flow rate $= 10\,kg/s$, inlet temperature $= 20^\circ C$. Neglecting heat transfer to the surroundings, the outlet temperature of the cold fluid in $^\circ C$ is

(A) 7.5

(B) 32.5

(C) 45.5

(D) 70.0

MCQ 7.43 Consider a laminar boundary layer over a heated flat plate. The free stream velocity is U_∞. At some distance x from the leading edge the velocity boundary layer thickness is δ_v and the thermal boundary layer thickness is δ_T. If the Prandtl number is greater than 1, then

(A) $\delta_v > \delta_T$

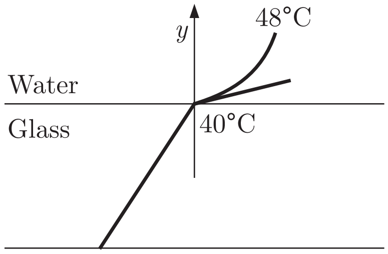
(B) $\delta_T > \delta_v$

(C) $\delta_v \approx \delta_T \sim (U_\infty x)^{-1/2}$

(D) $\delta_v \approx \delta_T \sim x^{-1/2}$

• Common Data For Q.44 and Q.45

Heat is being transferred by convection from water at $48^\circ C$ to a glass plate whose surface that is exposed to the water is at $40^\circ C$. The thermal conductivity of water is 0.6 W/mK and the thermal conductivity of glass is 1.2 W/mK. The spatial gradient of temperature in the water at the water-glass interface is $dT/dy = 1 \times 10^4\,K/m$.



MCQ 7.44 The value of the temperature gradient in the glass at the water-glass interface in K/m is

(A) -2×10^4
(B) 0.0
(C) 0.5×10^4
(D) 2×10^4

MCQ 7.45 The heat transfer coefficient h in $W/m^2 K$ is

(A) 0.0
(B) 4.8
(C) 6
(D) 750

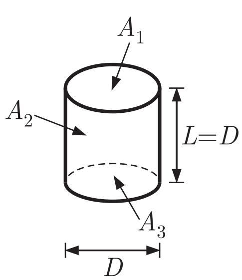
YEAR 2002 ONE MARK

MCQ 7.46 For the same inlet and outlet temperatures of hot and cold fluids, the Log mean Temperature Difference (LMTD) is

(A) greater for parallel flow heat exchanger than for counter flow heat exchanger
(B) greater for counter flow heat exchanger than for parallel flow heat exchanger
(C) same for both parallel and counter flow heat exchangers
(D) dependent on the properties of the fluids.

YEAR 2001 ONE MARK

MCQ 7.47 For the circular tube of equal length and diameter shown below, the view factor F_{13} is 0.17. The view factor F_{12} in this case will be



(A) 0.17
(B) 0.21
(C) 0.79
(D) 0.83

MCQ 7.48 In descending order of magnitude, the thermal conductivity of (a) pure iron, (b) liquid water, (c) saturated water vapour and (d) aluminum can be arranged as

(A) abcd
(B) bcad
(C) dabc
(D) dcba

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SOLUTION

SOL 7.1 Option (C) is correct.

The sum of the absorbed, reflected and transmitted radiation be equal to

$$\alpha + \rho + \tau = 1$$

$\alpha =$ Absorpivity, $\rho =$ Reflectivity, $\tau =$ Transmissivity

For an opaque surfaces such as solids and liquids

$$\tau = 0,$$

Thus, $\alpha + \rho = 1$

SOL 7.2 Option (A) is correct.

The performance of the fins is judged on the basis of the enhancement in heat transfer area relative to the no fin case. The fin effectiveness

$$\varepsilon_{fin} = \frac{\text{Heat transfer rate from the fin of base area}}{\text{Heat transfer rate from the surface area}}$$

When determining the rate of heat transfer from a finned surface, we must consider the unfinned portion of the surface as well as the fins and number of fins.

Thin and closed spaced fin configuration, the unfinned portion of surface is reduced and number of fins is increased. Hence the fin effectiveness will be maximum for thin and closely spaced fins.

SOL 7.3 Option (D) is correct.

According to the reciprocity relation.

$$A_1F_{12} = A_2F_{21}$$

Which yields $F_{21} = \frac{A_1}{A_2} \times F_{12} = \frac{\pi D_1 L}{\pi D_2 L} \times 1 = \left(\frac{D_1}{D_2}\right)$

$F_{11} = 0$ since no radiation leaving surface 1 and strikes 1

$F_{12} = 1$, since all radiation leaving surface 1 and strikes 2

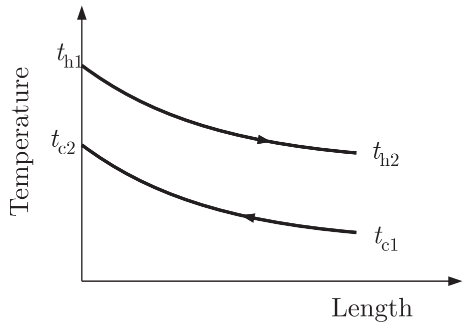
The view factor F_{22} is determined by applying summation rule to surface 2,

$$F_{21} + F_{22} = 1$$

Thus $F_{22} = 1 - F_{21} = 1 - \left(\frac{D_1}{D_2}\right)$

SOL 7.4 Option (C) is correct.

Given : $t_{h1} = 80^\circ\text{C}$, $t_{c1} = 30^\circ\text{C}$, $\dot{m}_h = 0.5\text{ kg/sec}$, $\dot{m}_c = 2.09\text{ kg/sec.}$, $\varepsilon = 0.8$



Capacity rate for hot fluid

$$C_h = 4.18 \times 0.5 = 2.09 \text{ kJ/Ksec.}$$

$$C_c = 1 \times 2.09 = 2.09 \text{ kJ/K sec.}$$

So, $$C_h = C_c$$

$$\text{Effectiveness } \varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{(t_{h1} - t_{h1})\,C_h}{(t_{h1} - t_{c1})\,C_c}$$

$$0.8 = \frac{80 - t_{h2}}{80 - 30}$$

or, $$80 - t_{h2} = 40$$

$$t_{h2} = 40^\circ\text{C}$$

From energy balance,

$$C_h(t_{h1} - t_{h1}) = C_c(t_{c2} - t_{c1})$$

$$80 - 40 = t_{c2} - 30$$

$$t_{c2} = 70^\circ\text{C}$$

Now LMTD $$\theta_m = \frac{\theta_1 - \theta_2}{\ln\frac{\theta_1}{\theta_2}} \qquad ...(i)$$

$$\theta_1 = t_{h1} - t_{c2} = 80 - 70 = 10^\circ\text{C}$$

$$\theta_2 = t_{h2} - t_{c1} = 40 - 30 = 10^\circ\text{C}$$

$$\theta_1 = \theta_2 \qquad ...(ii)$$

So LMTD is undefined

Let $$\frac{\theta_1}{\theta_2} = x \Rightarrow \theta_1 = x\theta_2$$

Put in equation (i), so

$$\theta_m = \lim_{x \to 1} \frac{x\theta_2 - \theta_2}{\ln\frac{x\theta_2}{\theta_2}} = \lim_{x \to 1} \frac{\theta_2(x-1)}{\ln x}$$

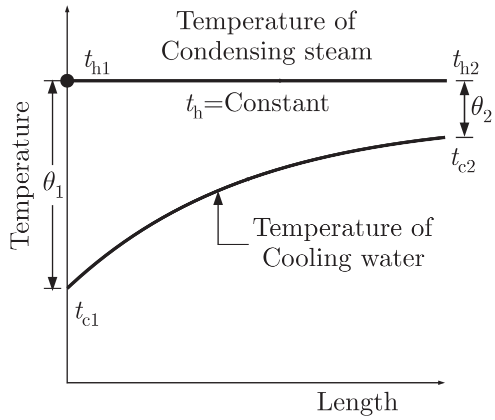
It is a $\left[\frac{0}{0}\right]$ form, applying L-Hospital rule

$$\theta_m = \lim_{x \to 1} \frac{\theta_2(1-0)}{\frac{1}{x}} = \lim_{x \to 1} x\theta_2$$

$$\theta_m = \theta_2 = \theta_1 \qquad \text{From equation (ii)}$$

$$\theta_m = \theta_1 = t_{h1} - t_{c2} = 80 - 70 = 10^\circ\text{C}$$

SOL 7.5 Option (B) is correct.



Given : $t_{h1} = t_{h2} = 60^\circ\,C,\ t_{c1} = 30^\circ\,C,\ t_{c2} = 45^\circ\,C$

From diagram, we have

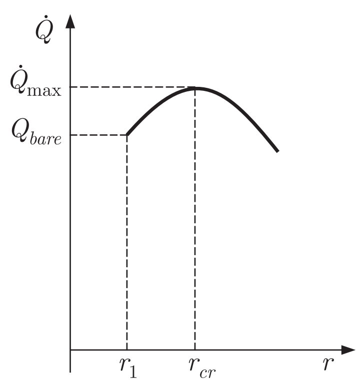
$$\theta_1 = t_{h1} - t_{c1} = 60 - 30 = 30^\circ\,C$$

And $$\theta_2 = t_{h2} - t_{c2} = 60 - 45 = 15^\circ\,C$$

Now LMTD, $$\theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} = \frac{30 - 15}{\ln\left(\frac{30}{15}\right)} = 21.6^\circ\,C$$

SOL 7.6 Option (C) is correct.

Given : $d_0 = 25\,\text{mm} = 0.025\,\text{m},\ r_0 = \frac{0.025}{2} = 0.0125\,\text{m},\ h = 5\,\text{W/m}^2\,\text{K},$
$k = 0.05\,\text{W/mK}$



Hence, Critical radius of insulation for the pipe is given by,

$$r_c = \frac{k}{h} = \frac{0.05}{5} = 0.01\,\text{m}$$

$$r_c < r_0 \text{ or } r_0 > r_c \qquad \text{...(i)}$$

So, from equation (i) option *a* and *b* is incorrect. The critical radius is less than the outer radius of the pipe and adding the insulation will not increase the heat loss. Hence the correct statement is adding the insulation will reduce the heat loss.

SOL 7.7 Option (D) is correct.

Given : $D = 12\,\text{mm} = 12 \times 10^{-3}\,\text{m}$, $h = 5\,\text{W/m}^2\,\text{K}$, $k = 20\,\text{W/m K}$

For spherical ball, $= \dfrac{12 \times 10^{-3}}{6} = 2 \times 10^{-3}\,\text{m}$

$$l = \frac{\text{volume}}{\text{surface area}} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{D}{6}$$

The non-dimensional factor (hl/k) is called Biot Number. It gives an indication of the ratio of internal (conduction) resistance to the surface (convection) resistance.

A small value of Bi implies that the system has a small conduction resistance i.e., relatively small temperature gradient or the existence of a practically uniform temperature within the system.

Biot Number, $Bi = \dfrac{hl}{k} = \dfrac{5 \times 2 \times 10^{-3}}{20} = 0.0005$

Since, Value of Biot Number is very less. Hence, conduction resistance is much less than convection resistance.

SOL 7.8 Option (A) is correct.

Given : $\left(\dfrac{\delta_H}{\delta_{Th}}\right)_P = \dfrac{1}{2}$ and $\left(\dfrac{\delta_H}{\delta_{Th}}\right)_Q = 2$

Here, δ_H →Thickness of laminar hydrodynamic boundary layer

And δ_{Th} →Thickness of thermal boundary layer

$$(\text{Re})_P = (\text{Re})_Q = 10^4$$

$$(\text{Pr})_P = \frac{1}{8}$$

$$(\text{Nu})_P = 35$$

For thermal boundary layer prandtl Number is given by, (For fluid Q)

$$(\text{Pr})_Q^{1/3} = \left(\frac{\delta_H}{\delta_{Th}}\right)_Q = 2$$

$$(\text{Pr})_Q = (2)^3 = 8$$

For laminar boundary layer on flat plate, relation between Reynolds Number, Prandtl Number and Nusselt Number is given by,

$$\text{Nu} = \frac{hl}{k} = (\text{Re})^{1/2}(\text{Pr})^{1/3}$$

Since, Reynolds Number is same for both P and Q.

So, $$\frac{(\text{Nu})_P}{(\text{Nu})_Q} = \frac{(\text{Pr})_P^{1/3}}{(\text{Pr})_Q^{1/3}}$$

$$(\text{Nu})_Q = \frac{(\text{Pr})_Q^{1/3}}{(\text{Pr})_P^{1/3}} \times (\text{Nu})_P = \frac{(8)^{1/3}}{(1/8)^{1/3}} \times (35) = \frac{2}{1/2} \times 35$$

$$= 140$$

SOL 7.9 Option (B) is correct.

Given, $d = 5\,\text{mm} = 0.005\,\text{m},\ l = 100\,\text{mm} = 0.1\,\text{m},\ k = 400\ \text{W/m K}$

$T_0 = 130^\circ\,\text{C},\ T_a = 30^\circ\,\text{C},\ h = 40\ \text{W/m}^2\,\text{K}$

Heat loss by the fin is given by,

$$Q_{fin} = mkA_c(T_0 - T_a)\tanh(ml) \qquad \text{...(i)}$$

$$\frac{\text{Perimeter}}{\text{Cross sectional Area}} = \frac{p}{A_c} = \frac{\pi d}{\frac{\pi}{4}d^2} = \frac{4}{d} = \frac{4}{0.005}$$

$$\frac{p}{A_c} = 800 \qquad \text{...(ii)}$$

And
$$m = \sqrt{\frac{h}{k}\left(\frac{p}{A_c}\right)} = \sqrt{\frac{40}{400} \times 800} = \sqrt{80}$$

From equation(i),

$$Q_{fin} = \sqrt{80} \times 400 \times \frac{\pi}{4} \times (0.005)^2(130 - 30) \times \tanh(\sqrt{80} \times 0.1)$$

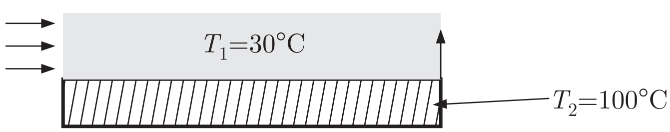
$$= 8.944 \times 400 \times 1.96 \times 10^{-5} \times 100 \times \tanh(0.8944)$$

$$= 7.012 \times 0.7135 \simeq 5\ \text{W}$$

SOL 7.10 Option (B) is correct.

Given : $T_1 = 30^\circ\,\text{C},\ T_2 = 100^\circ\,\text{C},\ k = 1.0\ \text{W/mK},$

$$T = 30 + 70\exp(-y) \qquad \text{...(i)}$$



Under steady state conditions,

Heat transfer by conduction = Heat transfer by convection

$$-kA\frac{dT}{dy} = hA\Delta T \qquad A \to \text{Area of plate}$$

$$-kA\frac{d}{dy}(30 + 70e^{-y}) = hA\Delta T$$

Solving above equation, we get

$$-kA(-70e^{-y}) = hA\Delta T$$

At the surface of plate, $y = 0$

Hence
$$70kA = hA\Delta T$$

$$h = \frac{70kA}{A\Delta T} = \frac{70k}{\Delta T} = \frac{70 \times 1}{(100 - 30)} = 1\ \text{W/m}^2\,\text{K}$$

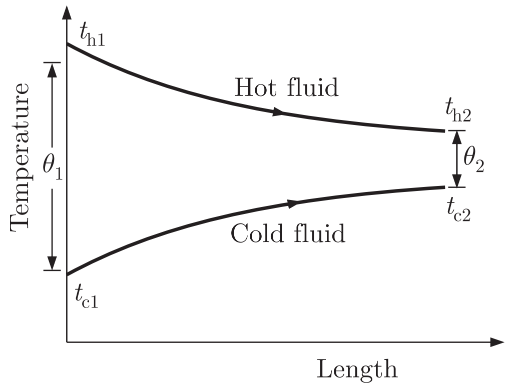
SOL 7.11 Option (B) is correct.

Given :

$\dot{C}_h = \dot{C}_c,\ \dot{m}_h = 1\ \text{kg/sec},\ c_{ph} = 4\ \text{kJ/kg K},\ t_{h1} = 102^\circ\,\text{C},\ t_{c1} = 15^\circ\,\text{C}$

$U = 1\ \text{kW/m}^2\text{K},\ A = 5\ \text{m}^2$

The figure shown below is for parallel flow.



$$\dot{C}_h = \dot{m}_h c_{ph} = 4 \text{ kJ/sK}$$

The heat exchanger is characterized by the following relation,

$$\varepsilon = \frac{1 - \exp(-2NTU)}{2} \quad ..(i)$$

For parallel flow heat exchanger effectiveness is given by

$$\varepsilon = \frac{1 - \exp[-\text{NTU}(1 + C)]}{1 + C} \quad ...(ii)$$

Comparing equation (i) and equation (ii), we get capacity ratio

$$C = \frac{C_c}{C_h} = \frac{C_{\min}}{C_{\max}} = 1 \quad ...(iii)$$

Applying energy balance for a parallel flow

$$C_h(t_{h1} - t_{h2}) = C_c(t_{c2} - t_{c1})$$

$$\frac{C_c}{C_h} = \frac{t_{h1} - t_{h2}}{t_{c2} - t_{c1}} = 1 \quad \text{From equation(iii)}$$

$$t_{h1} - t_{h2} = t_{c2} - t_{c1}$$

Number of transfer units is given by,

$$\text{NTU} = \frac{UA}{C_{\min}} = \frac{1 \times 5}{4} = 1.25$$

Effectiveness, $\varepsilon = \dfrac{1 - \exp(-2 \times 1.25)}{2} = \dfrac{1 - 0.0820}{2} = 0.46$

Maximum possible heat transfer is,

$$Q_{\max} = C_{\min}(t_{h1} - t_{c1})$$

$$= 4 \times [(273 + 102) - (273 + 15)] = 348 \text{ kW}$$

But Actual Heat transfer is,

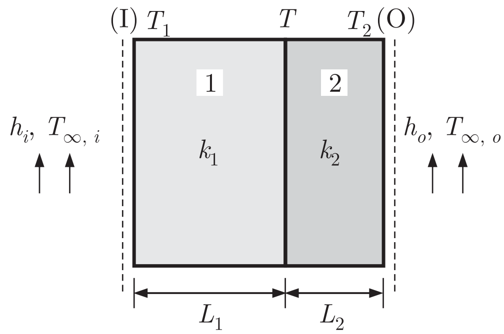
$$Q_a = \varepsilon Q_{\max} = 0.46 \times 348 = 160 \text{ kW}$$

And $Q_a = C_c(t_{c2} - t_{c1})$

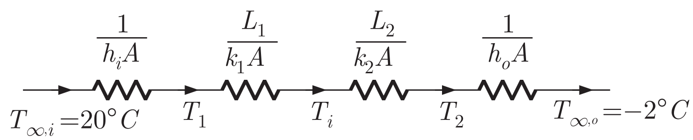
$$160 = 4(t_{c2} - 15)$$

$$t_{c2} = 40 + 15 = 55^\circ \text{C}$$

SOL 7.12 Option (C) is correct.



The equivalent resistance diagram for the given system is,



$$R_{eq} = \frac{1}{h_i A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_0 A}$$

$$R_{eq} \times A = \frac{1}{h_i} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_0} = \frac{1}{20} + \frac{0.3}{20} + \frac{0.15}{50} + \frac{1}{50}$$

$$= 0.05 + 0.015 + 0.003 + 0.02 = 0.088 \text{ m}^2 \text{ K/W}$$

Heat flux, $$q = \frac{Q}{A} = \frac{\Delta T}{AR_{eq}} \qquad Q = \frac{\Delta T}{\sum R}$$

Under steady state condition,

$$q = \frac{T_{\infty i} - T_{\infty o}}{AR_{eq}} = h_i (T_{\infty i} - T_1) = \frac{k_1(T_1 - T)}{L_1} = \frac{k_2(T - T_2)}{L_2} \qquad \text{...(i)}$$

$$= \frac{T_{\infty i} - T_{\infty o}}{AR_{eq}} = \frac{20 - (-2)}{0.088} = 250 \text{ W/m}^2 \qquad \text{...(ii)}$$

$$= \frac{T_{\infty i} - T_1}{\frac{1}{h_i}} = \frac{20 - T_1}{\frac{1}{20}} \qquad \text{From equation(i)}$$

$$250 = 20(20 - T_1)$$

$$12.5 = 20 - T_1 \quad \Rightarrow \quad T_1 = 20 - 12.5 = 7.5° \text{ C}$$

Again from equation(i),

$$q = \frac{k_1(T_1 - T)}{L_1}$$

$$250 = \frac{20}{0.3}(7.5 - T)$$

$$3.75 = 7.5 - T \quad \Rightarrow T = 3.75° \text{ C}$$

Alternative :

Under steady state conditions,

Heat flow from I to interface wall = Heat flow from interface wall to O

$$\frac{(T_{\infty,i}-T)}{\frac{1}{h_iA}+\frac{L_1}{k_1A}} = \frac{(T-T_{\infty,o})}{\frac{L_2}{k_2A}+\frac{1}{h_0A}}$$

$$\frac{T_{\infty,i}-T}{\frac{1}{h_i}+\frac{L_1}{k_1}} = \frac{T-T_{\infty,o}}{\frac{L_2}{k_2}+\frac{1}{h_o}}$$

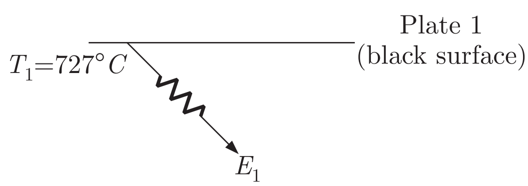
$$\frac{(20-T)}{\frac{1}{20}+\frac{0.3}{20}} = \frac{T-(-2)}{\frac{0.15}{50}+\frac{1}{50}}$$

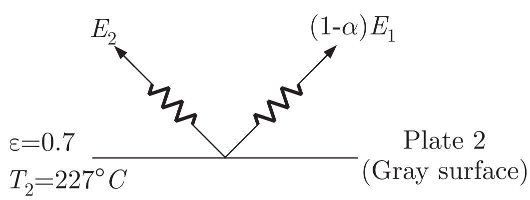
$$\frac{(20-T)}{\frac{1.3}{20}} = \frac{T+2}{\frac{1.15}{50}}$$

$$(20-T) = 2.826(T+2) = 2.826T+5.652$$

$$T = \frac{14.348}{3.826} = 3.75^\circ\text{ C}$$

SOL 7.13 Option (D) is correct.





Given : $\sigma_b = 5.67\times10^{-8}\text{ W/m}^2\text{ K}^4,\ T_2 = (227+273)\text{ K} = 500\text{ K}$

$T_1 = (727+273)\text{ K} = 1000\text{ K}$

Let, $\alpha \to$ The absorptivity of the gray surface

$E_1 \to$ The radiant energy of black surface

$E_2 \to$ The radiant energy of gray surface

Now, Plate 1 emits radiant energy E_1 which strikes the plate 2. From it a part αE_1 absorbed by the plate 2 and the remainder $(E_1-\alpha E_1)$ is reflected back to the plate 1. On reaching plate 1, all the part of this energy is absorbed by the plate 1, because the absorptivity of plate 1 is equal to one (it is a black surface).

Irradiation denotes the total radiant energy incident upon a surface per unit time per unit area.

Energy leaving from the plate 2 is,

$$E = E_2+(1-\alpha)E_1 \qquad \text{...(i)}$$

Hence, E_2 is the energy emitted by plate 2.

$$E_2 = \varepsilon\sigma_b T_2^4 = 0.7 \times 5.67 \times 10^{-8} \times (500)^4 \qquad E = \varepsilon\sigma_b T^4$$
$$= 0.7 \times 5.67 \times 10^{-8} \times 625 \times 10^8 = 2480.625 \text{ W/m}^2$$

And fraction of energy reflected from surface 2 is,

$$= (1-\alpha)E_1 = (1-\alpha)\sigma T_1^4$$
$$= 5.67 \times 10^{-8}(1-0.7) \times (1000)^4 = 17010 \text{ W/m}^2$$

Now, Total energy incident upon plate 1 is,

$$E = E_2 + (1-\alpha)E_1 = 2480.625 + 17010$$
$$= 19490.625 \text{ W/m}^2 = 19.49 \text{ kW/m}^2 \cong 19.5 \text{ kW/m}^2$$

SOL 7.14 Option (D) is correct.

Given : $\varepsilon_2 = 0.8$, $\varepsilon_1 = 0.7$

As both the plates are gray, the net heat flow from plate 1 to plate 2 per unit time is given by,

$$Q_{12} = \frac{\varepsilon_1\varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1\varepsilon_2}\sigma_b(T_1^4 - T_2^4) = \frac{1}{\frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_1} - 1}\sigma_b(T_1^4 - T_2^4)$$
$$= \frac{1}{\frac{1}{0.8} + \frac{1}{0.7} - 1} \times 5.67 \times 10^{-8}[(1000)^4 - (500)^4]$$
$$= \frac{1}{1.68} \times 5.67 \times 9375 = 31640.625 \text{ W/m}^2$$
$$\simeq 31.7 \text{ kW/m}^2$$

SOL 7.15 Option (C) is correct.

Given : $\mu = 0.001$ Pa-s, $c_p = 1$ kJ/kg K, $k = 1$ W/m K

The prandtl Number is given by,

$$\Pr = \frac{\mu c_p}{k} = \frac{0.001 \times 1 \times 10^3}{1} = 1$$

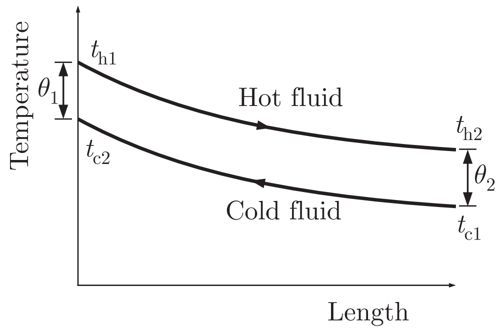
And
$$\frac{\delta}{\delta_t} = \frac{\text{hydrodynamic bondary layer thickness}}{\text{Thermal boundary layer thickness}} = (\Pr)^{1/3}$$

Given,
$$\delta = 1 \text{ m}$$
$$\frac{\delta}{\delta_t} = (1)^{1/3} = 1$$
$$\delta = \delta_t = 1 \text{ mm}$$

Hence, thermal boundary layer thickness at same location is 1 mm.

SOL 7.16 Option (C) is correct.

The $T - L$ curve shows the counter flow.



Given : $\theta_m = 20^\circ$ C, $t_{c1} = 20^\circ$ C, $t_{h1} = 100^\circ$ C

$$\dot{m}_c = 2\dot{m}_h \Rightarrow \frac{\dot{m}_c}{\dot{m}_h} = 2 \quad ...(i)$$

$$c_{ph} = 2c_{pc} \Rightarrow \frac{c_{ph}}{c_{pc}} = 2 \quad ...(ii)$$

Energy balance for counter flow is,

Heat lost by hot fluid = Heat gain by cold fluid

$$\dot{m}_h c_{ph}(t_{h1} - t_{h2}) = \dot{m}_c c_{pc}(t_{c2} - t_{c1})$$

$$\frac{c_{ph}}{c_{pc}}(t_{h1} - t_{h2}) = \frac{\dot{m}_c}{\dot{m}_h}(t_{c2} - t_{c1})$$

$$2(t_{h1} - t_{h2}) = 2(t_{c2} - t_{c1})$$

$$t_{h1} - t_{c2} = t_{h2} - t_{c1}$$

$$\theta_1 = \theta_2 \quad ...(iii)$$

And $$\theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} \quad ...(iv)$$

Substituting the equation (iii) in equation (iv), we get undetermined form.

Let $$\frac{\theta_1}{\theta_2} = x, \qquad \Rightarrow \theta_1 = \theta_2 x \quad ...(v)$$

Substitute θ_1 in equation(iv),

$$\theta_m = \lim_{x \to 1} \frac{\theta_2 x - \theta_2}{\ln\left(\frac{\theta_2 x}{\theta_2}\right)} = \lim_{x \to 1} \frac{\theta_2(x-1)}{\ln x} \quad ...(vi)$$

$\left[\frac{0}{0}\right]$ form, So we apply L-Hospital rule,

$$\theta_m = \lim_{x \to 1} \frac{\theta_2(1-0)}{\frac{1}{x}} = \lim_{x \to 1} x\,\theta_2$$

$$\theta_m = \theta_2 = \theta_1 \quad \text{From equation(iii)}$$

Now we have to find exit temperature of cold fluid (t_{c2}),

So, $$\theta_m = \theta_1 = t_{h1} - t_{c2}$$

$$t_{c2} = t_{h1} - \theta_m = 100 - 20 = 80^\circ \text{ C}$$

SOL 7.17 Option (D) is correct.

Given : $h = 10\ \text{W/m}^2\,\text{K}$, $T_i = 30^\circ\,\text{C}$, $q_g = 100\ \text{W/m}^3$

Five faces of the object are insulated, So no heat transfer or heat generation by these five faces. Only sixth face (PQRS) interacts with the surrounding and generates heat.

Hence, Heat generated throughout the volume

$$Q = \text{Rate of heat Generated} \times \text{Volume of object}$$
$$= 100 \times (1 \times 2 \times 2) = 400\ \text{W}$$

And heat transfer by convection is given by

$$Q = hA(T_f - T_i)$$
$$400 = 10 \times (2 \times 2)(T_f - 30)$$
$$T_f = 30 + 10 = 40^\circ\,\text{C}$$

SOL 7.18 Option (B) is correct.

Given : $D_1 = 1\ \text{m}$, $D_2 = 2\ \text{m}$

Hence, the small cylindrical surface (surface 1) cannot see itself and the radiation emitted by this surface strikes on the enclosing surface 2. From the conservation principal (summation rule).

For surface 1, $F_{12} + F_{11} = 1$ $\qquad F_{11} = 0$

$$F_{12} = 1 \qquad ...(i)$$

From the reciprocity theorem

$$A_1F_{12} = A_2F_{21}$$
$$F_{21} = \frac{A_1}{A_2} = \frac{\pi D_1 L}{\pi D_2 L} = \frac{D_1}{D_2} = \frac{1}{2} = 0.5$$

and from the conservation principal, for surface 2, we have

$$F_{21} + F_{22} = 1$$
$$F_{22} = 1 - F_{21} = 1 - 0.5 = 0.5$$

So, the fraction of the thermal radiation leaves the larger surface and striking itself is $F_{22} = 0.5$.

SOL 7.19 Option (D) is correct.

Given : $\left(\frac{\partial T}{\partial x}\right)_Q = 10\ \text{K/m}$, $(T)_P = (T)_Q$, $(k)_P = (k)_Q = 0.1\ \text{W/mK}$

Direction of heat flow is always normal to surface of constant temperature. So, for surface P,

$$\frac{\partial T}{\partial x} = 0$$

Because, $Q = -kA(\partial T/\partial x)$ and ∂T is the temperature difference for a short perpendicular distance dx. Let width of both the bodies are unity.

From the law of energy conservation,

$$\text{Heat rate at } P = \text{Heat rate at } Q$$

$$-0.1 \times 1 \times \left(\frac{\partial T}{\partial y}\right)_P = -0.1 \times 2 \times \left(\frac{\partial T}{\partial x}\right)_Q$$

Because for P heat flow in y direction and for Q heat flow in x direction

$$\left(\frac{\partial T}{\partial y}\right)_P = \frac{0.1 \times 2 \times 10}{0.1} = 20 \text{ K/m}$$

SOL 7.20 Option (B) is correct.

The region beyond the thermal entrance region in which the dimensionless temperature profile expressed as $\left(\frac{T - T_w}{T_\infty - T_w}\right)$ remains unchanged is called thermally fully developed region.

Nusselt Number is given by,

$$N_u = \frac{hL}{k} = \left(\frac{\partial T}{\partial y'}\right)_{\text{at } y' = 0} \quad \text{...(i)}$$

Here, $T = \frac{T - T_w}{T_\infty - T_w}$ and $y' = \frac{y}{\partial_t}$

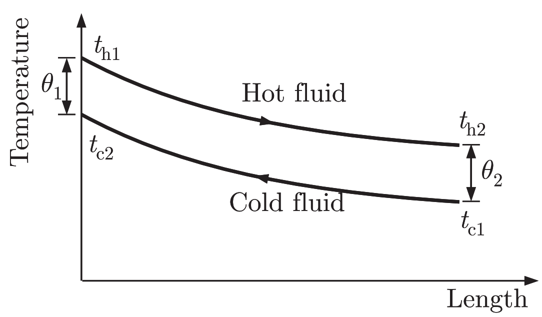
So,

$$N_u = \frac{\partial}{\partial y'}\left[\frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3\right]_{y'=0} = \frac{\partial}{\partial y}\left[\frac{3}{2}y' - \frac{1}{2}(y')^3\right]_{y'=0}$$

$$= \left[\frac{3}{2} - \frac{3}{2}\left(\frac{y}{\delta_t}\right)^2\right]_{y'=0} = \frac{3}{2} = 1.5$$

SOL 7.21 Option (B) is correct.

The counter flow arrangement of the fluid shown below :



Given: for hot fluid : $t_{h1} = 60^\circ \text{ C}$, $\dot{m}_h = 1 \text{ kg/sec}$, $c_h = 10 \text{ kJ/kg K}$

And for cold fluid : $t_{c2} = 30^\circ \text{ C}$, $\dot{m}_c = 2 \text{ kg/sec}$, $c_c = 5 \text{ kJ/kg K}$

Heat capacity of Hot fluid,

$$C_h = \dot{m}_h c_h = 1 \times 10 = 10 \text{ kJ/k. sec}$$

And heat capacity of cold fluid,

$$C_c = \dot{m}_c c_c = 2 \times 5 = 10 \text{ kJ/k sec}$$

By energy balance for the counter flow

$$\dot{m}_h c_h (t_{h1} - t_{h2}) = \dot{m}_c c_c (t_{c2} - t_{c1})$$

$$C_h (t_{h1} - t_{h2}) = C_c (t_{c2} - t_{c1}) \quad C_h = C_c$$

$$t_{h1} - t_{c2} = t_{h2} - t_{c1}$$

$$\theta_1 = \theta_2$$

LMTD,
$$\theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} \qquad ...(i)$$

Let,
$$\frac{\theta_1}{\theta_2} = x$$
θ_1 is equal to θ_2 and θ_m is undetermined

$$\theta_1 = x\theta_2$$

Substituting θ_1 in equation (i), we get,

$$\theta_m = \lim_{x \to 1} \frac{x\theta_2 - \theta_2}{\ln(x)} = \lim_{x \to 1} \frac{\theta_2(x-1)}{\ln(x)}$$

$\left(\frac{0}{0}\right)$form, So we apply L-hospital rule,

$$\theta_m = \lim_{x \to 1} \frac{\theta_2 \times 1}{\frac{1}{x}} = \lim_{x \to 1} x\theta_2$$

$$\theta_m = \theta_2 = \theta_1 \quad \Rightarrow \quad \theta_1 = t_{h1} - t_{c2} = 60 - 30 = 30^\circ \text{C}$$

SOL 7.22 Option (D) is correct.

Given : $T_1 = 25^\circ \text{C} = (273 + 25) = 298 \text{ K}$, $A = 0.1 \text{ m}^2$, $m = 4 \text{ kg}$,
$c = 2.5 \text{ kJ/kg K}$
$h = ?$, $T_2 = 225^\circ \text{C} = 273 + 225 = 498 \text{ K}$

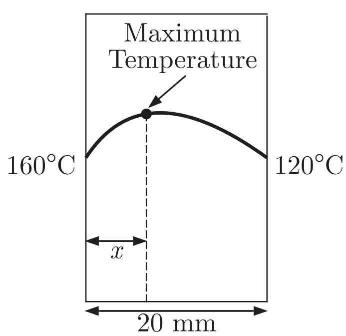
Temperature Gradient, $\frac{dT}{dt} = -0.02 \text{ K/s}$

Here negative sign shows that plate temperature decreases with the time.
From the given condition,
Heat transfer by convection to the plate = Rate of change of internal energy

$$hA(T_2 - T_1) = -mc\frac{dT}{dt}$$

$$h = -\frac{mc}{A(T_2 - T_1)} \times \frac{dT}{dt} = -\frac{4 \times 2.5 \times 10^3}{0.1(498 - 298)} \times (-0.02) = 10 \text{ W/m}^2 \text{ K}$$

SOL 7.23 Option (C) is correct.



Let the location of maximum temperature occurs at the distance x from the left face. We know that steady state heat flow equation in one dimension with a uniform heat generation is given by,

$$\frac{\partial^2 T}{\partial x^2} + \frac{q_g}{k} = 0 \qquad \text{...(i)}$$

Here q_g = Heat generated per unit volume and per unit time,

Given : $q_g = 80\,\text{MW/m}^2 = 80 \times 10^6\ \text{W/m}^2$, $k = 200\ \text{W/m K}$

Substituting the value of q_g and k in equation (i), we get

$$\frac{\partial^2 T}{\partial x^2} + \frac{80 \times 10^6}{200} = 0$$

$$\frac{\partial^2 T}{\partial x^2} + 4 \times 10^5 = 0$$

Integrating the above equation,

$$\frac{\partial T}{\partial x} + 4 \times 10^5 \times x + c_1 = 0 \qquad \text{...(ii)}$$

Again integrating, we get

$$T + 4 \times 10^5 \times \frac{x^2}{2} + c_1 x + c_2 = 0 \qquad \text{...(iii)}$$

Applying boundary conditions on equation (iii), we get

(1) At $x = 0$, $T = 160^\circ\,\text{C}$

$$160 + c_2 = 0$$

$$c_2 = -160 \qquad \text{...(iv)}$$

(2) At $x = 20\,\text{mm} = 0.020\,\text{m}$, $T = 120^\circ\,\text{C}$

$$120 + 4 \times 10^5 \times \frac{(0.020)^2}{2} + c_1 \times 0.020 + (-160) = 0 \qquad c_2 = -160$$

$$120 + 80 + 0.020c_1 - 160 = 0$$

$$0.020c_1 + 40 = 0$$

$$c_1 = -\frac{40}{0.020} = -2000 \qquad \text{...(v)}$$

To obtain the location of maximum temperature, applying maxima-minima principle and put $\frac{dT}{dx} = 0$ in equation (ii), we get

$$0 + 4 \times 10^5 x + (-2000) = 0 \qquad c_1 = -2000$$

$$x = \frac{2000}{4 \times 10^5} = 500 \times 10^{-5} = 5 \times 10^{-3}\,\text{m} = 5\,\text{mm}$$

SOL 7.24 Option (B) is correct.

From the previous part of the question, at $x = 5\,\text{mm}$ temperature is maximum.

So, put $x = 5\,\text{mm} = 5 \times 10^{-3}\,\text{m}$ in equation(iii), we get

$$T + 4 \times 10^5 \times \frac{(5 \times 10^{-3})^2}{2} + (-2000) \times 5 \times 10^{-3} + (-160) = 0$$

$$T + 5 \times 10^6 \times 10^{-6} - 10 - 160 = 0$$

$$T + 5 - 170 = 0 \quad \Rightarrow \quad T = 165^\circ \text{C}$$

SOL 7.25 Option (D) is correct.

Given : $T_{\text{inter}} = \frac{T_1 + T_2}{2}$

Heat transfer will be same for both the ends

So, $$Q = -\frac{k_1 A_1 (T_1 - T_{inter})}{2b} = -\frac{k_2 A_2 (T_{inter} - T_2)}{b} \quad Q = -kA\frac{dT}{dx}$$

There is no variation in the horizontal direction. Therefore, we consider portion of equal depth and height of the slab, since it is representative of the entire wall.

So, $$A_1 = A_2 \text{ and } T_{\text{inter}} = \frac{T_1 + T_2}{2}$$

So, we get $$\frac{k_1\left[T_1 - \left(\frac{T_1 + T_2}{2}\right)\right]}{2} = k_2\left[\frac{T_1 + T_2}{2} - T_2\right]$$

$$k_1\left[\frac{2T_1 - T_1 - T_2}{2}\right] = 2k_2\left[\frac{T_1 + T_2 - 2T_2}{2}\right]$$

$$\frac{k_1}{2}[T_1 - T_2] = k_2[T_1 - T_2]$$

$$k_1 = 2k_2$$

SOL 7.26 Option (D) is correct.

Given : $P = 100$ W, $\nu = 2.5 \times 3 \times 3 = 22.5$ m^3, $T_i = 20^\circ$ C

Now Heat generated by the bulb in 24 hours,

$$Q = 100 \times 24 \times 60 \times 60 = 8.64 \text{ MJ} \quad \text{...(i)}$$

Volume of the room remains constant.

Heat dissipated, $Q = mc_v dT = \rho\nu c_v (T_f - T_i)$ $\qquad m = \rho v$

Where, T_f = Final temperature of room

ρ = Density of air = 1.2 kg/m^3

c_v of air = 0.717 kJ/kg K

Substitute the value of Q from equation (i), we get

$$8640000 = 1.2 \times 22.5 \times 0.717 \times 10^3 (T_f - 20)$$

$$8640 = 1.2 \times 22.5 \times 0.717 (T_f - 20)$$

$$(T_f - 20) = 446.30$$

$$T_f = 446.30 + 20 = 466.30^\circ \text{C} \simeq 470^\circ \text{C}$$

SOL 7.27 Option (C) is correct.

Given : Relation humidity= 5% at temperature 20° C

Relative humidity,

$$\phi = \frac{\text{Actual mass of water vapour in a given volume of moist air}}{\text{mass of water vapour in the same volume of saturated air at same temperature \& pressure}}$$

$$\phi = \frac{m_v}{m_s} = \frac{p_v}{p_s} = 0.05 \qquad ...(i)$$

Where, $p_v =$ Partial pressure of vapor at 20° C

From given table at $T = 20^\circ$C, $p_s = 2.34$ kPa

From equation (i),

$$p_v = 0.05 \times p_s = 0.05 \times 2.34 = 0.117 \text{ kPa}$$

Phase equilibrium means, $p_s = p_v$

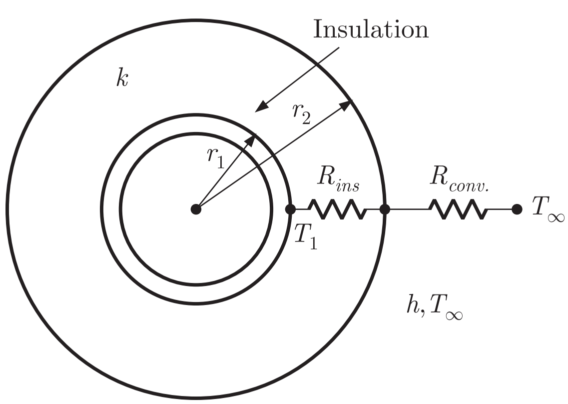
The temperature at which p_v becomes saturated pressure can be found by interpolation of values from table, for $p_s = 0.10$ to $p_s = 0.26$

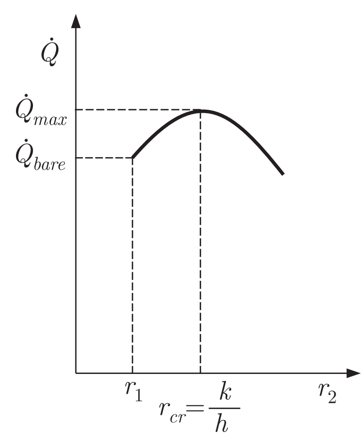
$$T = -15 + \left[\frac{-10-(-15)}{0.26-0.10}\right](0.117-0.10)$$

$$= -15 + \frac{5}{0.16} \times 0.017 = -14.47 \simeq -14.5^\circ \text{ C}$$

SOL 7.28 Option (B) is correct.

The variation of heat transfer with the outer radius of the insulation r_2, when $r_1 < r_{cr}$





The rate of heat transfer from the insulated pipe to the surrounding air can be expressed as

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv.}} = \frac{T_1 - T_\infty}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi Lk} + \frac{1}{h(2\pi r_2 L)}}$$

The value of r_2 at which $\dot{Q}$ reaches a maximum is determined from the requirement that $\frac{d\dot{Q}}{dr_2} = 0$. By solving this we get,

$$r_{cr,pipe} = \frac{k}{h} \qquad ...(i)$$

From equation (i), we easily see that by increasing the thickness of insulation, the value of thermal conductivity increases and heat loss by the conduction also increases.
But by increasing the thickness of insulation, the convection heat transfer co-efficient decreases and heat loss by the convection also decreases. These both cases are limited for the critical thickness of insulation.

SOL 7.29 Option (D) is correct.
The general heat equation in cartesian co-ordinates,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

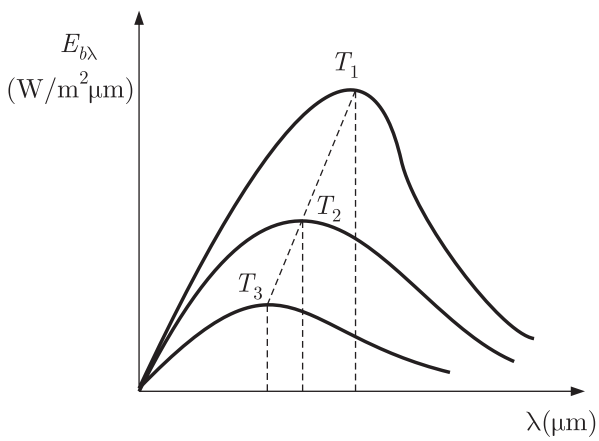
For one dimensional heat conduction,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\rho c_p}{k}\frac{\partial T}{\partial t} \qquad \alpha = \frac{k}{\rho c_p} = \text{Thermal Diffusitivity}$$

For constant properties of medium,

$$\frac{\partial T}{\partial t} \propto \frac{\partial^2 T}{\partial x^2}$$

SOL 7.30 Option (D) is correct.



Given : $T_1 > T_2 > T_3$
From, Wien's displacement law,

$$\lambda_{max} T = 0.0029 \text{ mK} = \text{Constant}$$

$$\lambda_{max} \propto \frac{1}{T}$$

If T increase, then λ_m decrease. But according the figure, when T increases, then λ_m also increases. So, the Wien's law is not satisfied.

SOL 7.31 Option (C) is correct.
Assumptions :

(1) Heat transfer is steady since there is no indication of change with time.

(2) Heat transfer can be approximated as being one-dimensional since it is predominantly in the x-direction.

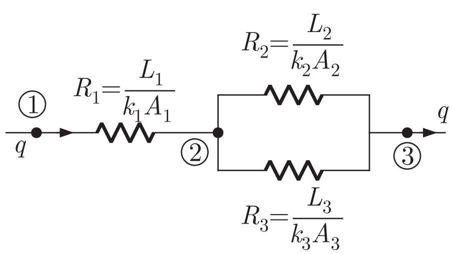
(3) Thermal conductivities are constant.

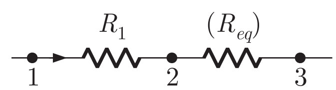
(4) Heat transfer by radiation is negligible.

Analysis :

There is no variation in the horizontal direction. Therefore, we consider a 1 m deep and 1 m high portion of the slab, since it representative of the entire wall.

Assuming any cross-section of the slab normal to the $x-$ direction to be isothermal, the thermal resistance network for the slab is shown in the figure.





$$R_1 = \frac{L_1}{k_1A_1} = \frac{0.5}{0.02(1\times 1)} = 25\text{ K/W}$$

$$R_2 = \frac{L_2}{k_2A_2} = \frac{0.25}{0.10\times(1\times 0.5)} = 5\text{ K/W}$$

$$R_3 = \frac{L_3}{k_3A_3} = \frac{0.25}{0.04\times(1\times 0.5)} = 12.5\text{ K/W}$$

Resistance R_2 and R_3 are in parallel. So the equivalent resistance R_{eq} will be

$$\frac{1}{R_{eq}} = \frac{1}{R_2}+\frac{1}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{R_3+R_2}{R_2R_3}$$

$$R_{eq} = \frac{R_2R_3}{R_2+R_3} = \frac{5\times 12.5}{5+12.5} = 3.6\text{ K/W}$$

Resistance R_1 and R_{eq} are in series. So total Resistance will be

$$R = R_1+R_{eq} = 25+3.6 = 28.6\text{ K/W}$$

SOL 7.32 Option (C) is correct.

Given : $D = 5\text{ mm} = 0.005\text{ m}$, $T_i = 500\text{ K}$, $T_a = 300\text{ K}$, $k = 400\text{ W/mK}$, $\rho = 9000\text{ kg/m}^3$, $c = 385\text{ J/kg K}$, $h = 250\text{ W/m}^2\text{K}$,

Given that lumped analysis is assumed to be valid.

So, $$\frac{T-T_a}{T_i-T_a} = \exp\left(-\frac{hAt}{\rho\nu c}\right) = \exp\left(-\frac{ht}{\rho lc}\right) \qquad \text{...(i)}$$

$$l = \frac{\nu}{A} = \frac{\text{Volume of ball}}{\text{Surface Area}} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} \qquad l = \frac{\nu}{A}$$

$$l = \frac{R}{3} = \frac{D}{6} = \frac{0.005}{6} = \frac{1}{1200} \text{ m}$$

On substituting the value of l and other parameters in equation. (i),

$$\frac{T-300}{500-300} = \exp\left(-\frac{250 \times t}{9000 \times \frac{1}{1200} 385}\right)$$

$$T = 300 + 200 \times e^{-0.08658t}$$

On differentiating the above equation w.r.t. t,

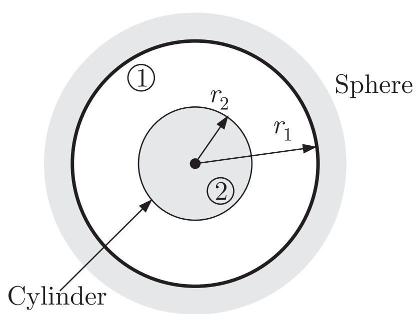
$$\frac{dT}{dt} = 200 \times (-0.08658) \times e^{-0.08658t}$$

Rate of fall of temperature of the ball at the beginning of cooling is (at beginning $t = 0$)

$$\left(\frac{dT}{dt}\right)_{t=0} = 200 \times (-0.08658) \times 1 = -17.316 \text{ K/sec}$$

Negative sign shows fall of temperature.

SOL 7.33 Option (C) is correct.



Given : $d_1 = 1\,\text{m}$, $d_2 = 0.5\,\text{m}$, $L = 0.5\,\text{m}$

The cylinder surface cannot see itself and the radiation emitted by this surface falls on the enclosing sphere. So, from the conservation principle (summation rule) for surface 2,

$$F_{21} + F_{22} = 1$$

$$F_{21} = 1 \qquad F_{22} = 0$$

From the reciprocity theorem,

$$A_1 F_{12} = A_2 F_{21}$$

$$F_{12} = \frac{A_2}{A_1} \times F_{21} = \frac{A_2}{A_1} \qquad \text{...(ii)}$$

For sphere, $F_{11} + F_{12} = 1$

$$F_{11} = 1 - F_{12} \qquad \text{...(iii)}$$

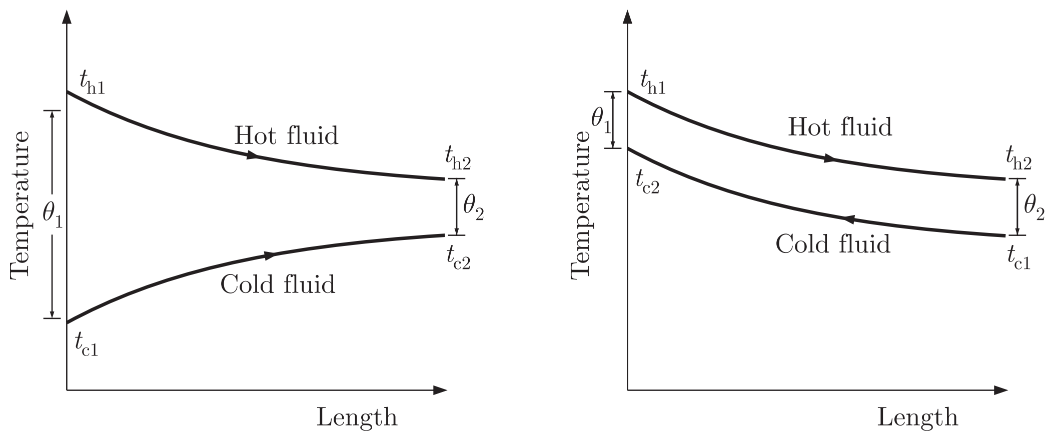
From equation (ii) and (iii), we get

$$F_{11} = 1 - \frac{A_2}{A_1} = 1 - \frac{2\pi r_2 l}{\pi d_1^2} = 1 - \frac{2r_2 l}{d_1^2}$$

$$= 1 - \frac{2 \times 0.250 \times 0.5}{1^2} = 1 - \frac{1}{4} = 0.75$$

SOL 7.34 Option (D) is correct.

The figure shown below are of parallel flow and counter flow respectively.



For parallel flow,

$t_{h1} = 80^\circ C,\ t_{h2} = 50^\circ C,\ t_{c1} = 30^\circ C,\ t_{c2} = 40^\circ C$

$$\theta_{mp} = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} = \frac{(t_{h1} - t_{c1}) - (t_{h2} - t_{c2})}{\ln\left(\frac{t_{h1} - t_{c1}}{t_{h2} - t_{c2}}\right)}$$

Where, θ_{mp} denotes the LMTD for parallel flow.

$$\theta_{mp} = \frac{(80 - 30) - (50 - 40)}{\ln\left(\frac{50}{10}\right)} = \frac{40}{\ln(5)} = 24.85^\circ C$$

For counter flow arrangement

$t_{h1} = 80^\circ C,\ t_{h2} = 50^\circ C,\ t_{c1} = 40^\circ C,\ t_{c2} = 30^\circ C$

Where, θ_{mc} denotes the LMTD for counter flow.

$$\theta_{mc} = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} = \frac{(t_{h1} - t_{c2}) - (t_{h2} - t_{c1})}{\ln\left(\frac{t_{h1} - t_{c2}}{t_{h2} - t_{c1}}\right)}$$

$$= \frac{(80 - 30) - (50 - 40)}{\ln\left(\frac{50}{10}\right)} = \frac{40}{\ln(5)} = 28.85^\circ C$$

Now for defining the type of flow, we use the correction factor.

$$\theta_m = F\theta_{mc} = F\theta_{mp} \qquad ...(i)$$

Where $F =$ correction factor, which depends on the geometry of the heat exchanger and the inlet and outlet temperatures of the of the hot and cold streams.

$F < 1$, for cross flow and $F = 1$, for counter and parallel flow

So, From equation (i),

$$F = \frac{\theta_m}{\theta_{mc}} = \frac{26}{28.85} = 0.90 < 1$$

and also $$F = \frac{\theta_m}{\theta_{mp}} = \frac{26}{24.85} = 1.04 > 1$$

So, cross flow in better for this problem.

SOL 7.35 Option (C) is correct.

Given : A duct of rectangular cross section. For which sides are $a = 1\text{ m}$ and $b = 0.5\text{ m}$

$T_1 = 30^\circ\text{C}$, $T_2 = 20^\circ\text{C}$, $V = 10\text{ m/sec}$, $k = 0.025\text{ W/m K}$

Viscosity $= 18\,\mu\text{Pas}$, $\text{Pr} = 0.73$, $\rho = 1.2\text{ kg/m}^3$, $\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.33}$

Hence, For a rectangular conduit of sides a and b,

Hydraulic diameter, $$D_H = \frac{4A}{p}$$

Where, A is the flow cross sectional area and p the wetted perimeter

$$D_H = \frac{4ab}{2(a+b)} = \frac{2ab}{(a+b)}$$

$$= \frac{2 \times 1 \times 0.5}{(1+0.5)} = \frac{1}{1.5} = 0.666\text{ m}$$

Reynolds Number, $$\text{Re} = \frac{\rho V D_H}{\mu}$$

$$= \frac{1.2 \times 10 \times 0.666}{18 \times 10^{-6}} = 4.44 \times 10^5$$

SOL 7.36 Option (D) is correct.

From the first part of the question,

$$\text{Re} = 4.44 \times 10^5$$

Which is greater than 3×10^5. So, flow is turbulent flow.

Therefore, $$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.33}$$

$$\frac{hL}{k} = 0.023(4.44 \times 10^5)^{0.8} \times (0.73)^{0.33} \qquad \text{Nu} = \frac{hL}{k}$$

$$= 0.023 \times 32954 \times 0.9013 = 683.133$$

$$h = 683.133 \times \frac{k}{L}$$

$$= 683.133 \times \frac{0.025}{0.666} = 25.64\text{ W/m}^2\text{ K}$$

$D_H = L = 0.666\text{ m}$

Total Area, $$A = 2(a+b)L = 2(1+0.5)L = 3L$$

Heat transfer by convection is given by,

$$Q = hA(T_1 - T_2)$$

$$= 25.64 \times 3L \times [(273 + 30) - (273 + 20)]$$

Heat transfer per meter length of the duct is given by

$$\frac{Q}{L} = 25.64 \times 3 \times 10 = 769.2 \text{ W} \simeq 769 \text{ W}$$

SOL 7.37 Option (B) is correct.

The one dimensional time dependent heat conduction equation can be written more compactly as a simple equation,

$$\frac{1}{r^n}\frac{\partial}{\partial r}\left[r^n \frac{\partial T}{\partial r}\right] + \frac{q}{k} = \frac{\rho c}{k}\frac{\partial T}{\partial t} \qquad \text{...(i)}$$

Where, $n = 0$, For rectangular coordinates

$n = 1$, For cylindrical coordinates

$n = 2$, For spherical coordinates

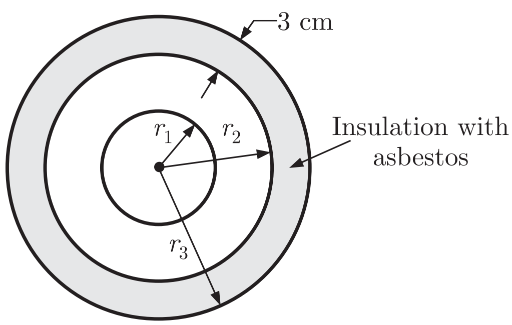
Further, while using rectangular coordinates it is customary to replace the r-variable by the x-variable.

For sphere, substitute $r = 2$ in equation (i)

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial T}{\partial r}\right] + \frac{q}{k} = \frac{\rho c}{k}\frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial T}{\partial r}\right] + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \alpha = \frac{k}{\rho c} = \text{thermal diffusivity}$$

SOL 7.38 Option (C) is correct.



Let Length of the tube $= l$

Given : $r_1 = \frac{d_1}{2} = 2/2\,\text{cm} = 1\,\text{cm}$, $r_2 = \frac{5}{2}\,\text{cm} = 2.5\,\text{cm}$

Radius of asbestos surface, $r_3 = \frac{d_2}{2} + 3 = 2.5 + 3 = 5.5\,\text{cm}$

$k_s = 19$ W/mK, $k_a = 0.2$ W/mK

And $T_1 - T_2 = 600^\circ$ C

From the given diagram heat is transferred from r_1 to r_2 and from r_2 to r_3. So Equivalent thermal resistance,

$$\Sigma R = \frac{1}{2\pi k_s l}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi k_a l}\ln\left(\frac{r_3}{r_2}\right) \qquad \text{For hollow cylinder } R_t = \frac{\log_e(r_2/r_1)}{2\pi kl}$$

$$\Sigma R \times l = \frac{1}{2\pi k_s}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi k_a}\ln\left(\frac{r_3}{r_2}\right)$$

$$= \frac{1}{2 \times 3.14 \times 19}\ln\left(\frac{2.5}{1}\right) + \frac{1}{2 \times 3.14 \times 0.2}\ln\left(\frac{5.5}{2.5}\right)$$

$$= \frac{0.916}{119.32} + \frac{0.788}{1.256} = 0.00767 + 0.627$$

$$= 0.635 \text{ mK/W} \qquad \text{...(i)}$$

Heat transfer per unit length,

$$Q = \frac{T_1 - T_2}{(\Sigma R \times l)} = \frac{600}{0.635} = 944.88 \simeq 944.72 \text{ W/m}$$

SOL 7.39 Option (B) is correct.

Given : $h = 400 \text{ W/m}^2\text{K}$, $k = 20 \text{ W/mK}$, $c = 400 \text{ J/kg K}$, $\rho = 8500 \text{ kg/m}^3$
$T_i = 30^\circ \text{C}$, $D = 0.706 \text{ mm}$, $T_a = 300^\circ \text{C}$, $T = 298^\circ \text{C}$

Biot Number, $B_i = \frac{hl}{k}$ ..(i)

And $l = \frac{\text{Volume}}{\text{Surface Area}} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{\frac{1}{6}\pi D^3}{\pi D^2}$

$$= \frac{D}{6} = \frac{0.706 \times 10^{-3}}{6} = 1.176 \times 10^{-4} \text{ m}$$

From equation (i), we have

$$Bi = \frac{hl}{k} = \frac{400 \times 1.176 \times 10^{-4}}{20} = 0.0023$$

$$Bi < 0.1$$

The value of Biot Number is less than one. So the lumped parameter solution for transient conduction can be conveniently stated as

$$\frac{T - T_a}{T_i - T_a} = e^{-\left(\frac{hAt}{\rho c \nu}\right)} = e^{-\left(\frac{ht}{\rho c l}\right)} \qquad \frac{\nu}{A} = l$$

$$\frac{298 - 300}{30 - 300} = \exp\left(\frac{-400t}{8500 \times 400 \times 1.176 \times 10^{-4}}\right)$$

$$\frac{-2}{-270} = e^{-t}$$

$$\frac{2}{270} = e^{-t}$$

Take natural logarithm both sides, we get

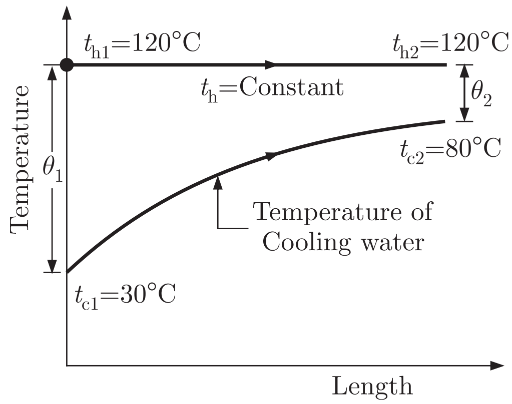
$$\ln\left(\frac{2}{270}\right) = -t \quad \rightarrow \quad t = 4.90 \text{ sec}$$

SOL 7.40 Option (A) is correct.

Given : $t_{c1} = 30^\circ\text{C}$, $\frac{dm}{dt} = \dot{m} = 1500 \text{ kg/hr} = \frac{1500}{3600} \text{ kg/sec} = 0.4167 \text{ kg/sec}$

$t_{h2} = t_{h1} = 120^\circ \text{C}$, $t_{c2} t_{c2} = 80^\circ\text{C}$, $c_w = 4.187 \text{ kJ/kg K}$, $U = 2000 \text{ W/m}^2\text{K}$.

Figure for condensation is given below :



Hence, $\theta_1 = t_{h1} - t_{c1} = 120 - 30 = 90^\circ \text{C}$

And $\theta_2 = t_{h2} - t_{c2} = 120 - 80 = 40^\circ \text{C}$

So, Log mean temperature difference (LMTD) is,

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} = \frac{90 - 40}{\ln\left(\frac{90}{40}\right)} = 61.66^\circ \text{C}$$

Energy transferred is given by,

$$Q = \dot{m}c_w \Delta T = UA\theta_m$$

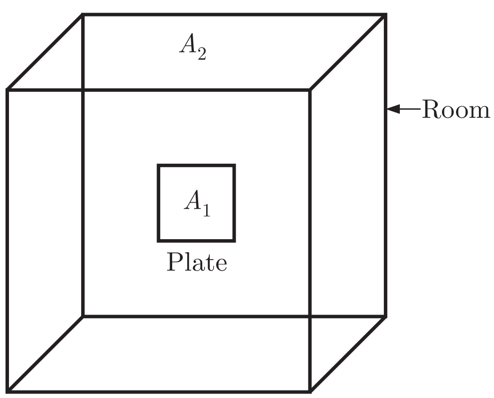
$$A = \frac{\dot{m}c_w \Delta T}{U\theta_m} = \frac{0.4167 \times 4.187 \times 1000 \times 50}{2000 \times 61.66} = 0.707 \text{ m}^2$$

SOL 7.41 Option (B) is correct.

Given, for plate :

$A_1 = 10 \text{ cm}^2 = 10 \times (10^{-2})^2 \text{ m}^2 = 10^{-3} \text{ m}^2,\ T_1 = 800 \text{ K},\ \varepsilon_1 = 0.6$

For Room : $A_2 = 100 \text{ m}^2, T_2 = 300 \text{ K}, \varepsilon_2 = 0.3$ and $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$



Total heat loss from one surface of the plate is given by,

$$(Q_{12}) = \frac{E_{b1} - E_{b2}}{\frac{(1-\varepsilon_1)}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{(1-\varepsilon_2)}{A_2\varepsilon_2}}$$

If small body is enclosed by a large enclosure, then $F_{12} = 1$ and from Stefan's

Boltzman law $E_b = \sigma T^4$. So we get

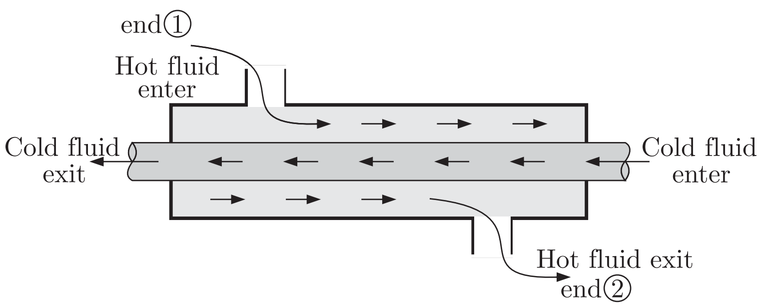
$$(Q_{12}) = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}} = \frac{5.67 \times 10^{-8}[(800)^4 - (300)^4]}{\frac{1-0.6}{10^{-3}\times 0.6} + \frac{1}{10^{-3}} + \frac{1-0.3}{100\times 0.3}}$$

$$= \frac{22.765 \times 10^3}{666.66 + 1000 + 0.0233} = 13.66 \text{ W}$$

Q_{12} is the heat loss by one surface of the plate. So, heat loss from the two surfaces is given by,

$$Q_{net} = 2 \times Q_{12} = 2 \times 13.66 = 27.32 \text{ W}$$

SOL 7.42 Option (B) is correct.



In counter flow, hot fluid enters at the point 1 and exits at the point 2 or cold fluid enter at the point 2 and exit at the point 1.

Given : for hot fluid,

$c_h = 2 \text{ kJ/kg K}, \ m_h = 5 \text{ kg/sec}, \ t_{h1} = 150^\circ \text{C}, \ t_{h2} = 100^\circ \text{C}$

and for cold fluid,

$c_c = 4 \text{ kJ/kg K}, \ m_c = 10 \text{ kg/sec}, \ t_{c2} = 20^\circ \text{C}, \ t_{c1} = ?$

From the energy balance,

Heat transferred by the hot fluid = Heat gain by the cold fluid

$$\dot{m}_h c_h (t_{h1} - t_{h2}) = \dot{m}_c c_c (t_{c1} - t_{c2})$$

$$5 \times 2 \times 10^3 (150 - 100) = 10 \times 4 \times 10^3 (t_{c1} - 20)$$

$$10^4 \times 50 = 4 \times 10^4 (t_{c1} - 20)$$

$$t_{c1} = \frac{130}{4} = 32.5^\circ \text{C}$$

Hence, outlet temperature of the cold fluid,

$$t_{c1} = 32.5^\circ \text{C}$$

SOL 7.43 Option (A) is correct.

The non-dimensional Prandtl Number for thermal boundary layer is,

$$\frac{\delta_v}{\delta_T} = (\text{Pr})^{1/3}$$

(i) When $\text{Pr} = 1$ $\quad \delta_v = \delta_T$

(ii) When $\Pr > 1$ $\delta_v > \delta_T$

(iii) When $\Pr < 1$ $\delta_v < \delta_T$

So for $\Pr > 1$, $\delta_v > \delta_T$

SOL 7.44 Option (C) is correct.

Given for water : $T_w = 48^\circ\,\text{C}$, $k_w = 0.6\,\text{W/mK}$

And for glass : $T_g = 40^\circ\,\text{C}$, $k_g = 1.2\,\text{W/mK}$

Spatial gradient $\left(\dfrac{dT}{dy}\right)_w = 1 \times 10^4\,\text{K/m}$

Heat transfer takes place between the water and glass interface by the conduction and convection. Heat flux would be same for water and glass interface. So, applying the conduction equation for water and glass interface.

$$k_w\left(\frac{dT}{dy}\right)_w = k_g\left(\frac{dT}{dy}\right)_g \qquad q = \frac{Q}{A} = \frac{-kA\frac{dT}{dx}}{A} = -k\frac{dT}{dx}$$

$$\left(\frac{dT}{dy}\right)_g = \frac{k_w}{k_g}\left(\frac{dT}{dy}\right)_w = \frac{0.6}{1.2} \times 10^4 = 0.5 \times 10^4\,\text{K/m}$$

SOL 7.45 Option (D) is correct.

From the equation of convection,

Heat flux, $q = h[T_w - T_g]$ ...(i)

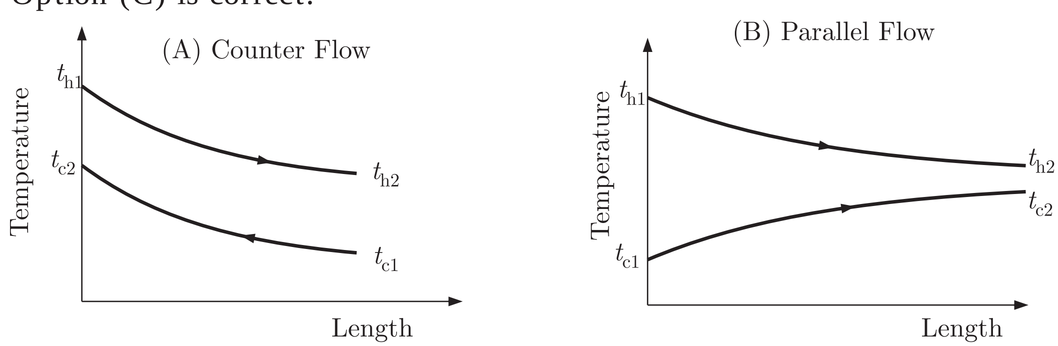
Where, $h =$ Heat transfer coefficient

First find q, $q = k_w\left(\dfrac{dT}{dy}\right)_w = k_g\left(\dfrac{dT}{dy}\right)_g = 0.6 \times 10^4 = 6000\,\text{W/m}^2$

Now from equation (i),

$$h = \frac{q}{T_w - T_g} = \frac{6000}{48 - 40} = \frac{6000}{8} = 750\,\text{W/m}^2\,\text{K}$$

SOL 7.46 Option (C) is correct.



Given : (A) For counter flow $t_{h1} = t_{C1}$, $t_{h2} = t_{C2}$

$$LMTD,\ \theta_{mc} = \frac{\theta_1 - \theta_2}{\ln\frac{\theta_1}{\theta_2}}$$

$$\theta_{mc} = \frac{(t_{h1} - t_{C2}) - (t_{h2} - t_{C1})}{\ln\left[\frac{t_{h1} - t_{C2}}{t_{h2} - t_{C1}}\right]} = \frac{(t_{h1} - t_{h2}) - (t_{h2} - t_{h1})}{\ln\left[\frac{t_{h1} - t_{h2}}{t_{h2} - t_{h1}}\right]} = \frac{2(t_{h1} - t_{h2})}{\ln\left[\frac{t_{h1} - t_{h2}}{t_{h2} - t_{h1}}\right]} \quad ...(i)$$

(B) For parallel flow given : $t_{h1} = t_{C2}$, $t_{h2} = t_{C1}$

$$LMTD,\ \theta_{mp} = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)}$$

$$\theta_{mp} = \frac{(t_{h1} - t_{C1}) - (t_{h2} - t_{C2})}{\ln\left[\frac{t_{h1} - t_{C1}}{t_{h2} - t_{C2}}\right]} = \frac{(t_{h1} - t_{h2}) - (t_{h2} - t_{h1})}{\ln\left[\frac{t_{h1} - t_{h2}}{t_{h2} - t_{h1}}\right]} = \frac{2(t_{h1} - t_{h2})}{\ln\left[\frac{t_{h1} - t_{h2}}{t_{h2} - t_{h1}}\right]} \quad ...(ii)$$

From equation (i) and (ii), we get

$$\theta_{mc} = \theta_{mp}$$

SOL 7.47 Option (D) is correct.

Given : $F_{13} = 0.17$

Applying summation rule :

$$F_{11} + F_{12} + F_{13} = 1$$

The flat surface cannot see itself.

So, $F_{11} = 0$

This gives, $F_{12} = 1 - F_{11} - F_{13} = 1 - 0 - 0.17 = 0.83$

SOL 7.48 Option (C) is correct.

S. No.	Materials	Thermal Conductivity (W/m – K)
1.	Aluminum	237
2.	Pure Iron	80.2
3.	Liquid Water	0.607
4.	Saturated Water Vapour	0.026

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CHAPTER 8

THERMODYNAMICS

YEAR 2012 ONE MARK

MCQ 8.1 Steam enters an adiabatic turbine operating at steady state with an enthalpy of $3251.0\,\text{kJ/kg}$ and leaves as a saturated mixture at $15\,\text{kPa}$ with quality (dryness fraction) 0.9. The enthalpies of the saturated liquid and vapour at $15\,\text{kPa}$ are $h_f = 225.94\,\text{kJ/kg}$ and $h_g = 2598.3\,\text{kJ/kg}$ respectively. The mass flow rate of steam is $10\,\text{kg/s}$. Kinetic and potential energy changes are negligible. The power output of the turbine in MW is

(A) 6.5 (B) 8.9

(C) 9.1 (D) 27.0

MCQ 8.2 A ideal gas of mass m and temperature T_1 undergoes a reversible isothermal process from an initial pressure p_1 to final pressure p_2. The heat loss during the process is Q. The entropy change Δs of the gas is

(A) $mR\ln\left(\frac{p_2}{p_1}\right)$ (B) $mR\ln\left(\frac{p_1}{p_2}\right)$

(C) $mR\ln\left(\frac{p_2}{p_1}\right) - \frac{Q}{T_1}$ (D) zero

YEAR 2012 TWO MARKS

- Common Data For Q.3 and Q.4

Air enters an adiabatic nozzle at $300\,\text{kPa}$, $500\,\text{K}$ with a velocity of $10\,\text{m/s}$. It leaves the nozzle at $100\,\text{kPa}$ with a velocity of $180\,\text{m/s}$. The inlet area is $80\,\text{cm}^2$. The specific heat of air c_p is $1008\,\text{J/kgK}$.

MCQ 8.3 The exit temperature of the air is

(A) 516 K (B) 532 K

(C) 484 K (D) 468 K

MCQ 8.4 The exit area of the nozzle in cm^2 is

(A) 90.1 (B) 56.3

(C) 4.4 (D) 12.9

YEAR 2011 ONE MARK

MCQ 8.5 Heat and work are

(A) intensive properties (B) extensive properties

(B) point functions (D) path functions

MCQ 8.6 The contents of a well-insulated tank are heated by a resistor of $23\,\Omega$ in which 10 A current is flowing. Consider the tank along with its contents as a thermodynamic system. The work done by the system and the heat transfer to the system are positive. The rates of heat (Q), work (W) and change in internal energy (ΔU) during the process in kW are

(A) $Q = 0, W = -2.3, \Delta U = +2.3$

(B) $Q = +2.3, W = 0, \Delta U + 2.3$

(C) $Q = -2.3, W = 0, \Delta U = -2.3$

(D) $Q = 0, W = +2.3, \Delta U = -2.3$

YEAR 2011 TWO MARKS

MCQ 8.7 The values of enthalpy of steam at the inlet and outlet of a steam turbine in a Rankine cycle are $2800\,\text{kJ/kg}$ and $1800\,\text{kJ/kg}$ respectively. Neglecting pump work, the specific steam consumption in kg/kW hour is

(A) 3.60 (B) 0.36

(C) 0.06 (D) 0.01

MCQ 8.8 The crank radius of a single-cylinder I.C. engine is 60 mm and the diameter of the cylinder is 80 mm. The swept volume of the cylinder in cm^3 is

(A) 48 (B) 96

(C) 302 (D) 603

MCQ 8.9 An ideal Brayton cycle, operating between the pressure limits of 1 bar and 6 bar, has minimum and maximum temperature of 300 K and 1500 K. The ratio of specific heats of the working fluid is 1.4. The approximate final temperatures in Kelvin at the end of compression and expansion processes are respectively

(A) 500 and 900 (B) 900 and 500

(C) 500 and 500 (D) 900 and 900

• Common Data For Q.10 and Q.11

In an experimental set up, air flows between two stations P and Q adiabatically. The direction of flow depends on the pressure and temperature conditions maintained at P and Q. The conditions at station P are 150 kPa and 350 K . The temperature at station Q is 300 K.

The following are the properties and relations pertaining to air :

Specific heat at constant pressure,	$c_p = 1.005$ kJ/kgK;
Specific heat at constant volume,	$c_v = 0.718$ kJ/kgK;
Characteristic gas constant,	$R = 0.287$ kJ/kgK
Enthalpy,	$h = c_p T$
Internal energy,	$u = c_v T$

MCQ 8.10 If the air has to flow from station P to station Q, the maximum possible value of pressure in kPa at station Q is close to

(A) 50 (B) 87

(C) 128 (D) 150

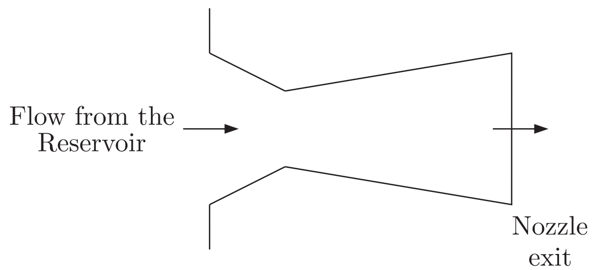
MCQ 8.11 If the pressure at station Q is 50 kPa, the change in entropy $(s_Q - s_P)$ in kJ/kgK is

(A) -0.155 (B) 0

(C) 0.160 (D) 0.355

• Common Data For Q.12 and Q.13

The temperature and pressure of air in a large reservoir are 400 K and 3 bar respectively. A converging diverging nozzle of exit area 0.005 m^2 is fitted to the wall of the reservoir as shown in the figure. The static pressure of air at the exit section for isentropic flow through the nozzle is 50 kPa . The characteristic gas constant and the ratio of specific heats of air are 0.287 kJ/kgK and 1.4 respectively.



MCQ 8.12 The density of air in kg/m^3 at the nozzle exit is

(A) 0.560 (B) 0.600

(C) 0.727 (D) 0.800

MCQ 8.13 The mass flow rate of air through the nozzle in kg/s is

(A) 1.30 (B) 1.77

(C) 1.85 (D) 2.06

YEAR 2010 ONE MARK

MCQ 8.14 A turbo-charged four-stroke direct injection diesel engine has a displacement volume of $0.0259\,m^3$ (25.9 litres). The engine has an output of $950\,kW$ at $2200\,rpm$. The mean effective pressure (in MPa) is closest to

(A) 2 (B) 1

(C) 0.2 (D) 0.1

MCQ 8.15 One kilogram of water at room temperature is brought into contact with a high temperature thermal reservoir. The entropy change of the universe is

(A) equal to entropy change of the reservoir

(B) equal to entropy change of water

(C) equal to zero

(D) always positive

YEAR 2010 TWO MARKS

MCQ 8.16 A mono-atomic ideal gas $(\gamma = 1.67, \text{molecular weight} = 40)$ is compressed adiabatically from $0.1\,MPa$, $300\,K$ to $0.2\,MPa$. The universal gas constant is $8.314\,kJ\,kg^{-1}mol^{-1}K^{-1}$. The work of compression of the gas (in $kJkg^{-1}$) is

(A) 29.7 (B) 19.9

(C) 13.3 (D) 0

MCQ 8.17 Consider the following two processes ;

(a) A heat source at 1200 K loses 2500 kJ of heat to a sink at 800 K

(b) A heat source at 800 K loses 2000 kJ of heat to a sink at 500 K

Which of the following statements is true ?

(A) Process I is more irreversible than Process II

(B) Process II is more irreversible than Process I

(C) Irreversibility associated in both the processes are equal

(D) Both the processes are reversible

• Common Data For Q.18 and Q.19

In a steam power plant operating on the Rankine cycle, steam enters the turbine at $4\,MPa$, $350^\circ C$ and exists at a pressure of $15\,kPa$. Then it enters the condenser and exits as saturated water. Next, a pump feeds back the water to the boiler. The adiabatic efficiency of the turbine is 90%. The thermodynamic states of water and steam are given in table.

State	$h(\text{kJkg}^{-1})$		$s(\text{kJkg}^{-1}\text{K}^{-1})$		$\nu(\text{m}^3\text{kg}^{-1})$	
Steam : 4 MPa, 350°C	3092.5		6.5821		0.06645	
Water : 15 kPa	h_f	h_g	s_f	s_g	ν_f	ν_g
	225.94	2599.1	0.7549	8.0085	0.001014	10.02

h is specific enthalpy, s is specific entropy and ν the specific volume; subscripts f and g denote saturated liquid state and saturated vapor state.

MCQ 8.18 The net work output (kJkg^{-1}) of the cycle is

(A) 498 (B) 775

(C) 860 (D) 957

MCQ 8.19 Heat supplied (kJkg^{-1}) to the cycle is

(A) 2372 (B) 2576

(C) 2863 (D) 3092

YEAR 2009 ONE MARK

MCQ 8.20 If a closed system is undergoing an irreversible process, the entropy of the system

(A) must increase

(B) always remains constant

(C) Must decrease

(D) can increase, decrease or remain constant

MCQ 8.21 A frictionless piston-cylinder device contains a gas initially at 0.8 MPa and 0.015 m^3. It expands quasi-statically at constant temperature to a final volume of 0.030 m^3. The work output (in kJ) during this process will be

(A) 8.32 (B) 12.00

(C) 554.67 (D) 8320.00

YEAR 2009 TWO MARKS

MCQ 8.22 A compressor undergoes a reversible, steady flow process. The gas at inlet and outlet of the compressor is designated as state 1 and state 2 respectively. Potential and kinetic energy changes are to be ignored. The following notations are used :

ν = Specific volume and p = pressure of the gas.

The specific work required to be supplied to the compressor for this gas compression process is

(A) $\int_1^2 p d\nu$ (B) $\int_1^2 \nu dp$

(C) $\nu_1(p_2 - p_1)$ (D) $-p_2(\nu_1 - \nu_2)$

MCQ 8.23 In an air-standard Otto-cycle, the compression ratio is 10. The condition at the beginning of the compression process is 100 kPa and 27° C. Heat added at constant volume is 1500 kJ/kg, while 700 kJ/kg of heat is rejected during the other constant volume process in the cycle. Specific gas constant for air = 0.287 kJ/kgK. The mean effective pressure (in kPa) of the cycle is

(A) 103 (B) 310

(C) 515 (D) 1032

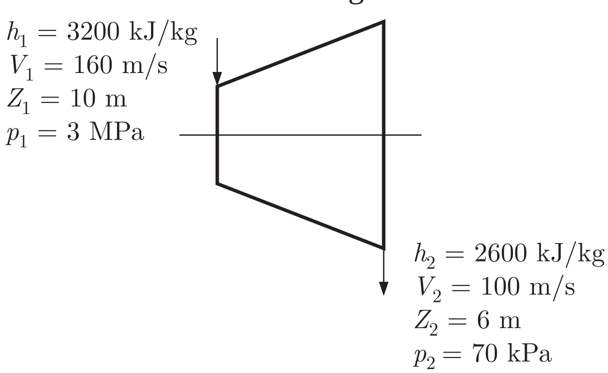
MCQ 8.24 An irreversible heat engine extracts heat from a high temperature source at a rate of 100 kW and rejects heat to a sink at a rate of 50 kW. The entire work output of the heat engine is used to drive a reversible heat pump operating between a set of independent isothermal heat reservoirs at 17°C and 75°C. The rate (in kW) at which the heat pump delivers heat to its high temperature sink is

(A) 50 (B) 250

(C) 300 (D) 360

• Common Data For Q.25 and Q.26

The inlet and the outlet conditions of steam for an adiabatic steam turbine are as indicated in the figure. The notations are as usually followed.



MCQ 8.25 If mass rate of steam through the turbine is 20 kg/s, the power output of the turbine (in MW) is

(A) 12.157 (B) 12.941

(C) 168.001 (D) 168.785

MCQ 8.26 Assume the above turbine to be part of a simple Rankine cycle. The density of water at the inlet to the pump is $1000\ kg/m^3$. Ignoring kinetic and potential energy effects, the specific work (in kJ/kg) supplied to the pump is

(A) 0.293 (B) 0.351

(C) 2.930 (D) 3.510

YEAR 2008 ONE MARK

MCQ 8.27 2 moles of oxygen are mixed adiabatically with another 2 moles of oxygen in mixing chamber, so that the final total pressure and temperature of the mixture become same as those of the individual constituents at their initial states. The universal gas constant is given as R. The change in entropy due to mixing, per mole of oxygen, is given by

(A) $-R\ln 2$ (B) 0

(C) $R\ln 2$ (D) $R\ln 4$

MCQ 8.28 Which one of the following is NOT a necessary assumption for the air-standard Otto cycle ?

(A) All processes are both internally as well as externally reversible.

(B) Intake and exhaust processes are constant volume heat rejection processes.

(C) The combustion process is a constant volume heat addition process.

(D) The working fluid is an ideal gas with constant specific heats.

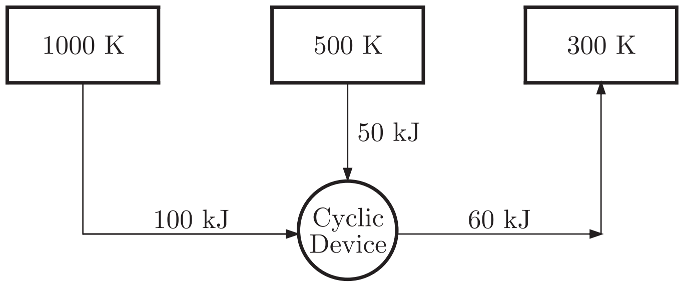
YEAR 2008 TWO MARKS

MCQ 8.29 A gas expands in a frictionless piston-cylinder arrangement. The expansion process is very slow, and is resisted by an ambient pressure of $100\ kPa$. During the expansion process, the pressure of the system (gas) remains constant at $300\ kPa$. The change in volume of the gas is $0.01\ m^3$. The maximum amount of work that could be utilized from the above process is

(A) 0 kJ (B) 1 kJ

(C) 2 kJ (D) 3 kJ

MCQ 8.30 A cyclic device operates between three reservoirs, as shown in the figure. Heat is transferred to/from the cycle device. It is assumed that heat transfer between each thermal reservoir and the cyclic device takes place across negligible temperature difference. Interactions between the cyclic device and the respective thermal reservoirs that are shown in the figure are all in the form of heat transfer.



The cyclic device can be

(A) a reversible heat engine

(B) a reversible heat pump or a reversible refrigerator

(C) an irreversible heat engine

(D) an irreversible heat pump or an irreversible refrigerator

MCQ 8.31 A balloon containing an ideal gas is initially kept in an evacuated and insulated room. The balloon ruptures and the gas fills up the entire room. Which one of the following statements is TRUE at the end of above process ?

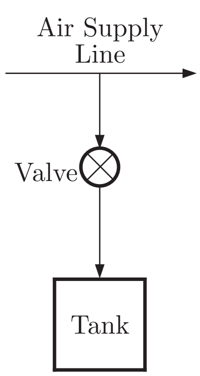
(A) The internal energy of the gas decreases from its initial value, but the enthalpy remains constant

(B) The internal energy of the gas increases from its initial value, but the enthalpy remains constant

(C) Both internal energy and enthalpy of the gas remain constant

(D) Both internal energy and enthalpy of the gas increase

MCQ 8.32 A rigid, insulated tank is initially evacuated. The tank is connected with a supply line through which air (assumed to be ideal gas with constant specific heats) passes at 1 MPa, 350°C. A valve connected with the supply line is opened and the tank is charged with air until the final pressure inside the tank reaches 1 MPa. The final temperature inside the tank.



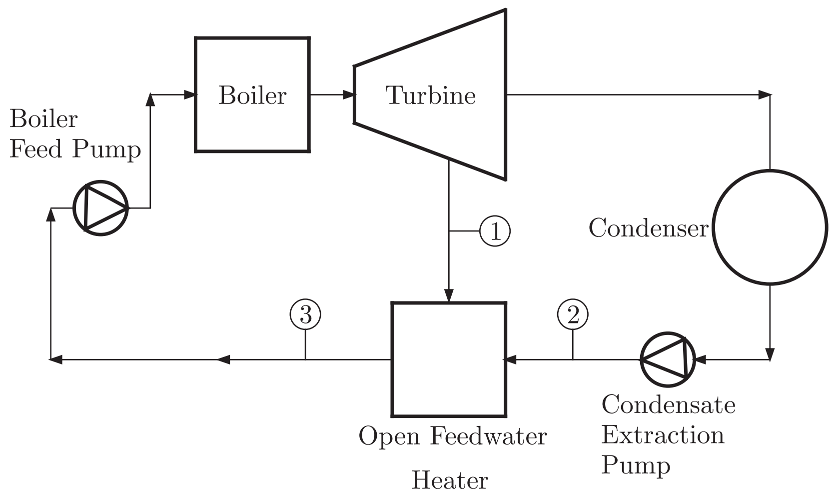
(A) is greater than 350°C

(B) is less than 350°C

(C) is equal to 350° C

(D) may be greater than, less than, or equal to, 350° C depending on the volume of the tank

MCQ 8.33 A thermal power plant operates on a regenerative cycle with a single open feed water heater, as shown in the figure. For the state points shown, the specific enthalpies are: $h_1 = 2800\,\text{kJ/kg}$ and $h_2 = 200\,\text{kJ/kg}$. The bleed to the feed water heater is 20% of the boiler steam generation rate. The specific enthalpy at state 3 is



(A) 720 kJ/kg (B) 2280 kJ/kg

(C) 1500 kJ/kg (D) 3000 kJ/kg

MCQ 8.34 In a steady state flow process taking place in a device with a single inlet and a single outlet, the work done per unit mass flow rate is given by $W = -\int_{\text{inlet}}^{\text{outlet}} \nu dp$, where ν is the specific volume and p is the pressure.
The expression for W given above

(A) is valid only if the process is both reversible and adiabatic

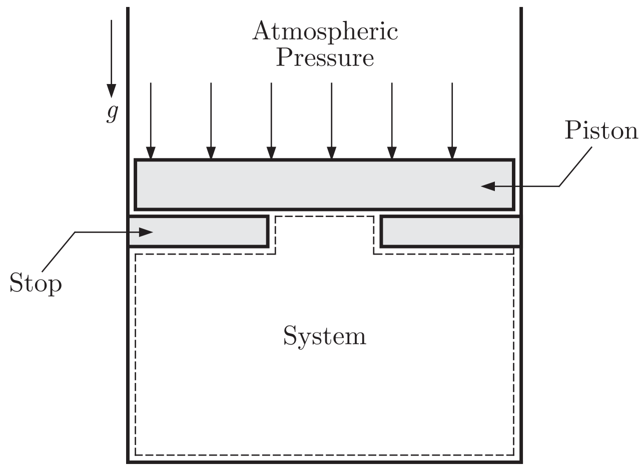
(B) is valid only if the process is both reversible and isothermal

(C) is valid for any reversible process

(D) is incorrect; it must be $W = \int_{\text{inlet}}^{\text{outlet}} p d\nu$

- Common Data For Q.35, 36 and Q.37

In the figure shown, the system is a pure substance kept in a piston-cylinder arrangement. The system is initially a two-phase mixture containing 1 kg of liquid and 0.03 kg of vapour at a pressure of 100 kPa. Initially, the piston rests on a set of stops, as shown in the figure. A pressure of 200 kPa is required to exactly balance the weight of the piston and the outside atmospheric pressure. Heat transfer takes place into the system until its volume increases by 50%. Heat transfer to the system occurs in such a manner that the piston, when allowed to move, does so in a very slow (quasi-static/quasi-equilibrium) process. The thermal reservoir from which heat is transferred to the system has a temperature of 400° C. Average temperature of the system boundary can be taken as 175° C. The heat transfer to the system is 1 kJ, during which its entropy increases by 10 J/K.



Specific volume of liquid (ν_f) and vapour (ν_g) phases, as well as values of saturation temperatures, are given in the table below.

Pressure (kPa)	Saturation temperature, T_{sat} (°C)	ν_f (m^3/kg)	ν_g (m^3/kg)
100	100	0.001	0.1
200	200	0.0015	0.002

MCQ 8.35 At the end of the process, which one of the following situations will be true ?

(A) superheated vapour will be left in the system

(B) no vapour will be left in the system

(C) a liquid + vapour mixture will be left in the system

(D) the mixture will exist at a dry saturated vapour state

MCQ 8.36 The work done by the system during the process is

(A) 0.1 kJ (B) 0.2 kJ

(C) 0.3 kJ (D) 0.4 kJ

MCQ 8.37 The net entropy generation (considering the system and the thermal reservoir together) during the process is closest to

(A) 7.5 J/K (B) 7.7 J/K

(C) 8.5 J/K (D) 10 J/K

YEAR 2007 ONE MARK

MCQ 8.38 Which of the following relationships is valid only for reversible processes undergone by a closed system of simple compressible substance (neglect changes in kinetic and potential energy ?)

(A) $\delta Q = dU + \delta W$
(B) $Tds = dU + pd\nu$
(C) $Tds = dU + \delta W$
(D) $\delta Q = dU + pd\nu$

MCQ 8.39 Water has a critical specific volume of $0.003155\,\text{m}^3/\text{kg}$. A closed and rigid steel tank of volume $0.025\,\text{m}^3$ contains a mixture of water and steam at $0.1\,\text{MPa}$. The mass of the mixture is $10\,\text{kg}$. The tank is now slowly heated. The liquid level inside the tank

(A) will rise
(B) will fall
(C) will remain constant
(D) may rise or fall depending on the amount of heat transferred

YEAR 2007 TWO MARKS

MCQ 8.40 The stroke and bore of a four stroke spark ignition engine are $250\,\text{mm}$ and $200\,\text{mm}$ respectively. The clearance volume is $0.001\,\text{m}^3$. If the specific heat ratio $\gamma = 1.4$, the air-standard cycle efficiency of the engine is

(A) 46.40%
(B) 56.10%
(C) 58.20%
(D) 62.80%

MCQ 8.41 Which combination of the following statements is correct ?
The incorporation of reheater in a steam power plant :
P : always increases the thermal efficiency of the plant.
Q : always increases the dryness fraction of steam at condenser inlet
R : always increases the mean temperature of heat addition.
S : always increases the specific work output.

(A) P and S
(B) Q and S
(C) P, R and S
(D) P, Q, R and S

MCQ 8.42 Which combination of the following statements is correct ?
P : A gas cools upon expansion only when its Joule-Thomson coefficient is positive in the temperature range of expansion.
Q : For a system undergoing a process, its entropy remains constant only when the process is reversible.
R : The work done by closed system in an adiabatic is a point function.
S : A liquid expands upon freezing when the slope of its fusion curve on

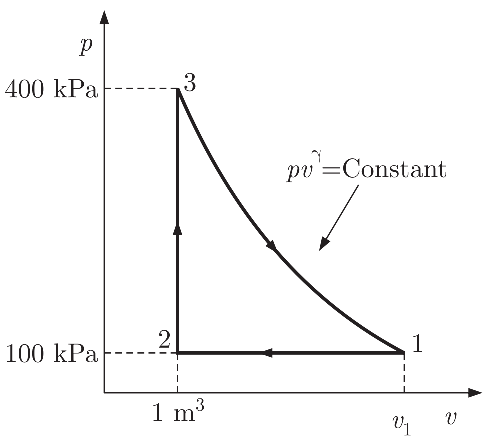
pressure-Temperature diagram is negative.

(A) R and S (B) P and Q

(C) Q, R and S (D) P, Q and R

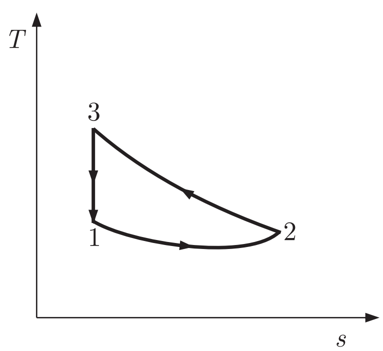
• Common Data For Q.43 and Q.44

A thermodynamic cycle with an ideal gas as working fluid is shown below.

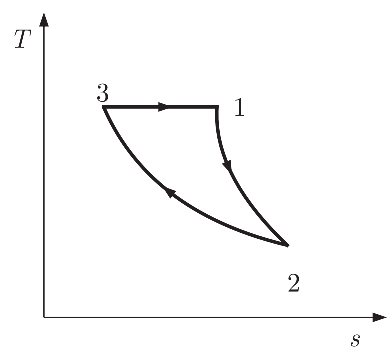


MCQ 8.43 The above cycle is represented on T-s plane by

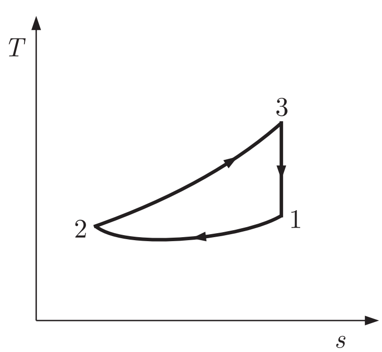
(A)



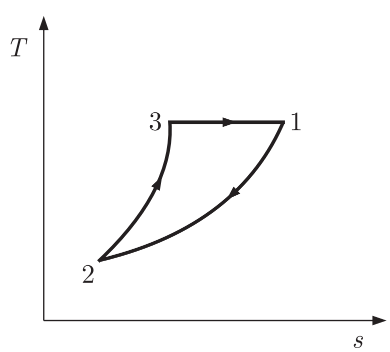
(B)



(C)



(D)



MCQ 8.44 If the specific heats of the working fluid are constant and the value of specific heat ratio is 1.4, the thermal efficiency (%) of the cycle is

(A) 21 (B) 40.9

(C) 42.6 (D) 59.7

MCQ 8.45 A heat transformer is device that transfers a part of the heat, supplied to it at an intermediate temperature, to a high temperature reservoir while rejecting the remaining part to a low temperature heat sink. In such a heat transformer, 100 kJ of heat is supplied at 350 K. The maximum amount of heat in kJ that can be transferred to 400 K, when the rest is rejected to a heat sink at 300 K is

(A) 12.50 (B) 14.29

(C) 33.33 (D) 57.14

YEAR 2006 TWO MARKS

MCQ 8.46 Given below is an extract from steam tables.

Temperature in °C	p_{sat} (Bar)	Specific volume m^3/kg		Enthalpy (kJ/ kg)	
		Saturated Liquid	Saturated vapour	Saturated liquid	Saturated vapour
45	0.09593	0.001010	15.26	188.45	2394.8
342.24	150	0.001658	0.010337	1610.5	2610.5

Specific enthalpy of water in kJ/kg at 150 bar and 45°C is

(A) 203.60 (B) 200.53

(C) 196.38 (D) 188.45

MCQ 8.47 Determine the correctness or otherwise Assertion (A) and the Reason (R)

Assertion (A) : In a power plant working on a Rankine cycle, the regenerative feed water heating improves the efficiency of the steam turbine.

Reason (R) : The regenerative feed water heating raises the average temperature of heat addition in the Rankine cycle.

(A) Both (A) and (R) are true and (R) is the correct reason for (A)

(B) Both (A) and (R) are true but (R) is NOT the correct reason for (A)

(C) Both (A) and (R) are false

(D) (A) is false but (R) is true

MCQ 8.48 Determine the correctness or otherwise of the following Assertion (A) and the Reason (R).

Assertion (A) : Condenser is an essential equipment in a steam power plant.

Reason (R) : For the same mass flow rate and the same pressure rise, a water pump requires substantially less power than a steam compressor.

(A) Both (A) and (R) are true and (R) is the correct reason for (A)

(B) Both (A) and (R) are true and (R) is NOT the correct reason for (A)

(C) Both (A) and (R) are false

(D) (A) is false but (R) is true

MCQ 8.49 Match items from groups I, II, III, IV and V.

Group I	Group II	Group III	Group IV	Group V
	When added to the system is	Differential	Function	Phenomenon
E Heat	G Positive	I Exact	K Path	M Transient
F Work	H Negative	J Inexact	L Point	N Boundary

(A) F-G-J-K-M
E-G-I-K-N

(B) E-G-I-K-M
F-H-I-K-N

(C) F-H-J-L-N
E-H-I-L-M

(D) E-G-J-K-N
F-H-J-K-M

MCQ 8.50 Group I shows different heat addition process in power cycles. Likewise, Group II shows different heat removal processes. Group III lists power cycles. Match items from Groups I, II and III.

Group I	Group II	Group III
P. Pressure constant	S. Pressure constant	1. Rankine Cycle
Q. Volume Constant	T. Volume Constant	2. Otto cycle
R. Temperature constant	U. Temperature Constant	3. Carnot cycle
		4. Diesel cycle
		5. Brayton cycle

(A) P-S-5
R-U-3
P-S-1
Q-T-2

(B) P-S-1
R-U-3
P-S-4
P-T-2

(C) R-T-3
P-S-1
P-T-4
Q-S-5

(D) P-T-4
R-S-3
P-S-1
P-S-5

• Common Data For Q.51 and Q.52

A football was inflated to a gauge pressure of 1 bar when the ambient temperature was 15° C. When the game started next day, the air temperature at the stadium was 5°C. Assume that the volume of the football remains constant at $2500\,cm^3$.

MCQ 8.51 The amount of heat lost by the air in the football and the gauge pressure of air in the football at the stadium respectively equal

(A) 30.6 J, 1.94 bar (B) 21.8 J, 0.93 bar

(C) 61.1 J, 1.94 bar (D) 43.7 J, 0.93 bar

MCQ 8.52 Gauge pressure of air to which the ball must have been originally inflated so that it would be equal 1 bar gauge at the stadium is

(A) 2.23 bar (B) 1.94 bar

(C) 1.07 bar (D) 1.00 bar

YEAR 2005 ONE MARK

MCQ 8.53 The following four figures have been drawn to represent a fictitious thermodynamic cycle, on the p-ν and T-s planes.

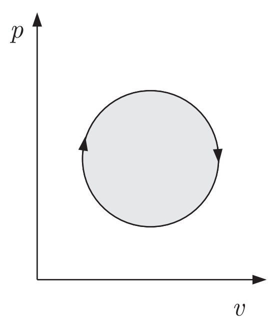


fig. 1

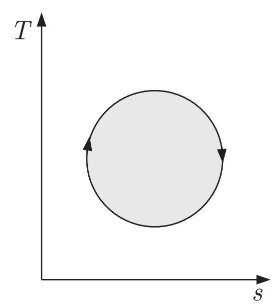


fig. 2

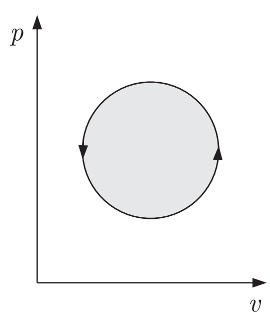


fig. 3

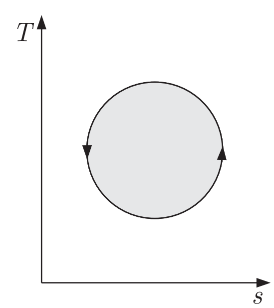


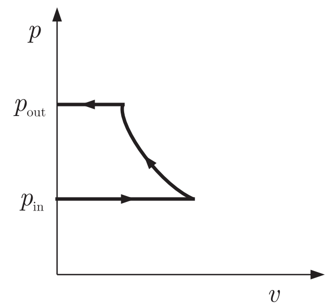
fig.4

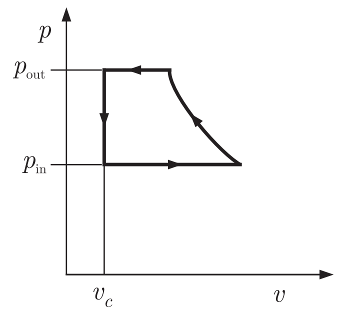
According to the first law of thermodynamics, equal areas are enclosed by

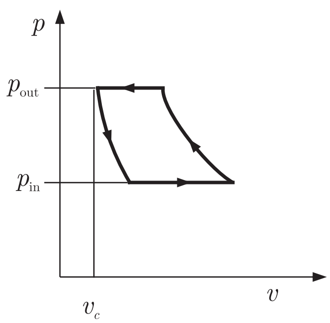
(A) figures 1 and 2 (B) figures 1 and 3

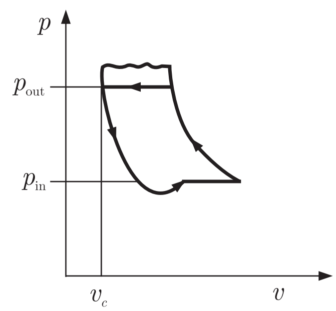
(C) figures 1 and 4 (D) figures 2 and 3

MCQ 8.54 A p-v diagram has been obtained from a test on a reciprocating compressor. Which of the following represents that diagram ?

(A)


(B)


(C)


(D)


YEAR 2005 TWO MARKS

MCQ 8.55 A reversible thermodynamic cycle containing only three processes and producing work is to be constructed. The constraints are

(i) there must be one isothermal process,

(ii) there must be one isentropic process,

(iii) the maximum and minimum cycle pressures and the clearance volume are fixed, and

(iv) polytropic processes are not allowed. Then the number of possible cycles are

(A) 1 (B) 2

(C) 3 (D) 4

MCQ 8.56 Nitrogen at an initial state of 10 bar, 1 m^3 and 300 K is expanded isothermally to a final volume of 2 m^3. The p-ν-T relation is $\left(p+\frac{a}{\nu^2}\right)\nu = RT$, where $a > 0$. The final pressure.

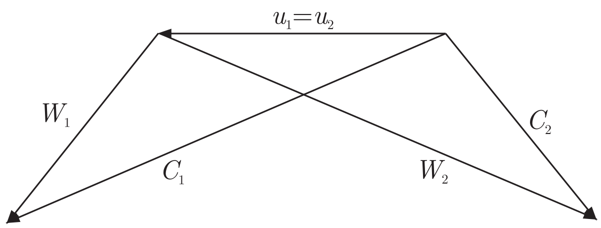
(A) will be slightly less than 5 bar

(B) will be slightly more than 5 bar

(C) will be exactly 5 bar

(D) cannot be ascertained in the absence of the value of a

MCQ 8.57 In the velocity diagram shown below, $u =$ blade velocity, $C =$ absolute fluid velocity and $W =$ relative velocity of fluid and the subscripts 1 and 2 refer to inlet and outlet. This diagram is for



(A) an impulse turbine

(B) a reaction turbine

(C) a centrifugal compressor

(D) an axial flow compressor

• Common Data For Q.58 and Q.59

In two air standard cycles-one operating in the Otto and the other on the Brayton cycle-air is isentropically compressed from 300 to 450 K. Heat is added to raise the temperature to 600 K in the Otto cycle and to 550 K in the Brayton cycle.

MCQ 8.58 In η_O and η_B are the efficiencies of the Otto and Brayton cycles, then

(A) $\eta_O = 0.25, \eta_B = 0.18$

(B) $\eta_O = \eta_B = 0.33$

(C) $\eta_O = 0.5, \eta_B = 0.45$

(D) it is not possible to calculate the efficiencies unless the temperature after the expansion is given

MCQ 8.59 If W_O and W_B are work outputs per unit mass, then

(A) $W_O > W_B$

(B) $W_O < W_B$

(C) $W_O = W_B$

(D) it is not possible to calculate the work outputs unless the temperature after the expansion is given

• Common Data For Q.90 and Q.61 :

The following table of properties was printed out for saturated liquid and saturated vapour of ammonia. The title for only the first two columns are available. All that we know that the other columns (column 3 to 8) contain data on specific properties, namely, internal energy (kJ/kg), enthalpy (kJ/kg) and entropy (kJ/kg.K)

$t(^\circ C)$	p(kPa)						
−20	190.2	88.76	0.3657	89.05	5.6155	1299.5	1418.0
0	429.6	179.69	0.7114	180.36	5.3309	1318.0	1442.2
20	587.5	272.89	1.0408	274.30	5.0860	1332.2	1460.2
40	1554.9	368.74	1.3574	371.43	4.8662	1341.0	1470.2

MCQ 8.60 The specific enthalpy data are in columns

(A) 3 and 7
(B) 3 and 8
(C) 5 and 7
(D) 5 and 8

MCQ 8.61 When saturated liquid at $40^\circ C$ is throttled to $-20^\circ C$, the quality at exit will be

(A) 0.189
(B) 0.212
(C) 0.231
(D) 0.788

YEAR 2004 ONE MARK

MCQ 8.62 A gas contained in a cylinder is compressed, the work required for compression being 5000 kJ. During the process, heat interaction of 2000 kJ causes the surroundings to be heated. The changes in internal energy of the gas during the process is

(A) −7000 kJ
(B) −3000 kJ
(C) +3000 kJ
(D) +7000 kJ

MCQ 8.63 The compression ratio of a gas power plant cycle corresponding to maximum work output for the given temperature limits of T_{min} and T_{max} will be

(A) $\left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{2(\gamma-1)}}$
(B) $\left(\frac{T_{min}}{T_{max}}\right)^{\frac{\gamma}{2(\gamma-1)}}$
(C) $\left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma-1}{\gamma}}$
(D) $\left(\frac{T_{min}}{T_{max}}\right)^{\frac{\gamma-1}{\gamma}}$

MCQ 8.64 At the time of starting, idling and low speed operation, the carburretor supplies a mixture which can be termed as

(A) Lean
(B) slightly leaner than stoichiometric
(C) stoichiometric
(D) rich

YEAR 2004 TWO MARKS

MCQ 8.65 A steel billet of 2000 kg mass is to be cooled from 1250 K to 450 K. The heat released during this process is to be used as a source of energy. The ambient temperature is 303 K and specific heat of steel is 0.5 kJ/kg K. The available energy of this billet is

(A) 490.44 MJ (B) 30.95 MJ

(C) 10.35 MJ (D) 0.10 MJ

MCQ 8.66 During a Morse test on a 4 cylinder engine, the following measurements of brake power were taken at constant speed.

All cylinders firing	3037 kW
Number 1 cylinder not firing	2102 kW
Number 2 cylinder not firing	2102 kW
Number 3 cylinder not firing	2100 kW
Number 4 cylinder not firing	2098 kW

The mechanical efficiency of the engine is

(A) 91.53% (B) 85.07%

(C) 81.07% (D) 61.22%

MCQ 8.67 A solar collector receiving solar radiation at the rate of $0.6\ \text{kW/m}^2$ transforms it to the internal energy of a fluid at an overall efficiency of 50%. The fluid heated to 250 K is used to run a heat engine which rejects heat at 315 K. If the heat engine is to deliver 2.5 kW power, the minimum area of the solar collector required would be

(A) 83.33 m^2 (B) 16.66 m^2

(C) 39.68 m^2 (D) 79.36 m^2

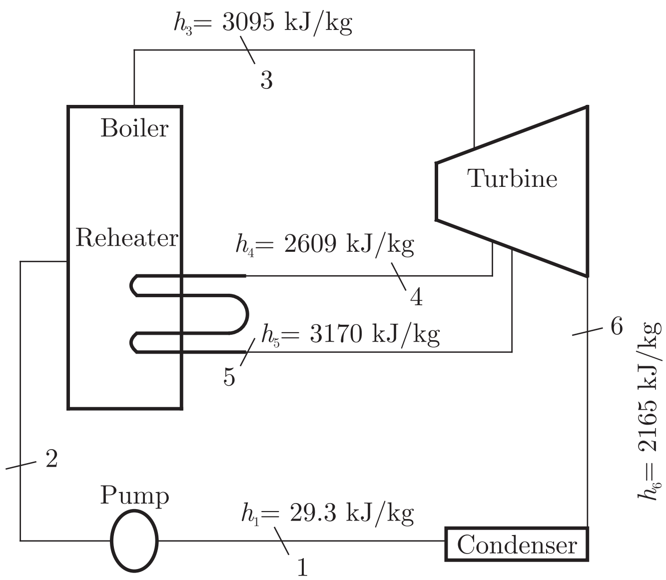
MCQ 8.68 An engine working on air standard Otto cycle has a cylinder diameter of 10 cm and stroke length of 15 cm. The ratio of specific heats for air is 1.4. If the clearance volume is 196.3 cc and the heat supplied per kg of air per cycle is 1800 kJ/kg, the work output per cycle per kg of air is

(A) 879.1 kJ (B) 890.2 kJ

(C) 895.3 kJ (D) 973.5 kJ

• Common Data For Q.69 and Q.70 :

Consider a steam power plant using a reheat cycle as shown . Steam leaves the boiler and enters the turbine at 4 MPa, 350° C ($h_3 = 3095$ kJ/kg). After expansion in the turbine to 400 kPa ($h_4 = 2609$ kJ/kg), and then expanded in a low pressure turbine to 10 kPa ($h_6 = 2165$ kJ/kg). The specific volume of liquid handled by the pump can be assumed to be



MCQ 8.69 The thermal efficiency of the plant neglecting pump work is

(A) 15.8% (B) 41.1%

(C) 48.5% (D) 58.6%

MCQ 8.70 The enthalpy at the pump discharge (h_2) is

(A) 0.33 kJ/kg (B) 3.33 kJ/kg

(C) 4.0 kJ/k (D) 33.3 kJ/kg

YEAR 2003 ONE MARK

MCQ 8.71 For a spark ignition engine, the equivalence ratio (ϕ) of mixture entering the combustion chamber has values

(A) $\phi < 1$ for idling and $\phi > 1$ for peak power conditions

(B) $\phi > 1$ for both idling and peak power conditions

(C) $\phi > 1$ for idling and $\phi < 1$ for peak power conditions

(D) $\phi < 1$ for both idling and peak power conditions

MCQ 8.72 A diesel engine is usually more efficient than a spark ignition engine because

(A) diesel being a heavier hydrocarbon, releases more heat per kg than gasoline

(B) the air standard efficiency of diesel cycle is higher than the Otto cycle, at a fixed compression ratio

(C) the compression ratio of a diesel engine is higher than that of an SI engine

(D) self ignition temperature of diesel is higher than that of gasoline

MCQ 8.73 In Ranking cycle, regeneration results in higher efficiency because

(A) pressure inside the boiler increases

(B) heat is added before steam enters the low pressure turbine

(C) average temperature of heat addition in the boiler increases

(D) total work delivered by the turbine increases

MCQ 8.74 Considering the variation of static pressure and absolute velocity in an impulse steam turbine, across one row of moving blades

(A) both pressure and velocity decreases

(B) pressure decreases but velocity increases

(C) pressure remains constant, while velocity increases

(D) pressure remains constant, while velocity decreases

MCQ 8.75 A 2 kW, 40 liters water heater is switched on for 20 minutes. The heat capacity c_p for water is 4.2 kJ/kgK. Assuming all the electrical energy has gone into heating the water, increase of the water temperature in degree centigrade is

(A) 2.7 (B) 4.0

(C) 14.3 (D) 25.25

YEAR 2003 TWO MARKS

MCQ 8.76 Considering the relationship $Tds = dU + pd\nu$ between the entropy (s), internal energy (U), pressure (p), temperature (T) and volume (ν), which of the following statements is correct ?

(A) It is applicable only for a reversible process

(B) For an irreversible process, $Tds > dU + pd\nu$

(C) It is valid only for an ideal gas

(D) It is equivalent to I^{st} law, for a reversible process

MCQ 8.77 In a gas turbine, hot combustion products with the specific heats $c_p = 0.98$ kJ/kgK, and $c_v = 0.7538$ kJ/kgK enters the turbine at 20 bar, 1500 K exit at 1 bar. The isentropic efficiency of the turbine is 0.94. The work developed by the turbine per kg of gas flow is

(A) 689.64 kJ/kg (B) 794.66 kJ/kg

(C) 1009.72 kJ/kg (D) 1312.00 kJ/kg

MCQ 8.78 An automobile engine operates at a fuel air ratio of 0.05, volumetric efficiency of 90% and indicated thermal efficiency of 30%. Given that the calorific value of the fuel is 45 MJ/kg and the density of air at intake is 1 kg/m^3, the indicated mean effective pressure for the engine is

(A) 6.075 bar (B) 6.75 bar
(C) 67.5 bar (D) 243 bar

MCQ 8.79 For an engine operating on air standard Otto cycle, the clearance volume is 10% of the swept volume. The specific heat ratio of air is 1.4. The air standard cycle efficiency is
(A) 38.3% (B) 39.8%
(C) 60.2% (D) 61.7%

• Common Data For Q.80 and 81

Nitrogen gas (molecular weight 28) is enclosed in a cylinder by a piston, at the initial condition of 2 bar, 298 K and 1 m^3. In a particular process, the gas slowly expands under isothermal condition, until the volume becomes 2 m^3. Heat exchange occurs with the atmosphere at 298 K during this process.

MCQ 8.80 The work interaction for the Nitrogen gas is
(A) 200 kJ (B) 138.6 kJ
(C) 2 kJ (D) -200 kJ

MCQ 8.81 The entropy changes for the Universe during the process in kJ/K is
(A) 0.4652 (B) 0.0067
(C) 0 (D) -0.6711

YEAR 2002 ONE MARK

MCQ 8.82 A positive value of Joule-Thomson coefficient of a fluid means
(A) temperature drops during throttling
(B) temperature remains constant during throttling
(C) temperature rises during throttling
(D) None of the above

MCQ 8.83 A correctly designed convergent-divergent nozzle working at a designed load is
(A) always isentropic (B) always choked
(C) never choked (D) never isentropic

YEAR 2002 TWO MARKS

MCQ 8.84 A Carnot cycle is having an efficiency of 0.75. If the temperature of the high temperature reservoir is 727°C, what is the temperature of low temperature reservoir ?

(A) 23° C (B) -23° C

(C) 0° C (D) 250° C

MCQ 8.85 An ideal air standard Otto cycle has a compression ratio of 8.5. If the ratio of the specific heats of air (γ) is 1.4, what is the thermal efficiency in percentage) of the Otto cycle ?

(A) 57.5 (B) 45.7

(C) 52.5 (D) 95

MCQ 8.86 The efficiency of superheat Rankine cycle is higher than that of simple Rankine cycle because

(A) the enthalpy of main steam is higher for superheat cycle

(B) the mean temperature of heat addition is higher for superheat cycle

(C) the temperature of steam in the condenser is high

(D) the quality of steam in the condenser is low.

YEAR 2001 ONE MARK

MCQ 8.87 The Rateau turbine belongs to the category of

(A) pressure compounded turbine (B) reaction turbine

(C) velocity compounded turbine (D) radial flow turbine

MCQ 8.88 A gas having a negative Joule-Thomson coefficient ($\mu < 0$), when throttled, will

(A) become cooler

(B) become warmer

(C) remain at the same temperature

(D) either be cooler or warmer depending on the type of gas

YEAR 2001 TWO MARKS

MCQ 8.89 A cyclic heat engine does 50 kJ of work per cycle. If the efficiency of the heat engine is 75%, the heat rejected per cycle is

(A) $16\frac{2}{3}$ kJ (B) $33\frac{1}{3}$ kJ

(C) $37\frac{1}{2}$ kJ (D) $66\frac{2}{3}$ kJ

MCQ 8.90 A single-acting two-stage compressor with complete intercooling delivers air at 16 bar. Assuming an intake state of 1 bar at 15° C, the pressure ratio per stage is

(A) 16 (B) 8

(C) 4 (D) 2

MCQ 8.91 A small steam whistle (perfectly insulated and doing no shaft work) causes a drop of 0.8 kJ/kg in the enthalpy of steam from entry to exit. If the kinetic energy of the steam at entry is negligible, the velocity of the steam at exit is

(A) 4 m/s (B) 40 m/s

(C) 80 m/s (D) 120 m/s

MCQ 8.92 In a spark ignition engine working on the ideal Otto cycle, the compression ratio is 5.5. The work output per cycle (i.e., area of the p-ν diagram) is equal to $23.625 \times 10^5 \times \nu_c$, where ν_c is the clearance volume in m^3. The indicated mean effective pressure is

(A) 4.295 bar (B) 5.250 bar

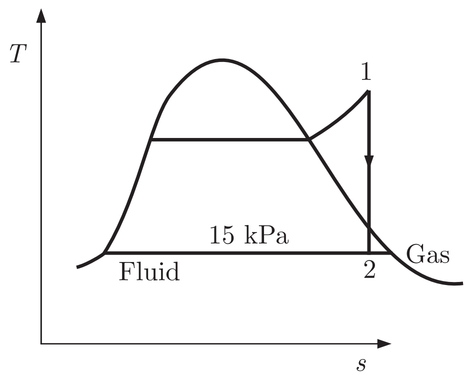
(C) 86.870 bar (D) 106.300 bar

****

SOLUTION

SOL 8.1 Option (B) is correct.

For adiabatic expansion steam in turbine.



Given $h_1 = 3251.0\,\text{kJ/kg},\ m = 10\,\text{kg/s},\ x = 0.9\,(\text{dryness fraction})$

At 15 kPa

Enthalpy of liquid, $h_f = 225.94\,\text{kJ/kg}$

Enthalpy of vapour, $h_g = 2598.3\,\text{kJ/kg}$

Since Power output of turbine.

$$P = \dot{m}(h_1 - h_2) \quad (\text{K.E and P.E are negligible}) \qquad ...(i)$$

$$h_2 = h_f + xh_{fg} = h_f + x(h_g - h_f)$$

$$= 225.94 + 0.9(2598.3 - 225.94) = 2361.064\,\text{kJ/kg}$$

From Eq. (i)

$$P = 10 \times (3251.0 - 2361.064) = 8899\,\text{kW} = 8.9\,\text{MW}$$

SOL 8.2 Option (B) is correct.

We know that $Tds = du + Pd\nu$ ...(i)

For ideal gas $p\nu = mRT$

For isothermal process

$$T = \text{constant}$$

For reversible process

$$du = 0$$

Then from equation (i)

$$ds = \frac{pd\nu}{T} = \frac{mRT}{T}\frac{d\nu}{\nu} = mR\frac{d\nu}{\nu}$$

$$\int ds = \Delta s = mR\int_{\nu_1}^{\nu_2}\frac{d\nu}{\nu} = mR\ln\frac{\nu_2}{\nu_1}$$

$$\Delta s = mR\ln\frac{p_1}{p_2} \qquad \left[\frac{p_1}{p_2} = \frac{\nu_2}{\nu_1}\right]$$

SOL 8.3 Option (C) is correct.

From energy balance for steady flow system.

$$E_{in} = E_{out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) \qquad ...(i)$$

As $h = c_p T$

Equation (1) becomes

$$c_p T_1 + \frac{V_1^2}{2} = c_p T_2 + \frac{V_2^2}{2}$$

$$T_2 = \left(\frac{V_1^2 - V_2^2}{2 \times c_p}\right) + T_1 = \frac{10^2 - 180^2}{2 \times 1008} + 500 = -16.02 + 500$$

$$= 483.98 \simeq 484\text{ K}$$

SOL 8.4 Option (D) is correct.

From Mass conservation.

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\frac{V_1 A_1}{\nu_1} = \frac{V_2 A_2}{\nu_2} \qquad ...(i)$$

where ν = specific volume of air $= \dfrac{RT}{p}$

Therefore Eq. (1) becomes

$$\frac{p_1 V_1 A_1}{RT_1} = \frac{p_2 V_2 A_2}{RT_2}$$

$$A_2 = \frac{p_1 \times V_1 \times A_1 \times T_2}{p_2 \times V_2 \times T_1} = \frac{300 \times 10 \times 80 \times 484}{100 \times 180 \times 500} = 12.9\text{ cm}^2$$

SOL 8.5 Option (D) is correct.

Work done is a quasi-static process between two given states depends on the path followed. Therefore,

$$\int_1^2 dW \neq W_2 - W_1 \qquad dW \text{ shows the inexact differential}$$

But, $\displaystyle\int_1^2 dW = W_{1-2} \text{ or } {}_1W_2$

So, Work is a path function and Heat transfer is also a path function. The amount of heat transferred when a system changes from state 1 to state 2 depends on the intermediate states through which the system passes i.e. the path.

$$\int_1^2 dQ = Q_{1-2} \text{ or } {}_1Q_2$$

dQ shows the inexact differential. So, Heat and work are path functions.

SOL 8.6 Option (A) is correct.

Given : $R = 23\,\Omega$, $i = 10$ A

Since work is done on the system. So,

$$W_{electrical} = -i^2R = -(10)^2 \times 23 = -2300\text{ W} = -2.3\text{ kW}$$

Here given that tank is well-insulated.

So, $\Delta Q = 0$

Applying the First law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta U + \Delta W = 0$$

$$\Delta W = -\Delta U$$

And $\Delta U = +2.3\text{ kW}$

Heat is transferred to the system

SOL 8.7 Option (A) is correct.

Given : $h_1 = 2800\text{ kJ/kg}$ = Enthalpy at the inlet of steam turbine

$h_2 = 1800\text{ kJ/kg}$ = Enthalpy at the outlet of a steam turbine

Steam rate or specific steam consumption

$$= \frac{3600}{W_T - W_p}\text{ kg/kWh}$$

Pump work W_p is negligible, therefore

$$\text{Steam rate} = \frac{3600}{W_T}\text{ kg/kWh}$$

And $W_T = h_1 - h_2$ From Rankine cycle

$$\text{Steam rate} = \frac{3600}{h_1 - h_2}\text{ kg/kWh} = \frac{3600}{2800 - 1800} = 3.60\text{ kg/kWh}$$

SOL 8.8 Option (D) is correct.

Given : $r = 60$ mm, $D = 80$ mm

Stroke length, $L = 2r = 2 \times 60 = 120$ mm (cylinder diameter)

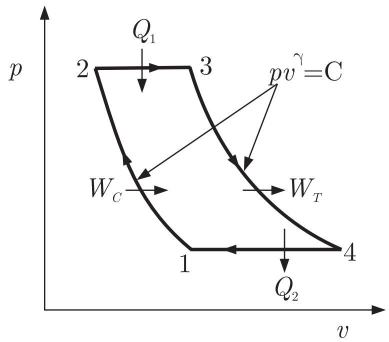
Swept Volume, $\nu_s = A \times L$

$$= \frac{\pi}{4}D^2 \times L = \frac{\pi}{4}(8.0)^2 \times 12.0$$

$$= \frac{\pi}{4}(8 \times 8) \times 12 = 602.88 \simeq 603\text{ cm}^3$$

SOL 8.9 Option (A) is correct.

Given p-ν curve shows the Brayton Cycle.



Given : $p_1 = 1\text{ bar} = p_4,\ p_2 = 6\text{ bar} = p_3,\ T_{minimum} = 300\text{ K},\ T_{maximum} = 1500\text{ K}$

$$\frac{c_p}{c_v} = \gamma = 1.4$$

We have to find T_2 (temperature at the end of compression) or T_4 (temperature at the end of expansion)

Applying adiabatic equation for process 1-2, we get

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{6}\right)^{\frac{1.4-1}{1.4}}$$

$$\frac{300}{T_2} = \left(\frac{1}{6}\right)^{0.286} \qquad T_1 = T_{minimum}$$

$$T_2 = \frac{300}{\left(\frac{1}{6}\right)^{0.286}} = 500.5\text{ K} \simeq 500\text{ K}$$

Again applying for the Process 3-4,

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{6}\right)^{\frac{1.4-1}{1.4}} = \left(\frac{1}{6}\right)^{0.286}$$

So, $$T_4 = T_3 \times \left(\frac{1}{6}\right)^{0.286} = 1500 \times \left(\frac{1}{6}\right)^{0.286} = 900\text{ K} \quad T_3 = T_{maximum}$$

SOL 8.10 Option (B) is correct.

Given : At station p :

$p_1 = 150\text{ kPa},\ T_1 = 350\text{ K}$

At station Q :

$p_2 = ?,\ T_2 = 300\text{ K}$

We know, $$\gamma = \frac{c_p}{c_v} = \frac{1.005}{0.718} = 1.39$$

Applying adiabatic equation for station P and Q,

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}} = \frac{p_1}{p_2}$$

$$p_2 = \frac{p_1}{\left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}} = \frac{150}{\left(\frac{350}{300}\right)^{\frac{1.39}{1.39-1}}} = \frac{150}{1.732} = 86.60\text{ kPa} \simeq 87\text{ kPa}$$

SOL 8.11 Option (C) is correct.

Given :

Pressure at Q $\quad p_2 = 50\,\text{kPa}$

Using the general relation to find the entropy changes between P and Q

$$Tds = dh - \nu dp$$

$$ds = \frac{dh}{T} - \frac{\nu}{T}dp \qquad \text{...(i)}$$

Given in the previous part of the question

$$h = c_p T$$

Differentiating both the sides, we get

$$dh = c_p dT$$

Put the value of dh in equation (i),

$$ds = c_p\frac{dT}{T} - \frac{\nu}{T}dp \qquad \text{From the gas equation } \nu/T = R/p$$

So,

$$= c_p\frac{dT}{T} - R\frac{dp}{p}$$

Integrating both the sides and putting the limits

$$\int_P^Q ds = c_p\int_P^Q \frac{dT}{T} - R\int_P^Q \frac{dp}{p}$$

$$[s]_P^Q = c_p[\ln T]_P^Q - R[\ln P]_P^Q$$

$$s_Q - s_P = c_p[\ln T_Q - \ln T_P] - R[\ln p_Q - \ln p_P]$$

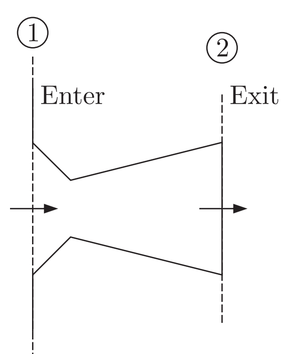
$$= c_p\ln\left(\frac{T_Q}{T_P}\right) - R\ln\left(\frac{p_Q}{p_P}\right)$$

$$= 1.005\ln\left(\frac{300}{350}\right) - 0.287\ln\left(\frac{50}{150}\right)$$

$$= 1.005 \times (-0.1541) - 0.287 \times (-1.099)$$

$$= 0.160\,\text{kJ/kg K}$$

SOL 8.12 Option (C) is correct.



Given :$T_1 = 400\,\text{K},\ p_1 = 3\,\text{bar},\ A_2 = 0.005\,\text{m}^2,\ p_2 = 50\,\text{kPa} = 0.5\,\text{bar},$

$R = 0.287$ kJ/kg K, $\gamma = \frac{c_p}{c_v} = 1.4,\ T_2 = ?$

Applying adiabatic equation for isentropic (reversible adiabatic) flow at section (1) and (2), we get

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 400\left(\frac{0.5}{3}\right)^{\frac{1.4-1}{1.4}}$$

$$= 400 \times (0.166)^{0.286} = 239.73 \text{ K}$$

Apply perfect Gas equation at the exit,

$$p_2\nu_2 = m_2 R T_2$$

$$p_2 = \frac{m_2}{\nu_2} R T_2 = \rho_2 R T_2 \qquad \left(\frac{m}{\nu} = \rho\right)$$

$$\rho_2 = \frac{p_2}{R T_2} = \frac{50 \times 10^3}{0.287 \times 10^3 \times 239.73} = 0.727 \text{ kg/m}^3$$

SOL 8.13 Option (D) is correct.

Given : $\rho_2 = 0.727$ kg/m^3, $A_2 = 0.005$ m^2, $V_2 = ?$

For isentropic expansion,

$$V_2 = \sqrt{2c_p(T_1 - T_2)}$$

$$= \sqrt{2 \times 1.005 \times 10^3 \times (400 - 239.73)}$$

for air $c_p = 1.005$ kJ/kg K

$$= \sqrt{322142.7} = 567.58 \text{ m/sec}$$

Mass flow rate at exit,

$$\dot{m} = \rho_2 A_2 V_2 = 0.727 \times 0.005 \times 567.58 = 2.06 \text{ kg/sec}$$

SOL 8.14 Option (A) is correct.

Given : $\nu = 0.0259$ m^3, Work output $= 950$ kW, $N = 2200$ rpm

Mean effective pressure

$$mep = \frac{\text{Net work for one cycle}}{\text{displacement volume}} \times 60$$

Number of power cycle

$$n = \frac{N}{2} = \frac{2200}{2} = 1100 \qquad \text{(for 4 stroke)}$$

Hence, net work for one cycle

$$= \frac{950 \times 10^3}{1100} = 863.64 \text{ W}$$

So,

$$mep = \frac{60 \times 863.64}{0.0259} = 2 \times 10^6 \text{ Pa} = 2 \text{ MPa}$$

SOL 8.15 Option (D) is correct.

We know that,

Entropy of universe is always increases.

$$\Delta s_{universe} > 0$$

$$(\Delta s)_{system} + (\Delta s)_{surrounding} > 0$$

SOL 8.16 Option (A) is correct.

Given : $\gamma = 1.67,\ M = 40,\ p_1 = 0.1\,\text{MPa} = 10^6 \times 0.1 = 10^5\ \text{Pa}$

$T_1 = 300\ \text{K},\ p_2 = 0.2\,\text{MPa} = 2\times 10^5\ \text{Pa},\ R_u = 8.314\ \text{kJ/kgmol K}$

$$\text{Gas constant} = \frac{\text{Universal Gas constant}}{\text{Molecular Weight}}$$

$$R = \frac{R_u}{M} = \frac{8.314}{40} = 0.20785\ \text{kJ/kg K}$$

For adiabatic process,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{300} = \left(\frac{0.2}{0.1}\right)^{\frac{1.67-1}{1.67}} = (2)^{0.4012}$$

$$T_2 = 300 \times (2)^{0.4012} = 300 \times 1.32 = 396\ \text{K}$$

Work done in adiabatic process is given by,

$$W = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} = \frac{R(T_1 - T_2)}{\gamma - 1}$$

$$= \frac{0.20785[300 - 396]}{1.67 - 1} = \frac{0.20785(-96)}{0.67} = -29.7\ \text{kJ/kg}$$

(Negative sign shows the compression work)

SOL 8.17 Option (B) is correct.

We know from the clausius Inequality,

If $\oint \frac{dQ}{T} = 0$, the cycle is reversible

$\oint \frac{dQ}{T} < 0$, the cycle is irreversible and possible

For case (a),

$$\oint_a \frac{dQ}{T} = \frac{2500}{1200} - \frac{2500}{800}$$

$$= \frac{25}{12} - \frac{25}{8} = -1.041\ \text{kJ/kg}$$

For case (b),

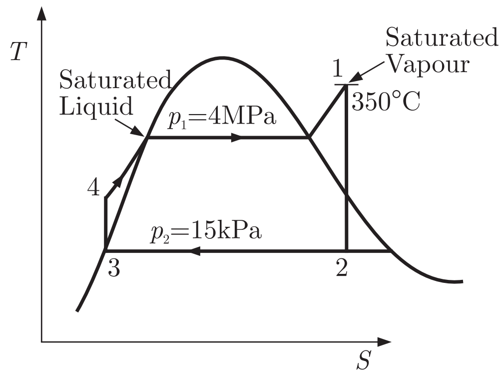
$$\oint_b \frac{dQ}{T} = \frac{2000}{800} - \frac{2000}{500} = \frac{20}{8} - \frac{20}{5} = -1.5\ \text{kJ/kg}$$

$$\oint_a \frac{dQ}{T} > \oint_b \frac{dQ}{T}$$

So, process (b) is more irreversible than process (a)

SOL 8.18 Option (C) is correct.

Given T - s curve is for the steam plant



Given : $p_1 = 4\,\text{MPa} = 4 \times 10^6\,\text{Pa}$, $T_1 = 350^\circ\,\text{C} = (273 + 350)\,\text{K} = 623\,\text{K}$

$p_2 = 15\,\text{kPa} = 15 \times 10^3\,\text{Pa}$, $\eta_{adiabatic} = 90\% = 0.9$

Now from the steam table,

Given data : $h_1 = 3092.5\,\text{kJ/kg}$, $h_3 = h_f = 225.94\,\text{kJ/kg}$, $h_g = 2599.1\,\text{kJ/kg}$

$$s_1 = s_2 = s_f + x(s_g - s_f) \qquad \text{...(i)}$$

Where, x = dryness fraction

From the table, we have

$$s_f = 0.7549\,\text{kJ/kg K}$$

$$s_g = 8.0085\,\text{kJ/kg K}$$

$$s_1 = s_2 = 6.5821$$

From equation (i),

$$x = \frac{s_2 - s_f}{s_g - s_f} = \frac{6.5821 - 0.7549}{8.0085 - 0.7549} = 0.8033$$

$$h_2 = h_f + x(h_g - h_f)$$

$$= 225.94 + 0.8033(2599.1 - 225.94)$$

$$= 225.94 + 1906.36 = 2132.3\,\text{kJ/kg}$$

Theoretical turbine work from the cycle is given by,

$$W_T = h_1 - h_2$$

$$= 3092.5 - 2132.3 = 960.2\,\text{kJ/kg}$$

Actual work by the turbine,

$$= \text{Theoretical work} \times \eta_{adiabatic}$$

$$= 0.9 \times 960.2 = 864.18\,\text{kJ/kg}$$

Pump work, $W_p = \nu_f(p_1 - p_2)$

$$= 0.001014(4000 - 15) = 4.04\,\text{kJ/kg}$$

$$W_{net} = W_T - W_p = 864.18 - 4.04 = 860.14\,\text{kJ/kg} \approx 860$$

SOL 8.19 Option (C) is correct.

Heat supplied $= h_1 - h_4$ From T-s diagram

From the pump work equation,

$$W_p = h_4 - h_3$$

$$h_4 = W_p + h_3 = 4.04 + 225.94 = 229.98\,\text{kJ/kg}$$

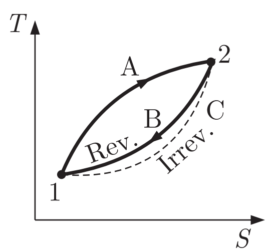
And Heat supplied,

$$Q = h_1 - h_4$$

$$= 3092.50 - 229.98 = 2862.53 \simeq 2863\,\text{kJ/kg}$$

SOL 8.20 Option (A) is correct.

We consider the cycle shown in figure, where A and B are reversible processes and C is an irreversible process. For the reversible cycle consisting of A and B.



$$\int_R \frac{dQ}{T} = \int_{A1}^{2} \frac{dQ}{T} + \int_{B2}^{1} \frac{dQ}{T} = 0$$

or $$\int_{A1}^{2} \frac{dQ}{T} = -\int_{B2}^{1} \frac{dQ}{T} \qquad \text{...(i)}$$

For the irreversible cycle consisting of A and C, by the inequality of clausius,

$$\oint \frac{dQ}{T} = \int_{A1}^{2} \frac{dQ}{T} + \int_{C2}^{1} \frac{dQ}{T} < 0 \qquad \text{...(ii)}$$

From equation (i) and (ii)

$$-\int_{B2}^{1} \frac{dQ}{T} + \int_{C2}^{1} \frac{dQ}{T} < 0$$

$$\int_{B2}^{1} \frac{dQ}{T} > \int_{C2}^{1} \frac{dQ}{T} \qquad \text{...(iii)}$$

Since the path B is reversible,

$$\int_{B2}^{1} \frac{dQ}{T} = \int_{B2}^{1} ds$$

Since entropy is a property, entropy changes for the paths B and C would be the same.

Therefore,

$$\int_{B2}^{1} ds = \int_{C2}^{1} ds \quad \text{...(iv)}$$

From equation (iii) and (iv),

$$\int_{C2}^{1} ds > \int_{C2}^{1} \frac{dQ}{T}$$

Thus, for any irreversible process.

$$ds > \frac{dQ}{T}$$

So, entropy must increase.

SOL 8.21 Option (A) is correct.

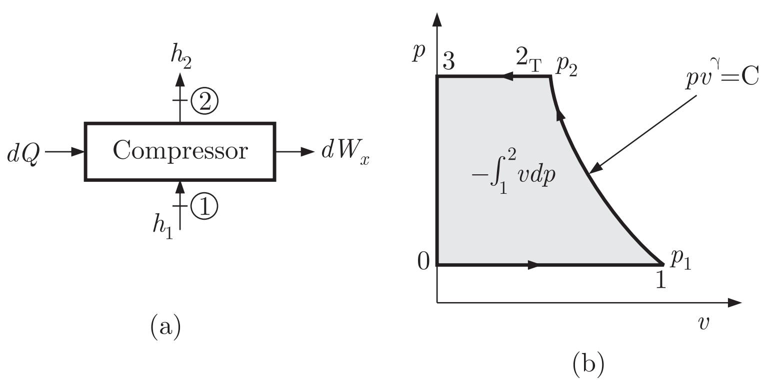
Given : $p_1 = 0.8\,\text{MPa}$, $\nu_1 = 0.015\,\text{m}^3$, $\nu_2 = 0.030\,\text{m}^3$, $T =$ Constant

We know work done in a constant temperature (isothermal) process

$$W = p_1\nu_1 \ln\left(\frac{\nu_2}{\nu_1}\right) = (0.8 \times 10^6)(0.015)\ln\left(\frac{0.030}{0.015}\right)$$

$$= (0.012 \times 10^6) \times 0.6931 = 8.32\,\text{kJ}$$

SOL 8.22 Option (B) is correct.



Steady flow energy equation for a compressor (Fig a) gives,

$$h_1 + dQ = h_2 + dW_x \quad \text{...(i)}$$

Neglecting the changes of potential and kinetic energy. From the property relation

$$Tds = dh - \nu dp$$

For a reversible process,

$$Tds = dQ$$

So, $$dQ = dh - \nu dp \quad \text{...(ii)}$$

If consider the process is reversible adiabatic then $dQ = 0$

From equation (i) and (ii),

$$h_1 - h_2 = dW_x \qquad \Rightarrow dh = h_2 - h_1 = -dW_x \quad \text{...(iii)}$$

And $$dh = \nu dp \quad \text{...(iv)}$$

From equation (iii) and (iv),

$$-dW_x = \nu dp$$

$$W_x = -\int \nu dp$$

Negative sign shows the work is done on the system (compression work) for initial and Final Stage

$$W_x = \int_1^2 \nu dp$$

SOL 8.23 Option (D) is correct.

Given : $r = 10,\ p_1 = 100\,\text{kPa},\ T_1 = 27^\circ\,\text{C} = (27+273)\,\text{K} = 300\,\text{K}$
$Q_s = 1500\,\text{kJ/kg},\ Q_r = 700\,\text{kJ/kg},\ R = 0.287\,\text{kJ/kg K}$
Mean Effective pressure

$$p_m = \frac{\text{Net work output}}{\text{Swept Volume}} \qquad \text{...(i)}$$

Swept volume, $\nu_1 - \nu_2 = \nu_2(r-1)$
where $\nu_1 =$ Total volume and $\nu_2 =$ Clearance volume

$$r = \frac{\nu_1}{\nu_2} = 10 \quad \Rightarrow \quad \nu_1 = 10\nu_2 \qquad \text{...(ii)}$$

Applying gas equation for the beginning process,

$$p_1\nu_1 = RT_1$$

$$\nu_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 300}{100} = 0.861\,\text{m}^3/\text{kg}$$

$$\nu_2 = \frac{\nu_1}{10} = \frac{0.861}{10} = 0.0861\,\text{m}^3/\text{kg}$$

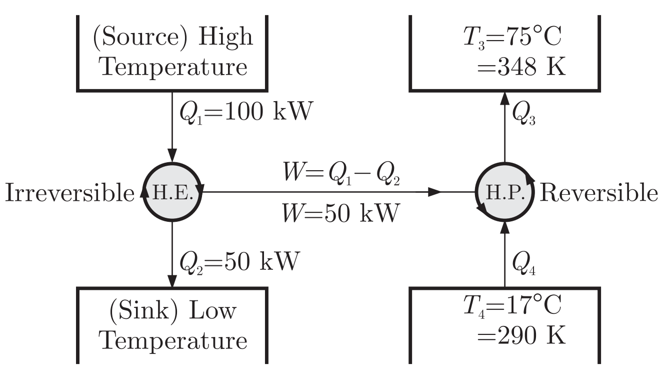
$$W_{net} = Q_s - Q_r = (1500-700)\,\text{kJ/kg K} = 800\,\text{kJ/kg K}$$

From equation (i)

$$p_m = \frac{800}{\nu_2(r-1)} = \frac{800}{0.0861\,(10-1)}$$

$$= \frac{800}{0.7749} = 1032.391\,\text{kPa} \backsimeq 1032\,\text{kPa}$$

SOL 8.24 Option (C) is correct.



We know that coefficient of performance of a Heat pump for the given system is,

$$(COP)_{H.P.} = \frac{Q_3}{Q_3 - Q_4} = \frac{Q_3}{W}$$

For a reversible process,

$$\frac{Q_3}{Q_4} = \frac{T_3}{T_4}$$

$$(COP)_{H.P.} = \frac{T_3}{T_3 - T_4} = \frac{Q_3}{W}$$

$$\frac{348}{348 - 290} = \frac{Q_3}{50}$$

$$Q_3 = \frac{348 \times 50}{58} = 300 \text{ K}$$

SOL 8.25 Option (A) is correct.

Given : $h_1 = 3200 \text{ kJ/kg}, V_1 = 160 \text{ m/sec}, z_1 = 10 \text{ m}$

$$p_1 = 3 \text{ mpA}, \dot{m} = -\frac{dM}{dt} = 20 \text{ kg/sec}$$

It is a adiabatic process, So $dQ = 0$

Apply steady flow energy equation [$S.F.E.E.$] at the inlet and outlet section of steam turbine,

$$h_1 + \frac{V_1^2}{2} + z_1 g + \frac{dQ}{dm} = h_2 + \frac{V_2^2}{2} + z_2 g + \frac{dW}{dm}$$

$$dQ = 0$$

So $\frac{dQ}{dm} = 0$

And $$h_1 + \frac{V_1^2}{2} + z_1 g = h_2 + \frac{V_2^2}{2} + z_2 g + \frac{dW}{dm}$$

$$\frac{dW}{dm} = (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + (z_1 - z_2) g$$

$$= (3200 - 2600) \times 10^3 + \left[\frac{(160)^2 - (100)^2}{2}\right] + (10 - 6) 9.8$$

$$= 600000 + 7800 + 39.20$$

$$\frac{dW}{dm} = 607839.2 \text{ J/kg} = 607.84 \text{ kJ/kg}$$

Power output of turbine

$$P = \text{Mass flow rate} \times \frac{dW}{dm}$$

$$= 20 \times 607.84 \times 10^3 \qquad \dot{m} = 20 \text{ kg/sec}$$

$$P = 12.157 \text{ MJ/sec} = 12.157 \text{ MW}$$

SOL 8.26 Option (C) is correct.

Given : $\rho = 1000\,\text{kg/m}^3$

Here given that ignoring kinetic and potential energy effects, So in the steady flow energy equation the terms $V^2/2, Z_1 g$ are equal to zero and dQ is also zero for adiabatic process.

S.F.E.E. is reduces to,

$$h_4 = h_3 + \frac{dW_p}{dm}$$ Here, W_p represents the pump work

where $h_3 =$ Enthalpy at the inlet of pump and $h_4 =$ Enthalpy at the outlet of the pump.

$$\frac{dW_p}{dm} = h_4 - h_3 = dh \quad \text{...(i)}$$

For reversible adiabatic compression,

$$dQ = dh - \nu dp \quad (dQ = 0)$$

$$dh = \nu dp \quad \text{...(ii)}$$

From equation (i) and (ii), we get

$$\frac{dW_p}{dm} = \nu dp$$

$$\frac{dW_p}{dm} = \frac{1}{\rho}(p_1 - p_2) \quad v = \frac{1}{\rho}$$

$$\frac{dW_p}{dm} = \frac{(3000-70)\,\text{kPa}}{1000} = \frac{2930}{1000}\,\text{kPa} = 2.930\,\text{kPa}$$

SOL 8.27 Option (B) is correct.

Given : $T_1 = T_2,\ p_1 = p_2$

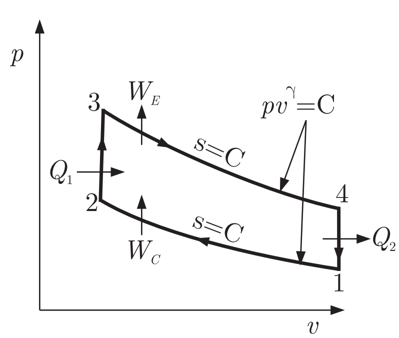
Universal Gas constant $= R$

Here given oxygen are mixed adiabatically

So, $dQ = 0$

We know, $ds = \dfrac{dQ}{T} = \dfrac{0}{T} = 0$

SOL 8.28 Option (B) is correct.



Assumptions of air standard otto cycle :-

(A) All processes are both internally as well as externally reversible.

(B) Air behaves as ideal gas

(C) Specific heats remains constant (c_p & c_v)

(D) Intake process is constant volume heat addition process and exhaust process is constant volume heat rejection process.

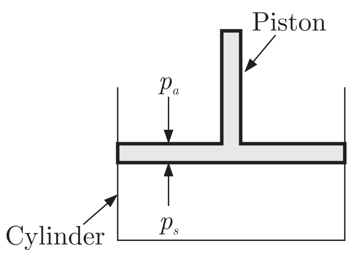
Intake process is a constant volume heat addition process, From the given options, option (2) is incorrect.

SOL 8.29 Option (C) is correct.

Given : $p_a = 100\,\text{kPa},\ p_s = 300\,\text{kPa},\ \Delta\nu = 0.01\,\text{m}^3$

Net pressure work on the system,

$$p = p_s - p_a = 300 - 100 = 200\,\text{kPa}$$



For constant pressure process work done is given by

$$W = p\Delta\nu = 200 \times 0.01 = 2\,\text{kJ}$$

SOL 8.30 Option (A) is correct.

A heat engine cycle is a thermodynamic cycle in which there is a net Heat transfer from higher temperature to a lower temperature device. So it is a Heat Engine.

Applying Clausius theorem on the system for checking the reversibility of the cyclic device.

$$\oint_R \frac{dQ}{T} = 0$$

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} - \frac{Q_3}{T_3} = 0$$

$$\frac{100 \times 10^3}{1000} + \frac{50 \times 10^3}{500} - \frac{60 \times 10^3}{300} = 0$$

$$100 + 100 - 200 = 0$$

Here, the cyclic integral of dQ/T is zero. This implies, it is a reversible Heat engine.

SOL 8.31 Option (C) is correct.

We know enthalpy,

$$h = U + p\nu \quad \text{...(i)}$$

Where, U = Internal energy

p = Pressure of the room

ν = Volume of the room

It is given that room is insulated, So there is no interaction of energy (Heat) between system (room) and surrounding (atmosphere).

It means Change in internal Energy $dU = 0$ and $U =$ Constant

And temperature is also remains constant.

Applying the perfect gas equation,

$$p\nu = nRT$$

$$p\nu = \text{Constant}$$

Therefore, from equation (i)

$$h = \text{Constant}$$

So this process is a constant internal energy and constant enthalpy process.

Alternate Method :

We know that enthalpy,

$$h = U + p\nu$$

Given that room is insulated, So there is no interaction of Energy (Heat) between system (room) and surrounding (atmosphere).

It means internal Energy $dU = 0$ and $U =$ constant.

Now flow work $p\nu$ must also remain constant thus we may conclude that during free expansion process $p\nu$ i.e. product of pressure and specific volume change in such a way that their product remains constant.

So, it is a constant internal energy and constant enthalpy process.

SOL 8.32 Option (A) is correct.

Given : $p_1 = 1\,\text{MPa}$, $T_1 = 350^\circ\text{C} = (350 + 273)\,\text{K} = 623\,\text{K}$

For air $\gamma = 1.4$

We know that final temperature (T_2) inside the tank is given by,

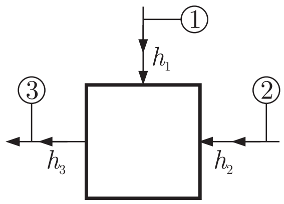
$$T_2 = \gamma T_1 = 1.4 \times 623 = 872.2\,\text{K} = 599.2^\circ\,\text{C}$$

T_2 is greater than 350° C.

SOL 8.33 Option (A) is correct.

Given : $h_1 = 2800\,\text{kJ/kg}$, $h_2 = 200\,\text{kJ/kg}$

From the given diagram of thermal power plant, point 1 is directed by the Boiler to the open feed water heater and point 2 is directed by the pump to the open feed water Heater. The bleed to the feed water heater is 20% of the boiler steam generation i.e. 20% of h_1



Open Feed Water Heater

So, $$h_3 = 20\% \text{ of } h_1 + 80\% \text{ of } h_2$$
$$= 0.2 \times 2800 + 0.8 \times 200 = 720 \text{ kJ/kg}$$

SOL 8.34 Option (C) is correct.

From the first law of thermodynamic,

$$dQ = dU + dW$$
$$dW = dQ - dU \qquad ...(i)$$

If the process is complete at the constant pressure and no work is done other than the $pd\nu$ work. So

$$dQ = dU + pd\nu$$

At constant pressure

$$pd\nu = d(p\nu)$$
$$(dQ) = dU + d(p\nu) = d(U + p\nu) = (dh) \qquad h = U + p\nu$$

From equation (i)

$$dW = -dh + dQ = -dh + Tds \qquad ds = dQ/T \quad ...(ii)$$

For an reversible process,

$$Tds = dh - \nu dp$$
$$-\nu dp = -dh + Tds \qquad ...(iii)$$

From equation (ii) and (iii)

$$dW = -\nu dp$$

On integrating both sides, we get

$$W = -\int \nu dp$$

It is valid for reversible process.

SOL 8.35 Option (A) is correct.

When the vapour is at a temperature greater than the saturation temperature, it is said to exist as super heated vapour. The pressure and Temperature of superheated vapour are independent properties, since the temperature may increase while the pressure remains constant. Here vapour is at 400° C and saturation temperature is 200° C.

So, at 200 kPa pressure superheated vapour will be left in the system.

SOL 8.36 Option (D) is correct.

Given : $p_1 = 100\,\text{kPa},\ p_2 = 200\,\text{kPa}$

Let, $\nu_1 = \nu$

Now, given that Heat transfer takes place into the system until its volume increases by 50%

So, $\nu_2 = \nu + 50\%\text{ of }\nu$

Now, for work done by the system, we must take pressure is $p_2 = 200\,\text{kPa}$, because work done by the system is against the pressure p_2 and it is a positive work done.

From first law of thermodynamics,

$$dQ = dU + dW \quad \text{...(i)}$$

But for a quasi-static process,

$$T = \text{Constant}$$

Therefore, change in internal energy is

$$dU = 0$$

From equation (i)

$$dQ = dW = pd\nu \qquad dW = pd\nu$$

$$= p[\nu_2 - \nu_1]$$

For initial condition at $100\,\text{kPa}$,volume

$$\nu_1 = m_{liquid} \times \frac{1}{\rho_f} + m_{vapour} \times \frac{1}{\rho_g}$$

Here $\frac{1}{\rho_f} = \nu_f = 0.001,\ \frac{1}{\rho_g} = \nu_g = 0.1$

$$m_{liquid} = 1\,\text{kg},\ m_{vapour} = 0.03\,\text{kg}$$

So $\nu_1 = 1 \times 0.001 + 0.03 \times 0.1 = 4 \times 10^{-3}\,\text{m}^3$

$$\nu_2 = \frac{3}{2}\nu_1 = \frac{3}{2} \times 4 \times 10^{-3} = 6 \times 10^{-3}\,\text{m}^3$$

$$= 200 \times 10^3\left[\frac{3\nu}{2} - \nu\right]$$

$$= 200 \times [6 \times 10^{-3} - 4 \times 10^{-3}]$$

$$= 200 \times 2 \times 10^{-3} = 0.4\,\text{kJ}$$

SOL 8.37 Option (C) is correct.

$$\Delta s_{net} = (\Delta s)_{system} + (\Delta s)_{surrounding} \quad \text{...(i)}$$

And it is given that,

$$(\Delta s)_{system} = 10\,\text{kJ}$$

Also, $(\Delta s)_{surrounding} = \left(\frac{Q}{T}\right)_{surrounding}$

Heat transferred to the system by thermal reservoir,

$$T = 400^\circ\,\text{C} = (400 + 273)\,\text{K} = 673\,\text{K}$$

$$Q = 1\,\text{kJ}$$
$$(\Delta s)_{surrounding} = \frac{1000}{673} = 1.485\,\text{J/K}$$

From equation (i)

$$(\Delta s)_{net} = 10 - 1.485 = 8.515\,\text{J/K}$$

(Take Negative sign, because the entropy of surrounding decrease due to heat transfer to the system.)

SOL 8.38 Option (D) is correct.

In this question we discuss on all the four options.

(A) $\delta Q = dU + \delta W$ This equation holds good for any process undergone by a closed stationary system.

(B) $Tds = dU + pd\nu$ This equation holds good for any process reversible or irreversible, undergone by a closed system.

(C) $Tds = dU + \delta W$ This equation holds good for any process, reversible or irreversible, and for any system.

(D) $\delta Q = dU + pd\nu$ This equation holds good for a closed system when only $pd\nu$ work is present. This is true only for a reversible (quasi-static) process.

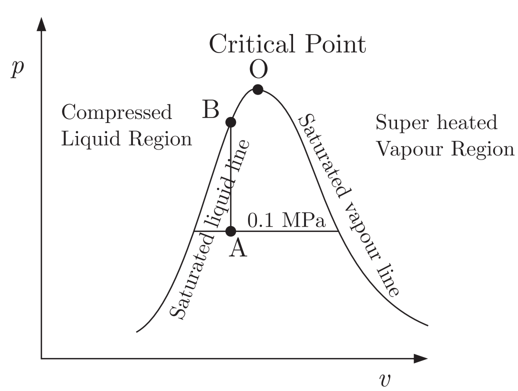
SOL 8.39 Option (A) is correct.

Given : $\nu_{cri} = 0.003155\,\text{m}^3/\text{kg}$, $\nu = 0.025\,\text{m}^3$, $p = 0.1\,\text{MPa}$ and $m = 10\,\text{kg}$

We know, Rigid means volume is constant.

Specific volume, $$\nu_S = \frac{\nu}{m} = \frac{0.025}{10} = 0.0025\,\text{m}^3/\text{kg}$$

We see that the critical specific volume is more than the specific volume and during the heating process, both the temperature and the pressure remain constant, but the specific volume increases to the critical volume (i.e. critical point). The critical point is defined as the point at which the saturated liquid and saturated vapour states are identical.



So, point (B) will touch the saturated liquid line and the liquid line will rise at the point O.

SOL 8.40 Option (C) is correct.

Given : $L = 250\text{ mm} = 0.25\text{ m},\ D = 200\text{ mm} = 0.2\text{ m},$

$$\nu_c = 0.001\text{ m}^3,\ \gamma = \frac{c_p}{c_v} = 1.4$$

Swept volume $\nu_s = A \times L = \frac{\pi}{4}(D)^2 \times L$

$$= \frac{\pi}{4}(0.2)^2 \times 0.25 = 0.00785\text{ m}^3$$

Compression ratio $r = \frac{\nu_T}{\nu_c} = \frac{\nu_c + \nu_s}{\nu_c} = \frac{0.001 + 0.00785}{0.001} = 8.85$

Air standard efficiency $\eta = 1 - \frac{1}{(r)^{\gamma - 1}} = 1 - \frac{1}{(8.85)^{1.4-1}}$

$$= 1 - \frac{1}{2.39} = 1 - 0.418 = 0.582 \text{ or } 58.2\%$$

SOL 8.41 Option (B) is correct.

We know, dryness fraction or quality of the liquid vapour mixture,

$$x = \frac{m_v}{m_v + m_l} = \frac{1}{1 + m_l / m_v} \quad ...(i)$$

Where, $m_v \to$ Mass of vapour and $m_l \to$ Mass of liquid

The value of x varies between 0 to 1. Now from equation (i) if incorporation of reheater in a steam power plant adopted then Mass of vapour m_v increase and Mass of liquid m_l decreases So, dryness fraction x increases.

In practice the use of reheater only gives a small increase in cycle efficiency, but it increases the net work output by making possible the use of higher pressure.

SOL 8.42 Option (A) is correct.

Following combination is correct

(R) The work done by a closed system in an adiabatic is a point function.

(S) A liquid expands upon freezing when the slope of its fusion curve on pressure-temperature diagram is negative.

SOL 8.43 Option (C) is correct.

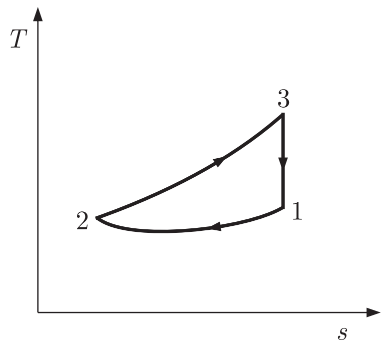
In the given p-ν diagram, three processes are occurred.

(i) Constant pressure (Process 1 – 2)

(ii) Constant Volume (Process 2 – 3)

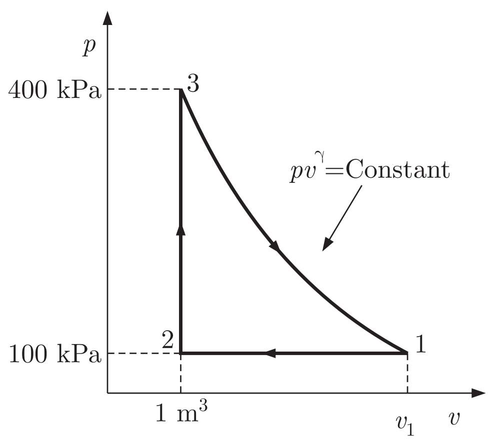
(iii) Adiabatic (Process 3 – 1)

We know that, Constant pressure and constant volume lines are inclined curves in the T-s curve, and adiabatic process is drawn by a vertical line on a T-s curve.



Given p-ν curve is clock wise. So T-s curve must be clockwise.

SOL 8.44 Option (A) is correct.



This cycle shows the Lenoir cycle.
For Lenoir cycle efficiency is given by

$$\eta_L = 1 - \gamma\left(\frac{r_p^{\frac{1}{\gamma}} - 1}{r_p - 1}\right)$$

Where, $$r_p = \frac{p_2}{p_1} = \frac{400}{100} = 4$$

And $$\gamma = \frac{c_p}{c_v} = 1.4 \text{ (Given)}$$

So, $$\eta_L = 1 - 1.4\left[\frac{(4)^{\frac{1}{1.4}} - 1}{4 - 1}\right] = 1 - 0.789 = 0.211$$

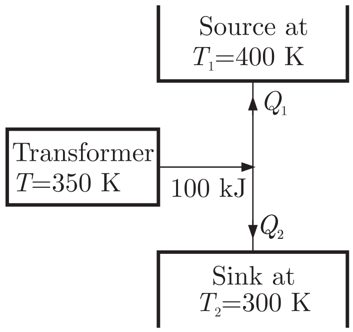
$$\eta_L = 21.1\% \simeq 21\%$$

SOL 8.45 Option (D) is correct.

Given : $T_1 = 400\,\text{K}$, $T_2 = 300\,\text{K}$, $T = 350\,\text{K}$, $Q = 100\,\text{kJ}$

Q_1 →Heat transferred to the source by the transformer

Q_2 → Heat transferred to the sink by the transformer



Applying energy balance on the system,

$$Q = Q_1 + Q_2$$

$$Q_2 = Q - Q_1 = 100 - Q_1 \qquad \text{...(i)}$$

Apply Clausicus inequality on the system,

$$\frac{Q}{T} = \frac{Q_1}{T_1} + \frac{Q_2}{T_2}$$

$$\frac{100}{350} = \frac{Q_1}{400} + \frac{Q_2}{300}$$

Substitute the value of Q_2 from equation (i),

$$\frac{100}{350} = \frac{Q_1}{400} + \left(\frac{100 - Q_1}{300}\right) = \frac{Q_1}{400} + \frac{100}{300} - \frac{Q_1}{300}$$

$$\frac{100}{350} - \frac{100}{300} = Q_1\left[\frac{1}{400} - \frac{1}{300}\right]$$

$$-\frac{1}{21} = -\frac{Q_1}{1200}$$

So, $$Q_1 = \frac{1200}{21} = 57.14\,\text{kJ}$$

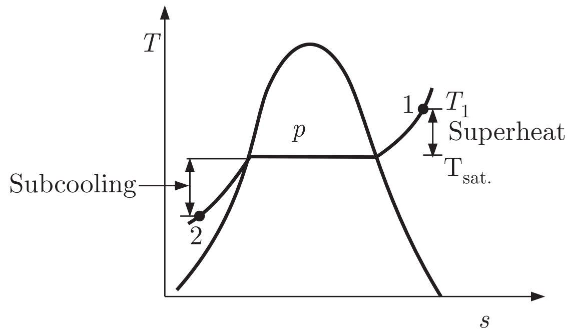
Therefore the maximum amount of heat that can be transferred at $400\,\text{K}$ is $57.14\,\text{kJ}$.

SOL 8.46 Option (D) is correct.

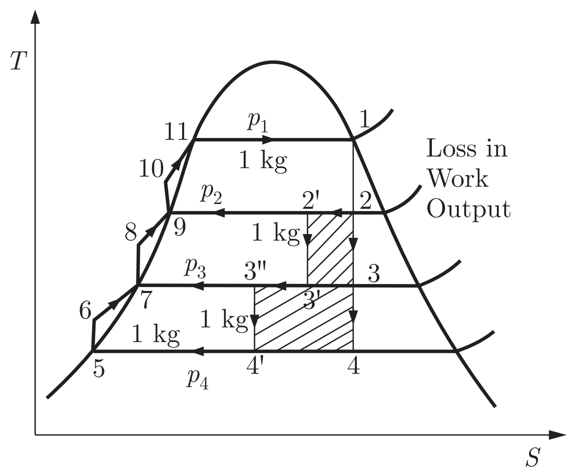
When the temperature of a liquid is less than the saturation temperature at the given pressure, the liquid is called compressed liquid (state 2 in figure). The pressure and temperature of compressed liquid may vary independently and a table of properties like the superheated vapor table could be arranged, to give the properties at any p and T.

The properties of liquids vary little with pressure. Hence, the properties are taken from the saturation table at the temperature of the compressed liquid. So, from the given table at $T = 45^\circ\text{C}$, Specific enthalpy of water

$= 188.45 \text{ kJ/kg}.$



SOL 8.47 Option (A) is correct.



The thermal efficiency of a power plant cycle increases by increase the average temperature at which heat is transferred to the working fluid in the boiler or decrease the average temperature at which heat is rejected from the working fluid in the condenser. Heat is transferred to the working fluid with the help of the feed water heater.

So, (A) and (R) are true and (R) is the correct reason of (A).

SOL 8.48 Option (D) is correct.

(A) Condenser is an essential equipment in a steam power plant because when steam expands in the turbine and leaves the turbine in the form of super saturated steam. It is not economical to feed this steam directly to the boiler. So, condenser is used to condensed the steam into water and it is a essential part (equipment) in steam power plant.

Assertion (A) is correct.

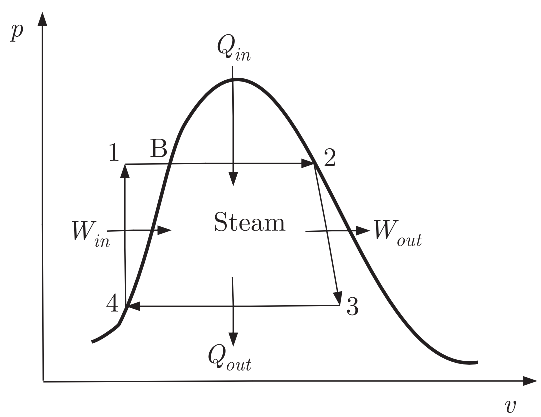
(R) The compressor and pumps require power input. The compressor is capable of compressing the gas to very high pressures. Pump work very much like compressor except that they handle liquid instead of gases. Now for same mass flow rate and the same pressure rise, a water pump require very less power because the specific volume of liquid is very less as compare to specific volume of vapour.

SOL 8.49 Option (D) is correct

Group (I)	Group (II)	Group (III)	Group (IV)	Group (V)
	When added to the system	Differential	Function	Phenomenon
E	G	J	K	N
F	H	J	K	M

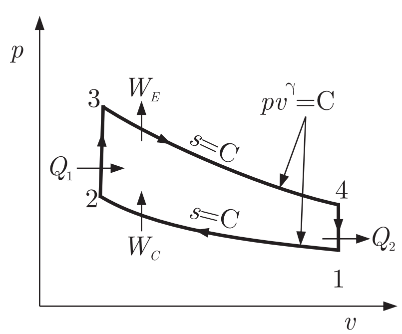
So correct pairs are
E-G-J-K-N and F-H-J-K-M

SOL 8.50 Option (A) is correct.
We draw p-v diagram for the cycles.
(a) Rankine cycle



Constant Pressure Process
$Q_1 =$ Heat addition at constant p and $Q_2 =$ Heat Rejection at constant p

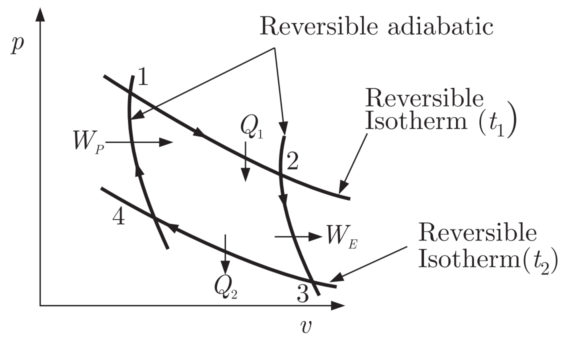
(b) Otto cycle



Constant Volume Process

$Q_1 =$ Heat addition at constant ν and $Q_2 =$ Heat Rejection at constant ν

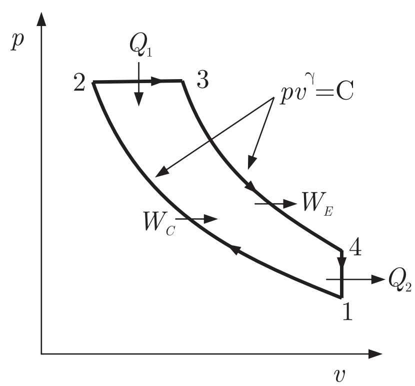
(c) Carnot cycle



Constant Temperature Process (Isothermal)

$Q_1 =$ Heat addition at constant T and $Q_2 =$ Heat Rejection at constant T

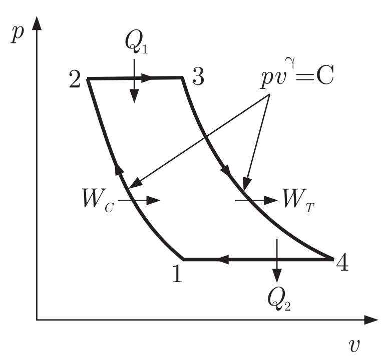
(d) Diesel cycle



Constant Pressure and constant volume process

$Q_1 =$ Heat addition at constant p and $Q_2 =$ Heat rejection at constant V

(e) Brayton cycle



Constant pressure Process

$Q_1 =$ Heat addition at constant p and $Q_2 =$ Heat rejection at constant p

From the Five cycles, we see that P - S - 5, R - U - 3, P - S - 1, Q - T - 2 are the correct pairs.

SOL 8.51 Option (D) is correct.

Given :
$$p_{gauge} = 1 \text{ bar}$$
$$p_{absolute} = p_{atm} + p_{gauge}$$
So,
$$p_{abs} = 1.013 + 1 = 2.013 \text{ bar} \qquad p_{atm} = 1.013 \text{ bar}$$
$$T_1 = 15^\circ \text{C} = (273 + 15) \text{K} = 288 \text{ K}$$
$$T_2 = 5^\circ \text{C} = (273 + 5) \text{K} = 278 \text{ K}$$
$$\text{Volume} = \text{Constant}$$
$$\nu_1 = \nu_2 = 2500 \text{ cm}^3 = 2500 \times (10^{-2})^3 \text{ m}^3$$
From the perfect gas equation,
$$p\nu = mRT$$
$$2.013 \times 10^5 \times 2500 \times (10^{-2})^3 = m \times 287 \times 288$$
$$2.013 \times 2500 \times 10^{-1} = m \times 287 \times 288$$
$$m = \frac{2.013 \times 250}{287 \times 288} = 0.0060 \text{ kg}$$
For constant Volume, relation is given by,
$$Q = mc_v dT \qquad c_v = 0.718 \text{ J/kg K}$$
$$= 0.0060 \times 0.718 \times (278 - 288) \qquad dT = T_2 - T_1$$
$$Q = -0.0437 = -43.7 \times 10^{-3} \text{ kJ}$$
$$= -43.7 \text{ Joule}$$ Negative sign shows the heat lost

As the process is isochoric i.e. constant volume, So from the prefect gas equation,
$$\frac{p}{T} = \text{Constant}$$
And
$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$
$$p_2 = \frac{T_2}{T_1} \times p_1 = \frac{278}{288} \times 2.013 = 1.943 \text{ bar} \qquad p_1 = p_{abs}$$
So,
$$\text{Gauge Pressure} = \text{Absolute pressure} - \text{atmospheric pressure}$$
$$p_{gauge} = 1.943 - 1.013 = 0.93 \text{ bar}$$

SOL 8.52 Option (C) is correct.

It is a constant volume process, it means
$$\frac{p}{T} = \text{Constant}$$
$$\frac{p_1}{p_2} = \frac{T_1}{T_2}$$

Substitute, $T_1 = 288$ and $T_2 = 278$

$$p_2 = p_{2,gauge} + p_{atm.} = 1 + 1.013$$

$$p_2 = 2.013\text{ bar}$$

So, $$p_1 = \frac{T_1}{T_2} \times p_2 = \frac{288}{278} \times 2.013 = 2.08\text{ bar}$$

Gauge pressure, $$p_{gauge} = 2.08 - 1.013 = 1.067 \simeq 1.07\text{ bar}$$

SOL 8.53 Option (A) is correct.

From the first law of thermodynamics for a cyclic process,

$$\Delta U = 0$$

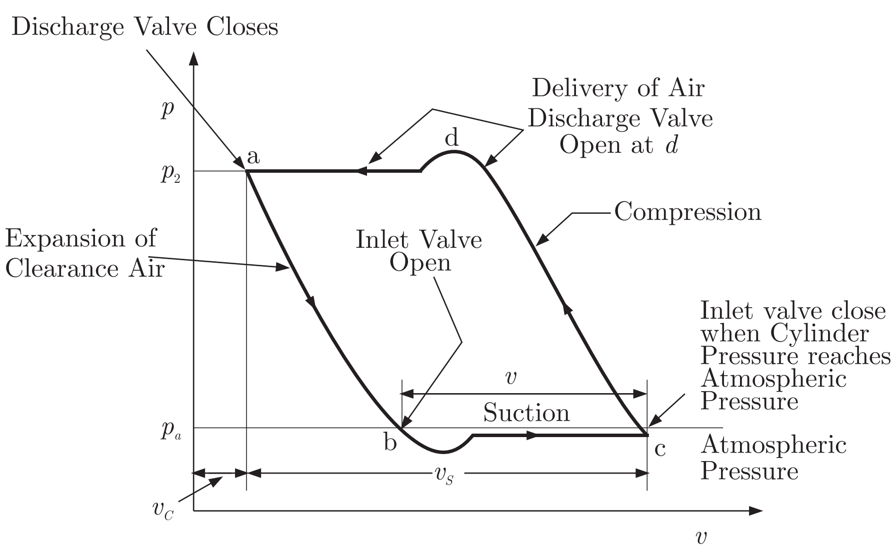
And $$\oint \delta Q = \oint \delta W$$

The symbol $\oint \delta Q$, which is called the cyclic integral of the heat transfer represents the heat transfer during the cycle and $\oint \delta W$, the cyclic integral of the work, represents the work during the cycle.

We easily see that figure 1 and 2 satisfies the first law of thermodynamics. Both the figure are in same direction (clockwise) and satisfies the relation.

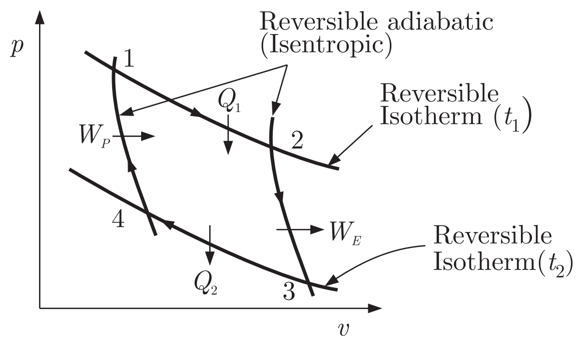
$$\oint \delta Q = \oint \delta W$$

SOL 8.54 Option (D) is correct.



From above figure, we can easily see that option (D) is same.

SOL 8.55 Option (A) is correct.



Now check the given processes :-

(i) Show in p-ν curve that process 1-2 and process 3-4 are Reversible isothermal process.

(ii) Show that process 2-3 and process 4-1 are Reversible adiabatic (isentropic) processes.

(iii) In carnot cycle maximum and minimum cycle pressure and the clearance volume are fixed.

(iv) From p-ν curve there is no polytropic process.

So, it consists only one cycle [carnot cycle]

SOL 8.56 Option (B) is correct.

Given : $p_1 = 10\text{ bar},\ \nu_1 = 1\text{ m}^3,\ T_1 = 300\text{ K},\ \nu_2 = 2\text{ m}^3$

Given that Nitrogen Expanded isothermally.

So, RT = Constant

And from given relation,

$$\left(p + \frac{a}{\nu^2}\right)\nu = RT = \text{Constant}$$

$$p_1\nu_1 + \frac{a}{\nu_1} = p_2\nu_2 + \frac{a}{\nu_2}$$

$$p_2\nu_2 = p_1\nu_1 + \frac{a}{\nu_1} - \frac{a}{\nu_2}$$

$$p_2 = p_1\left(\frac{\nu_1}{\nu_2}\right) + a\left(\frac{1}{\nu_1\nu_2} - \frac{1}{\nu_2^2}\right) = 10\left(\frac{1}{2}\right) + a\left(\frac{1}{2} - \frac{1}{4}\right) = 5 + \frac{a}{4}$$

Here $a > 0$, so above equation shows that p_2 is greater than 5 and +ve.

SOL 8.57 Option (B) is correct.

Velocity of flow, $u = u_1 = u_2 = \text{constant}$

& $W_2 >> W_1$ $W =$ Whirl velocity

Hence, it is a diagram of reaction turbine.

SOL 8.58 Option (B) is correct.

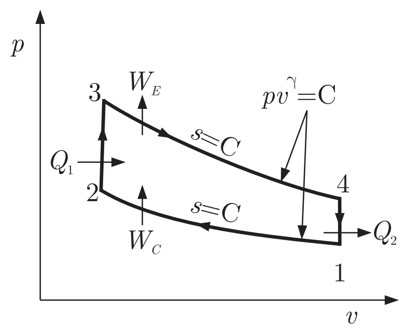
We know that efficiency,

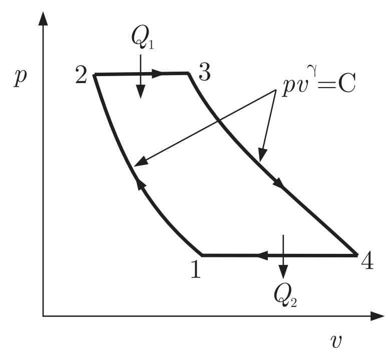
$$\eta_{Otto} = \eta_{Brayton} = 1 - \frac{T_1}{T_2}$$

$$\eta_{Otto} = \eta_{Brayton} = 1 - \frac{300}{450} = 1 - \frac{6}{9} = 0.33$$

So, $\eta_{Otto} = \eta_{Brayton} = 33\%$

SOL 8.59 Option (A) is correct.





From the previous part of the question

$T_{3(Otto)} = 600$ K, $T_{3(Brayton)} = 550$ K

From the p-v diagram of Otto cycle, we have

$$W_O = Q_1 - Q_2 = c_v(T_3 - T_2) - c_v(T_4 - T_1) \qquad ...(i)$$

For process 3 - 4,

$$\frac{T_3}{T_4} = \left(\frac{\nu_4}{\nu_3}\right)^{\gamma-1} = \left(\frac{\nu_1}{\nu_2}\right)^{\gamma-1} \qquad \nu_4 = \nu_1,\ \nu_3 = \nu_2$$

For process 1 - 2,

$$\frac{T_2}{T_1} = \left(\frac{\nu_1}{\nu_2}\right)^{\gamma-1}$$

So, $$\frac{T_3}{T_4} = \frac{T_2}{T_1}$$

$$T_4 = \frac{T_3}{T_2} \times T_1 = \frac{600}{450} \times 300 = 400 \text{ K}$$

And $$W_O = c_v(600 - 450) - c_v(400 - 300)$$

$$= c_v(150) - 100c_v = 50c_v \qquad ...(ii)$$

From p-ν diagram of brayton cycle, work done is,

$$W_B = Q_1 - Q_2 = c_p(T_3 - T_2) - c_p(T_4 - T_1)$$

And $$T_4 = \frac{T_1}{T_2} \times T_3 = \frac{300}{450} \times 550 = 366.67 \text{ K}$$

$$W_B = c_p(550 - 450) - c_p(366.67 - 300) = 33.33c_p \qquad ...(iii)$$

Dividing equation (ii) by (iii), we get

$$\frac{W_O}{W_B} = \frac{50c_v}{33.33c_p} = \frac{50}{33.33\gamma} \qquad \frac{c_p}{c_v} = \gamma, \gamma = 1.4$$

$$= \frac{50}{33.33 \times 1.4} = \frac{50}{46.662} > 1$$

From this, we see that,

$$W_O > W_B$$

SOL 8.60 Option (D) is correct.

From saturated ammonia table column 5 and 8 are the specific enthalpy data column.

SOL 8.61 Option (B) is correct.

The enthalpy of the fluid before throttling is equal to the enthalpy of fluid after throttling because in throttling process enthalpy remains constant.

$$h_1 = h_2$$

$$371.43 = 89.05 + x(1418 - 89.05) \qquad h = h_f + x(h_g - h_f)$$

$$= 89.05 + x(1328.95)$$

$$x = \frac{282.38}{1328.95} = 0.212$$

SOL 8.62 Option (C) is correct.

$W = -5000$ kJ (Negative sign shows that work is done on the system)

$Q = -2000$ kJ (Negative sign shows that heat rejected by the system)

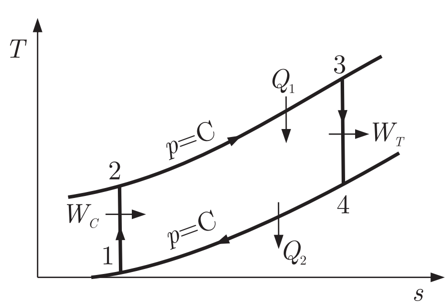
From the first law of thermodynamics,

$$\Delta Q = \Delta W + \Delta U$$

So, $$\Delta U = \Delta Q - \Delta W = -2000 - (-5000) = 3000 \text{ kJ}$$

SOL 8.63 Option (A) is correct.

The T-s curve for simple gas power plant cycle (Brayton cycle) is shown below :



From the T-s diagram, Net work output for Unit Mass,

$$W_{net} = W_T - W_c = c_p[(T_3 - T_4) - (T_2 - T_1)] \qquad \text{...(i)}$$

And from the T-s diagram,

$$T_3 = T_{max} \text{ and } T_1 = T_{min}$$

Apply the general relation for reversible adiabatic process, for process 3-4 and 1-2,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$T_4 = T_3(r_p)^{-\left(\frac{\gamma-1}{\gamma}\right)} \qquad \frac{p_3}{p_4} = \frac{p_2}{p_1} = r_p = \text{Pressure ratio}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1(r_p)^{\frac{\gamma-1}{\gamma}}$$

$$W_{net} = c_p\left[T_3 - T_3(r_p)^{-\left(\frac{\gamma-1}{\gamma}\right)} - T_1(r_p)^{\frac{\gamma-1}{\gamma}} + T_1\right] \qquad \text{...(ii)}$$

Differentiating equation (ii) w.r.t. (r_p) and on equating it to the zero, we get

$$\frac{dW_{net}}{dr_p} = c_p\left[-T_3\left(-\frac{\gamma-1}{\gamma}\right)r_p^{-\left(\frac{\gamma-1}{\gamma}\right)-1} - T_1\left(\frac{\gamma-1}{\gamma}\right)r_p^{\left(\frac{\gamma-1}{\gamma}-1\right)}\right]$$

$$= c_p\left[-T_3\left(-\frac{\gamma-1}{\gamma}\right)r_p^{\left(\frac{-\gamma+1-\gamma}{\gamma}\right)} - T_1\left(\frac{\gamma-1}{\gamma}\right)r_p^{\left(-\frac{1}{\gamma}\right)}\right]$$

$$= c_p\left[-T_3\left(-\frac{\gamma-1}{\gamma}\right)r_p^{\left(\frac{1-2\gamma}{\gamma}\right)} - T_1\left(\frac{\gamma-1}{\gamma}\right)r_p^{\left(-\frac{1}{\gamma}\right)}\right]$$

$$T_3 r_p^{\left(\frac{1}{\gamma}-2\right)} - T_1 r_p^{\left(-\frac{1}{\gamma}\right)} = 0$$

$$T_3 r_p^{\left(\frac{1}{\gamma}-2\right)} = T_1 r_p^{-\frac{1}{\gamma}}$$

$$\frac{T_3}{T_1} = \frac{(r_p)^{-\frac{1}{\gamma}}}{r_p^{\frac{1}{\gamma}-2}} = (r_p)^{-\frac{1}{\gamma}-\frac{1}{\gamma}+2} = r_p^{\frac{2(\gamma-1)}{\gamma}}$$

So, $$(r_p)_{opt} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}} = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{2(\gamma-1)}}$$

SOL 8.64 Option (C) is correct.

Stoichiometric mixture :

The S.M. is one in which there is just enough air for complete combustion of fuel.

SOL 8.65 Option (A) is correct.

Given : $m = 2000\text{ kg}$, $T_1 = 1250\text{ K}$, $T_2 = 450\text{ K}$, $T_0 = 303\text{ K}$, $c = 0.5\text{ kJ/kg K}$

$$Q_1 = \text{Available Energy} + \text{Unavailable energy}$$

$$A.E. = Q_1 - U.E. \qquad \text{...(i)}$$

And $$Q_1 = mc\Delta T$$

$$= 2000 \times 0.5 \times 10^3 \times (1250 - 450)$$

$$Q_1 = 800 \times 10^6 = 800 \text{ MJoule}$$

We know $\quad U.E. = T_0(\Delta s) \qquad$ ...(ii)

$$\Delta S = mc \ln\frac{T_1}{T_2} = 2000 \times 0.5 \times 10^3 \ln\frac{1250}{450}$$

$$= 10^6 \ln\frac{1250}{450} = 1.021 \times 10^6 \text{ J/kg}$$

Now, Substitute the value of Q_1 and $U.E.$ in equation (i),

$$A.E. = 800 \times 10^6 - 303 \times 1.021 \times 10^6 \quad \text{From equation (ii)}$$

$$= 10^6 \times [800 - 309.363]$$

$$= 490.637 \times 10^6 = 490.637 \backsimeq 490.44 \text{ MJ}$$

SOL 8.66 Option (C) is correct.

When all cylinders are firing then, power is 3037 kW = Brake Power

Power supplied by cylinders (Indicated power) is given below :

Cylinder No.	Power supplied (I.P.)
1.	I.P.$_1$ = 3037 − 2102 = 935 kW
2.	I.P.$_2$ = 3037 − 2102 = 935 kW
3.	I.P.$_3$ = 3037 − 2100 = 937 kW
4.	I.P.$_4$ = 3037 − 2098 = 939 kW

$$I.P._{Total} = \text{I.P.}_1 + \text{I.P.}_2 + \text{I.P.}_3 + \text{I.P.}_4$$

$$= 935 + 935 + 937 + 939 = 3746 \text{ kW}$$

And, $\quad \eta_{mech} = \dfrac{B.P.}{I.P.} = \dfrac{3037}{3746} = 0.8107 \text{ or } 81.07\%$

SOL 8.67 Option (D) is correct.

Given : $D = 10 \text{ cm} = 0.1 \text{ meter}, \ L = 15 \text{ cm} = 0.15 \text{ meter}$

$\gamma = \dfrac{c_p}{c_v} = 1.4, \ \nu_c = 196.3 \text{ cc}, \ Q = 1800 \text{ kJ/kg}$

$$\nu_s = A \times L = \frac{\pi}{4} D^2 \times L = \frac{\pi}{4} \times (10)^2 \times 15 = \frac{1500\pi}{4} = 1177.5 \text{ cc}$$

And Compression ratio, $\quad r = \dfrac{\nu_T}{\nu_c} = \dfrac{\nu_c + \nu_s}{\nu_c} = \dfrac{196.3 + 1177.5}{196.3} = 6.998 \simeq 7$

Cycle efficiency, $\quad \eta_{Otto} = 1 - \dfrac{1}{(r)^{\gamma - 1}} = 1 - \dfrac{1}{(7)^{1.4-1}}$

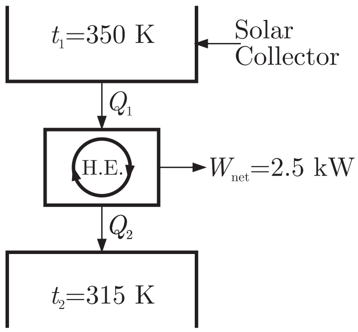
$$= 1 - \frac{1}{2.1779} = 1 - 0.4591 = 0.5409$$

$$\eta_{Otto} = 54.09\%$$

We know that, $\quad \eta = \dfrac{\text{Work output}}{\text{Heat Supplied}}$

$$\text{Work output} = \eta \times \text{Heat supplied}$$
$$= 0.5409 \times 1800 = 973.62\,\text{kJ} \simeq 973.5\,\text{kJ}$$

SOL 8.68 Option (A) is correct.



Solar collector receiving solar radiation at the rate of $0.6\,\text{kW/m}^2$. This radiation is stored in the form of internal energy. Internal energy of fluid after absorbing

Solar radiation, $\Delta U = \frac{1}{2} \times 0.6$ Efficiency of absorbing radiation is 50%

$$= 0.3\,\text{kW/m}^2$$

$$\eta_{Engine} = 1 - \frac{T_2}{T_1} = \frac{W_{net}}{Q_1}$$

$$Q_1 = \frac{W_{net} \times T_1}{T_1 - T_2} = \frac{2.5 \times 350}{350 - 315} = 25\,\text{kW}$$

Let, A is the minimum area of the solar collector.

So, $Q_1 = A \times \Delta U = A \times 0.3\,\text{kW/m}^2$

$$A = \frac{Q_1}{0.3} = \frac{25}{0.3} = \frac{250}{3} = 83.33\,\text{m}^2$$

SOL 8.69 Option (B) is correct.

Given : $h_1 = 29.3\,\text{kJ/kg}$, $h_3 = 3095\,\text{kJ/kg}$, $h_4 = 2609\,\text{kJ/kg}$, $h_5 = 3170\,\text{kJ/kg}$ $h_6 = 2165\,\text{kJ/kg}$

Heat supplied to the plant,

$$Q_S = (h_3 - h_1) + (h_5 - h_4) \quad \text{At boiler and reheater}$$
$$= (3095 - 29.3) + (3170 - 2609) = 3626.7\,\text{kJ}$$

Work output from the plant,

$$W_T = (h_3 - h_4) + (h_5 - h_6) = (3095 - 2609) + (3170 - 2165) = 1491\,\text{kJ}$$

Now, $\eta_{thermal} = \frac{W_T - W_p}{Q_s} = \frac{W_T}{Q_s}$ Given, $W_p = 0$

$$= \frac{1491}{3626.7} = 0.411 = 41.1\%$$

SOL 8.70 Option (D) is correct.

From the figure, we have enthalpy at exit of the pump must be greater than at inlet of pump because the pump supplies energy to the fluid.

$$h_2 > h_1$$

So, from the given four options only one option is greater than h_1

$$h_2 = 33.3\ \text{kJ/kg}$$

SOL 8.71 Option (B) is correct.

Equivalence Ratio or Fuel Air Ratio $\left(\frac{F}{A}\right)$

$$\phi = \frac{\text{Actual Fuel - Air ratio}}{\text{stoichiometric Fuel air Ratio}} = \frac{\left(\frac{F}{A}\right)_{actual}}{\left(\frac{F}{A}\right)_{stoichiometric}}$$

If $\phi = 1$, $\Rightarrow$ stoichiometric (Chemically correct) Mixture.

If $\phi > 1$, $\Rightarrow$ rich mixture.

If $\phi < 1$, $\Rightarrow$ lean mixture.

Now, we can see from these three conditions that $\phi > 1$, for both idling and peak power conditions, so rich mixture is necessary.

SOL 8.72 Option (C) is correct.

The compression ratio of diesel engine ranges between 14 to 25 where as for S.I, engine between 6 to 12. Diesel Engine gives more power but efficiency of diesel engine is less than compare to the S.I. engine for same compression ratio.

SOL 8.73 Option (C) is correct.

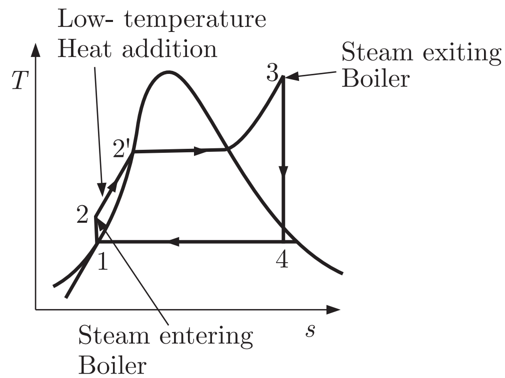


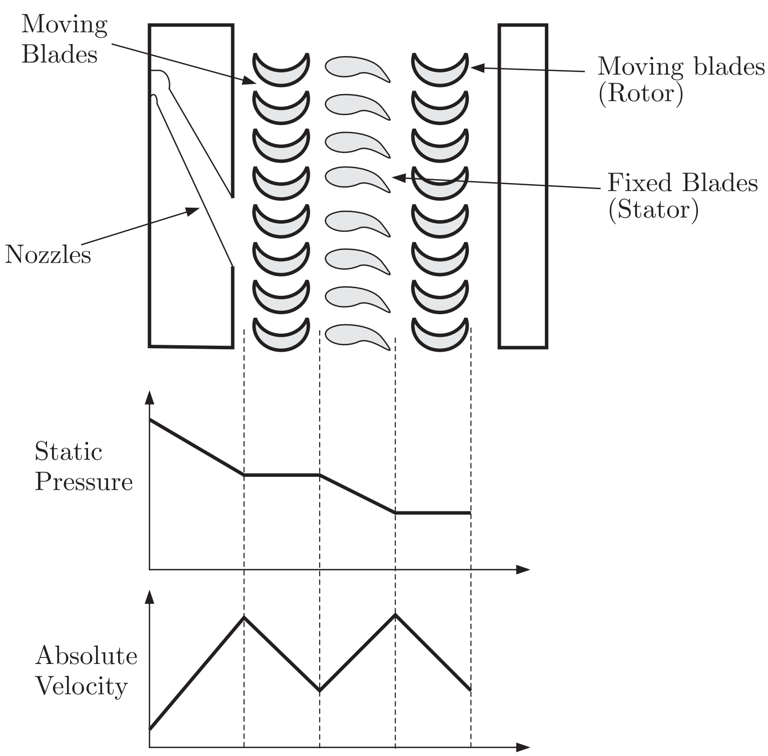
Fig : T - s curve of simple Rankine cycle

From the observation of the T - s diagram of the rankine cycle, it reveals that heat is transferred to the working fluid during process 2 – 2' at a relatively low temperature. This lowers the average heat addition temperature and thus the cycle efficiency.

To remove this remedy, we look for the ways to raise the temperature of the liquid leaving the pump (called the feed water) before it enters the boiler. One possibility is to transfer heat to the feed water from the expanding steam in a counter flow heat exchanger built into the turbine, that is, to use regeneration.

A practical regeneration process in steam power plant is accomplished by extracting steam from the turbine at various points. This steam is used to heat the feed water and the device where the feed water is heated by regeneration is called feed water heater. So, regeneration improves cycle efficiency by increasing the average temperature of heat addition in the boiler.

SOL 8.74 Option (D) is correct.



Easily shows that the diagram that static pressure remains constant, while velocity decreases.

SOL 8.75 Option (C) is correct.

Given : $p = 2\,\text{kW} = 2 \times 10^3\,\text{W}$, $t = 20\,\text{minutes} = 20 \times 60\,\text{sec}$,

$c_p = 4.2\,\text{kJ/kgK}$

Heat supplied, $Q = \text{Power} \times \text{Time}$

$$= 2 \times 10^3 \times 20 \times 60 = 24 \times 10^5 \text{ Joule}$$

And Specific heat at constant pressure,

$$Q = mc_p \Delta T$$

$$\Delta T = \frac{24 \times 10^5}{40 \times 4.2 \times 1000} = \frac{24 \times 100}{40 \times 4.2} = 14.3^\circ \text{C}$$

SOL 8.76 Option (D) is correct.

The Tds equation considering a pure, compressible system undergoing an internally reversible process.

From the first law of thermodynamics

$$(\delta Q)_{rev.} = dU + (\delta W)_{rev} \qquad ...(i)]$$

By definition of simple compressible system, the work is

$$(\delta W)_{rev} = pd\nu$$

And entropy changes in the form of

$$ds = \left(\frac{\delta Q}{T}\right)_{rev}$$

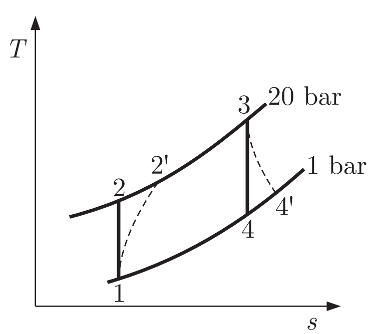
$$(\delta Q)_{rev} = Tds$$

From equation (i), we get

$$Tds = dU + pd\nu$$

This equation is equivalent to the I^{st} law, for a reversible process.

SOL 8.77 Option (A) is correct.



Given : $c_p = 0.98 \text{ kJ/kgK}, \ \eta_{isen} = 0.94, \ c_v = 0.7538 \text{ kJ/kgK}, \ T_3 = 1500 \text{ K}$
$p_3 = 20 \text{ bar} = 20 \times 10^5 \text{ N/m}^2, \ p_4 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$

$$\gamma = \frac{c_p}{c_v} = \frac{0.98}{0.7538} = 1.3$$

Apply general Equation for the reversible adiabatic process between point 3 and 4 in T-s diagram,

$$\left(\frac{T_3}{T_4}\right) = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{1500}{T_4} = \left(\frac{20 \times 10^5}{1 \times 10^5}\right)^{\frac{1.3-1}{1.3}} = (20)^{\frac{0.3}{1.3}}$$

$$T_4 = \frac{1500}{(20)^{\frac{0.3}{1.3}}} = 751.37 \text{ K}$$

And $\quad \eta_{isentropic} = \dfrac{\text{Actual output}}{\text{Ideal output}} = \dfrac{T_3 - T_4'}{T_3 - T_4}$

$$0.94 = \frac{1500 - T_4'}{1500 - 751.37}$$

$$0.94 \times 748.63 = 1500 - T_4'$$

$$T_4' = 1500 - 703.71 = 796.3 \text{ K}$$

Turbine work, $\quad W_t = c_p(T_3 - T_4') = 0.98(1500 - 796.3) = 698.64 \text{ kJ/kg}$

SOL 8.78 Option (A) is correct.

Given : $\phi = \dfrac{F}{A} = \dfrac{m_f}{m_a} = 0.05,\ \eta_v = 90\% = 0.90,\ \eta_{ith} = 30\% = 0.3$

$CV_{fuel} = 45 \text{ MJ/kg},\ \rho_{air} = 1 \text{ kg/m}^3$

We know that, volumetric efficiency is given by,

$$\eta_v = \frac{\text{Actual Volume}}{\text{Swept Volume}} = \frac{\nu_{ac}}{\nu_s}$$

$$\nu_{ac} = \eta_v \nu_s = 0.90 V_s \qquad \text{...(i)}$$

Mass of air, $\quad m_a = \rho_{air} \times \nu_{ac} = 1 \times 0.9\nu_s = 0.9\nu_s$

$$m_f = 0.05 \times m_a = 0.045\nu_s$$

$$\eta_{ith} = \frac{I.P.}{m_f \times CV} = \frac{p_{im} LAN}{m_f \times CV} \qquad I.P. = p_{im} LAN$$

$$p_{im} = \frac{\eta_{ith} \times m_f \times CV}{LAN} \qquad LAN = \nu_s$$

$$\frac{0.30 \times 0.045 \times \nu_s \times 45 \times 10^6}{\nu_s} = 0.6075 \times 10^6$$

$$= 6.075 \times 10^5 \text{ Pa} = 6.075 \text{ bar} \qquad 1 \text{ bar} = 10^5 \text{ Pa}$$

SOL 8.79 Option (D) is correct.

Given: $\quad \nu_c = 10\% \text{ of } \nu_s = 0.1\nu_s$

$$\frac{\nu_s}{\nu_c} = \frac{1}{0.1} = 10$$

And specific heat ratio $c_p/c_v = \gamma = 1.4$

We know compression ratio,

$$r = \frac{\nu_T}{\nu_c} = \frac{\nu_c + \nu_s}{\nu_c} = 1 + \frac{\nu_s}{\nu_c} = 1 + 10 = 11$$

Efficiency of Otto cycle,

$$\eta_{Otto} = 1 - \frac{1}{(r)^{\gamma - 1}} = 1 - \frac{1}{(11)^{1.4 - 1}}$$

$$= 1 - \frac{1}{(11)^{0.4}} = 1 - 0.3832 = 0.6168 \simeq 61.7\%$$

SOL 8.80 Option (B) is correct.

Given : $p_1 = 2\text{ bar} = 2 \times 10^5\text{ N/m}^2,\ T_1 = 298\text{ K} = T_2,\ \nu_1 = 1\text{ m}^3,\ \nu_2 = 2\text{ m}^3$

The process is isothermal,

So,
$$W = p_1\nu_1 \ln\frac{p_1}{p_2} = p_1\nu_1\ln\left(\frac{\nu_2}{\nu_1}\right) = 2 \times 10^5 \times 1\ln\left[\frac{2}{1}\right]$$
$$= 2 \times 0.6931 \times 10^5 = 138.63\text{ kJ} \simeq 138.6\text{ kJ}$$

SOL 8.81 Option (A) is correct.

Entropy,
$$\Delta S = \frac{\Delta Q}{T} \qquad \text{...(i)}$$

From first law of thermodynamics,
$$\Delta Q = \Delta U + \Delta W$$

For isothermal process,
$$\Delta U = 0$$
$$\Delta Q = \Delta W$$

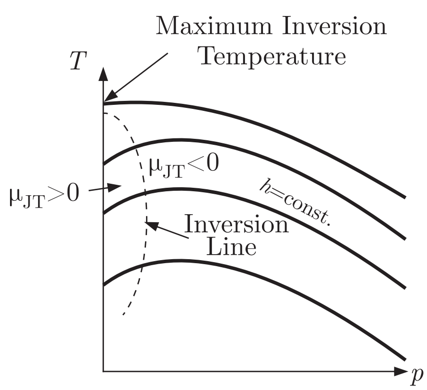
From equation (i),
$$\Delta S = \frac{\Delta W}{T} = \frac{138.63\text{ kJ}}{298\text{ K}} = 0.4652\text{ kJ/K}$$

SOL 8.82 Option (A) is correct.

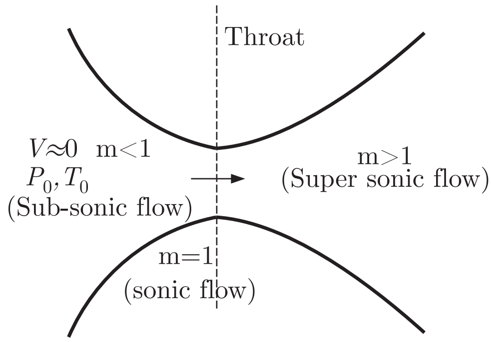
The Joule-Thomson coefficient is a measure of the change in temperature with pressure during a constant enthalpy process.

$$\mu = \left(\frac{\partial T}{\partial p}\right)_h$$

If
$$\mu_{JT} = \begin{cases} < 0 & \text{temperature increases} \\ = 0 & \text{Temperature remains constan t} \\ > 0 & \text{Temperature decreases during a throttling process} \end{cases}$$



SOL 8.83 Option (B) is correct.



The greatest velocity and lowest pressure occurs at the throat and the diverging portion remains a subsonic diffuser. For correctly designed convergent divergent nozzle, the throat velocity is sonic and the nozzle is now chocked.

SOL 8.84 Option (B) is correct.

Given : $\eta = 0.75$, $T_1 = 727^\circ \text{C} = (727 + 273) = 1000\,\text{K}$

The efficiency of Otto cycle is given by,

$$\eta = \frac{W_{net}}{Q_1} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

$$\frac{T_2}{T_1} = 1 - \eta \qquad \Rightarrow T_2 = (1 - \eta)\,T_1$$

$$T_2 = (1 - 0.75)\,1000 = 250\,\text{K or } -23^\circ\,\text{C}$$

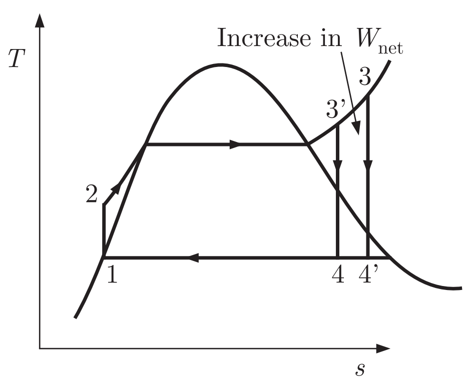
SOL 8.85 Option (A) is correct.

Given : $r = 8.5$, $\gamma = 1.4$

The efficiency of Otto cycle is,

$$\eta = 1 - \frac{1}{(r)^{\gamma - 1}} = 1 - \frac{1}{(8.5)^{1.4-1}} = 1 - \frac{1}{2.35} = 57.5\%$$

SOL 8.86 Option (B) is correct.

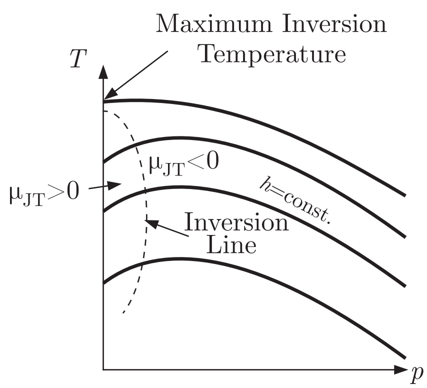


The average temperature at which heat is transferred to steam can be increased without increasing the boiler pressure by superheating the steam to high temperatures. The effect of superheating on the performance of vapour power cycle is shown on a T-s diagram the total area under the process curve $3-3'$ represents the increase in the heat input. Thus both the net work and heat input increase as a result of superheating the steam to a higher temperature. The overall effect is an increase in thermal efficiency, since the average temperature at which heat is added increases.

SOL 8.87 Option (A) is correct.

The Rateau turbine is a pressure compounded turbine.

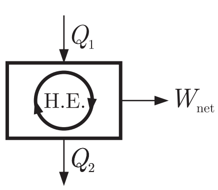
SOL 8.88 Option (B) is correct.



When $\mu < 0$ then temperature increases and become warmer.

SOL 8.89 Option (A) is correct.

Given : $W_{net} = 50\,\text{kJ}$, $\eta = 75\% = 0.75$



We know, efficiency of heat engine is,

$$\eta = \frac{W_{net}}{Q_1} \Rightarrow Q_1 = \frac{W_{net}}{\eta}$$

Where $Q_1 =$ Heat transferred by the source to the system.

$$Q_1 = \frac{50}{0.75} = 66.67\,\text{kJ}$$

From the figure heat rejected Q_2

(From the energy balance)

$$Q_1 = Q_2 + W_{net}$$
$$Q_2 = Q_1 - W_{net} = 66.67 - 50 = 16.67 = 16\frac{2}{3}\text{ kJ}$$

SOL 8.90 Option (C) is correct.

Given : $p_1 = 1\text{ bar},\ p_2 = 16\text{ bar}$

The intermediate pressure p_x (pressure ratio per stage) has an optimum value for minimum work of compression.

And $$p_x = \sqrt{p_1 p_2} = \sqrt{1 \times 16} = 4\text{ bar}$$

SOL 8.91 Option (B) is correct.

Let h_1 and h_2 are the enthalpies of steam at the inlet and at the outlet.

Given : $$h_1 - h_2 = 0.8\text{ kJ/kg}$$
$$V_1 = 0$$

From the energy balance for unit mars of steam, the total energy at inlet must be equal to total energy at outlet.

So, $$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$
$$V_2^2 = 2(h_1 - h_2)$$
$$V_2 = \sqrt{2 \times 0.8 \times 10^3} = 40\text{ m/sec}$$

SOL 8.92 Option (B) is correct.

Given : $$r = 5.5,\ W = 23.625 \times 10^5 \times \nu_c$$

We know, $$p_{mep} = \frac{W_{net}}{\nu_s} = \frac{23.625 \times 10^5}{\nu_s/\nu_c} \quad ...(i)$$

Where ν_s = swept volume

And $$r = \frac{\nu}{\nu_c} = \frac{\nu_c + \nu_s}{\nu_c} = 1 + \frac{\nu_s}{\nu_c}$$
$$\frac{\nu_s}{\nu_c} = (r - 1)$$

Where ν_t = Total volume

ν_c = clearance volume

Substitute this value in equation (i), we get

$$p_{mep} = \frac{23.625 \times 10^5}{r - 1} = \frac{23.625 \times 10^5}{5.5 - 1} = 5.25 \times 10^5 = 5.25\text{ bar}$$

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CHAPTER 9

REFRIGERATION & AIR-CONDITIONING

YEAR 2012 ONE MARK

• **Common Data For Q.1 and Q.2**

A refrigerator operates between 120 kPa and 800 kPa in an ideal vapour compression cycle with R-134a as the refrigerant. The refrigerant enters the compressor as saturated vapour and leaves the condenser as saturated liquid. The mass flow rate of the refrigerant is 0.2 kg/s. Properties for R134a are as follows :

Saturated R-134a					
p(kPa)	T($^\circ$C)	h_f(kJ/kg)	h_g(kJ/kg)	s_f(kJ/kgK)	s_g(kJ/kgK)
120	−22.32	22.5	237	0.093	0.95
800	31.31	95.5	267.3	0.354	0.918
Superheated R-134a					
p(kPa)		T($^\circ$C)	h(kJ/kg)		s(kJ/kgK)
800		40	276.45		0.95

MCQ 9.1 The rate at which heat is extracted, in kJ/s from the refrigerated space is

(A) 28.3 (B) 42.9

(C) 34.4 (D) 14.6

YEAR 2012 TWO MARKS

MCQ 9.2 The power required for the compressor in kW is

(A) 5.94 (B) 1.83

(C) 7.9 (D) 39.5

YEAR 2011 ONE MARK

MCQ 9.3 If a mass of moist air in an airtight vessel is heated to a higher temperature, then

(A) specific humidity of the air increases

(B) specific humidity of the air decreases

(C) relative humidity of the air increases

(D) relative humidity of the air decreases

YEAR 2010 ONE MARK

MCQ 9.4 A moist air sample has dry bulb temperature of 30°C and specific humidity of 11.5 g water vapour per kg dry air. Assume molecular weight of air as 28.93. If the saturation vapour pressure of water at 30°C is 4.24 kPa and the total pressure is 90 kPa, then the relative humidity (in %) of air sample is

(A) 50.5 (B) 38.5

(C) 56.5 (D) 68.5

YEAR 2009 ONE MARK

MCQ 9.5 In an ideal vapour compression refrigeration cycle, the specific enthalpy of refrigerant (in kJ/kg) at the following states is given as:

Inlet of condenser :283

Exit of condenser :116

Exit of evaporator :232

The COP of this cycle is

(A) 2.27 (B) 2.75

(C) 3.27 (D) 3.75

YEAR 2008 TWO MARKS

MCQ 9.6 Moist air at a pressure of 100 kPa is compressed to 500 kPa and then cooled to 35°C in an aftercooler. The air at the entry to the aftercooler is unsaturated and becomes just saturated at the exit of the aftercooler. The saturation pressure of water at 35°C is 5.628 kPa. The partial pressure of water vapour (in kPa) in the moist air entering the compressor is closest to

(A) 0.57 (B) 1.13

(C) 2.26 (D) 4.52

MCQ 9.7 Air (at atmospheric pressure) at a dry bulb temperature of 40°C and wet bulb temperature of 20°C is humidified in an air washer operating with continuous water recirculation. The wet bulb depression (i.e. the difference between the dry and wet bulb temperature) at the exit is 25% of that at the inlet. The dry bulb temperature at the exit of the air washer is closest to

(A) 10°C (B) 20°C

(C) 25°C (D) 30°C

YEAR 2007 TWO MARKS

MCQ 9.8 A building has to be maintained at 21°C (dry bulb) and 14.5°C (wet bulb). The dew point temperature under these conditions is 10.17°C. The outside temperature is -23°C (dry bulb) and the internal and external surface heat transfer coefficients are 8 W/m^{2}K and 23 W/m^{2}K respectively. If the building wall has a thermal conductivity of 1.2 W/m K, the minimum thickness (in m) of the wall required to prevent condensation is

(A) 0.471 (B) 0.407

(C) 0.321 (D) 0.125

MCQ 9.9 Atmospheric air at a flow rate of 3 kg/s (on dry basis) enters a cooling and dehumidifying coil with an enthalpy of 85 kJ/ kg of dry air and a humidity ratio of 19 grams/kg of dry air. The air leaves the coil with an enthalpy of 43 kJ/kg of dry air and a humidity ratio of 8 grams/kg of dry air. If the condensate water leaves the coil with an enthalpy of 67 kJ/kg, the required cooling capacity of the coil in kW is

(A) 75.0 (B) 123.8

(C) 128.2 (D) 159.0

YEAR 2006 ONE MARK

MCQ 9.10 Dew point temperature is the temperature at which condensation begins when the air is cooled at constant

(A) volume (B) entropy

(C) pressure (D) enthalpy

YEAR 2006 TWO MARKS

MCQ 9.11 The statements concern psychrometric chart.

1. Constant relative humidity lines are uphill straight lines to the right
2. Constant wet bulb temperature lines are downhill straight lines to the right

3. Constant specific volume lines are downhill straight lines to the right
4. Constant enthalpy lines are coincident with constant wet bulb temperature lines

Which of the statements are correct ?

(A) 2 and 3 (B) 1 and 2

(C) 1 and 3 (D) 2 and 4

YEAR 2005 ONE MARK

MCQ 9.12 For a typical sample of ambient air (at 35°C, 75% relative humidity and standard atmosphere pressure), the amount of moisture in kg per kg of dry air will be approximately

(A) 0.002 (B) 0.027

(C) 0.25 (D) 0.75

MCQ 9.13 Water at 42°C is sprayed into a stream of air at atmospheric pressure, dry bulb temperature of 40°C and a wet bulb temperature of 20°C. The air leaving the spray humidifier is not saturated. Which of the following statements is true ?

(A) Air gets cooled and humidified

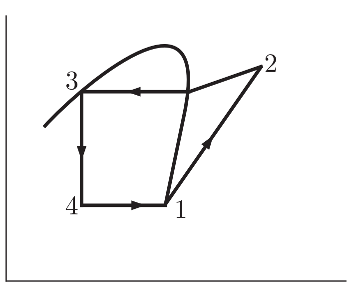
(B) Air gets heated and humidified

(C) Air gets heated and dehumidified

(D) Air gets cooled and dehumidified

YEAR 2005 TWO MARKS

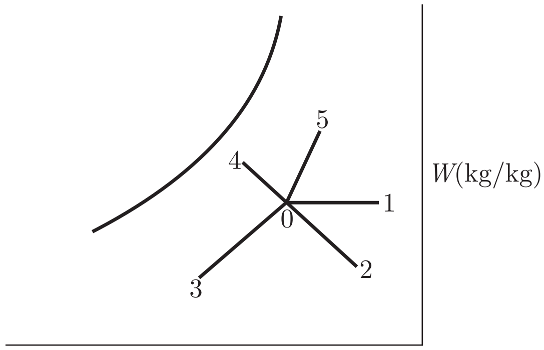
MCQ 9.14 The vapour compression refrigeration cycle is represented as shown in the figure below, with state 1 being the exit of the evaporator. The coordinate system used in this figure is



(A) p-h (B) T-s

(C) p-s (D) T-h

MCQ 9.15 Various psychometric processes are shown in the figure below.



	Process in Figure		Name of the process
P.	0 - 1	(i).	Chemical dehumidification
Q.	0 - 2	(ii).	Sensible heating
R.	0 - 3	(iii).	Cooling and dehumidification
S.	0 - 4	(iv).	Humidification with steam injection
T.	0 - 5	(v).	Humidification with water injection

The matching pairs are

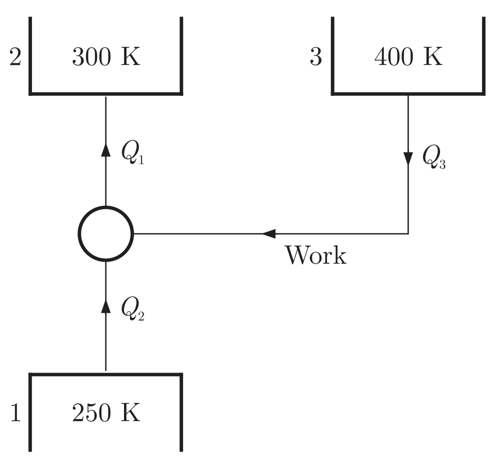
(A) P-(i), Q-(ii), R-(iii), S-(iv), T-(v)

(B) P-(ii), Q-(i), R-(iii), S-(v), T-(iv)

(C) P-(ii), Q-(i), R-(iii), S-(iv), T-(v)

(D) P-(iii), Q-(iv), R-(v), S-(i), T-(ii)

MCQ 9.16 A vapour absorption refrigeration system is a heat pump with three thermal reservoirs as shown in the figure. A refrigeration effect of 100 W is required at 250 K when the heat source available is at 400 K. Heat rejection occurs at 300 K. The minimum value of heat required (in W) is



(A) 167 (B) 100

(C) 80 (D) 20

YEAR 2004 ONE MARK

MCQ 9.17 In the window air conditioner, the expansion device used is

(A) capillary tube (B) thermostatic expansion valve

(C) automatic expansion valve (D) float valve

MCQ 9.18 During the chemical dehumidification process of air

(A) dry bulb temperature and specific humidity decreases

(B) dry bulb temperature increases and specific humidity decreases

(C) dry bulb temperature decreases and specific humidity increases

(D) dry bulb temperature and specific humidity increases

MCQ 9.19 Environment friendly refrigerant R134 is used in the new generation domestic refrigerators. Its chemical formula is

(A) $CHClF_2$ (B) $C_2Cl_3F_3$

(C) $C_2Cl_2F_4$ (D) $C_2H_2F_4$

YEAR 2004 TWO MARKS

MCQ 9.20 A heat engine having an efficiency of 70% is used to drive a refrigerator having a coefficient of performance of 5. The energy absorbed from low temperature reservoir by the refrigerator for each kJ of energy absorbed from high temperature source by the engine is

(A) 0.14 kJ (B) 0.71 kJ

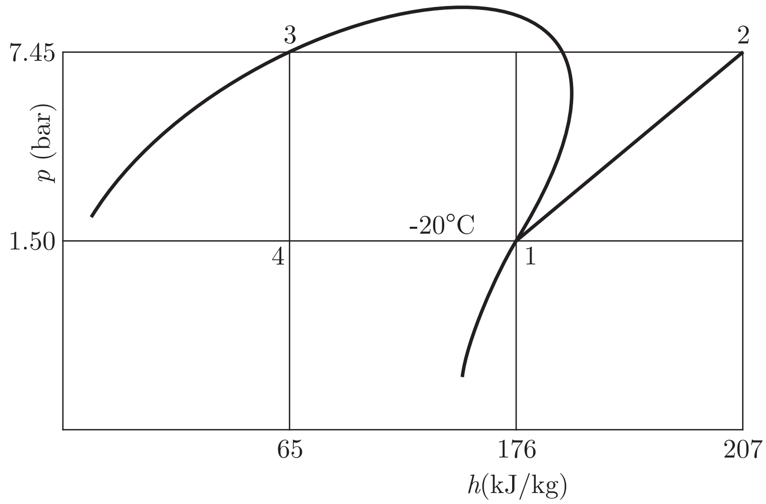
(C) 3.5 kJ (D) 7.1 kJ

MCQ 9.21 Dew point temperature of air at one atmospheric pressure (1.013 bar) is 18°C. The air dry bulb temperature is 30°C. The saturation pressure of water at 18°C and 30°C are 0.02062 bar and 0.04241 bar respectively. The specific heat of air and water vapour respectively are 1.005 and 1.88 kJ/kg K and the latent heat of vaporization of water at 0°C is 2500 kJ/kg. The specific humidity (kg/kg of dry air) and enthalpy (kJ/kg or dry air) of this moist air respectively, are

(A) 0.01051, 52.64 (B) 0.01291, 63.15

(C) 0.01481, 78.60 (D) 0.01532, 81.40

MCQ 9.22 A R-12 refrigerant reciprocating compressor operates between the condensing temperature of 30°C and evaporator temperature of -20°C. The clearance volume ratio of the compressor is 0.03. Specific heat ratio of the vapour is 1.15 and the specific volume at the suction is 0.1089 m^3/kg. Other properties at various states are given in the figure. To realize 2 tons of refrigeration, the actual volume displacement rate considering the effect of clearance is



(A) 6.35×10^{-3} m^3/s (B) 63.5×10^{-3} m^3/s

(C) 635×10^{-3} m^3/s (D) 4.88×10^{-3} m^3/s

YEAR 2003 ONE MARK

MCQ 9.23 An industrial heat pump operates between the temperatures of 27° C and $-13°$ C. The rates of heat addition and heat rejection are 750 W and 1000 W, respectively. The COP for the heat pump is

(A) 7.5 (B) 6.5

(C) 4.0 (D) 3.0

MCQ 9.24 For air with a relative humidity of 80%

(A) the dry bulb temperature is less than the wet bulb temperature

(B) the dew point temperature is less than wet bulb temperature

(C) the dew point and wet bulb temperature are equal

(D) the dry bulb and dew point temperature are equal

YEAR 2003 TWO MARKS

• Common Data For Q.25 and Q.26

A refrigerator based on ideal vapour compression cycle operates between the temperature limits of $-20°$ C and 40° C. The refrigerant enters the condenser as saturated vapour and leaves as saturated liquid. The enthalpy and entropy values for saturated liquid and vapour at these temperatures are given in the table below.

T ($^\circ$C)	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kg K)	s_g (kJ/kg K)
-20	20	180	0.07	0.7366
40	80	200	0.3	0.67

MCQ 9.25 If refrigerant circulation rate is 0.025 kg/s, the refrigeration effect is equal to

(A) 2.1 kW (B) 2.5 kW

(C) 3.0 kW (D) 4.0 kW

MCQ 9.26 The COP of the refrigerator is

(A) 2.0 (B) 2.33

(C) 5.0 (D) 6.0

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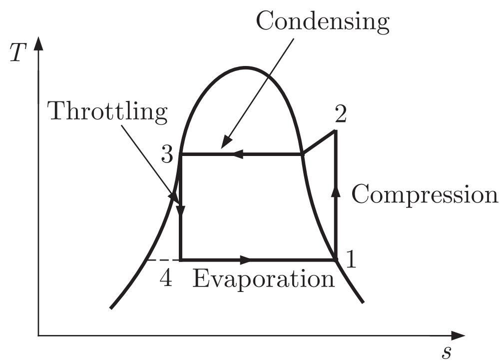
SOLUTION

SOL 9.1 Option (A) is correct.



T-s diagram for given Refrigeration cycle is given above

Since Heat is extracted in evaporation process.

So rate of heat extracted $= \dot{m}(h_1 - h_4)$

From above diagram $(h_3 = h_4)$ for throttling process, so

$$\text{Heat extracted} = \dot{m}(h_1 - h_3)$$

From given table

$h_1 = h_g$ at $120\,\text{kPa}$, $h_g = 237\,\text{kJ/kg}$

$h_3 = h_f$ at $120\,\text{kPa}$, $h_f = 95.5\,\text{kJ/kg}$

Hence $\quad \text{Heat extracted} = \dot{m}(h_g - h_f) = 0.2 \times (237 - 95.5) = 28.3\,\text{kJ/s}$

SOL 9.2 Option (C) is correct.

Since power is required for compressor in refrigeration is in compression cycle (1-2)

Hence, $\quad \text{Power required} = \dot{m}(h_2 - h_1) = \dot{m}(h_2 - h_f)$

Since for isentropic compression process.

$$s_1 = s_2 \text{ from figure.} = 0.95$$

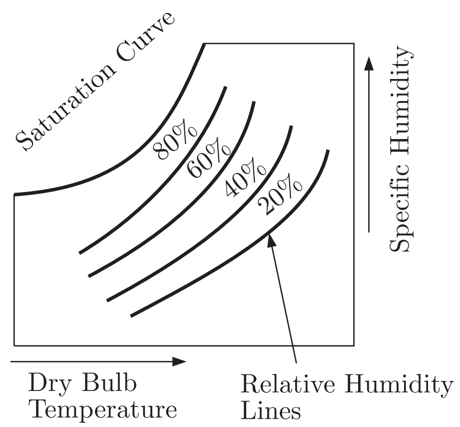
For entropy $s = 0.95$ the enthalpy $h = 276.45\,\text{kJ/kg}$

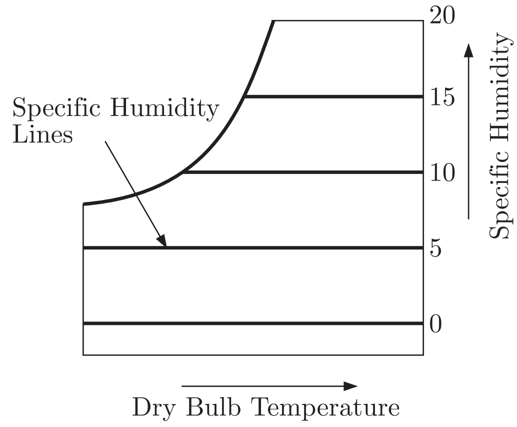
$$h = h_2 = 276.45 \text{ (From table)}$$

Hence $\quad \text{Power} = 0.2(276.45 - 237) = 7.89 \simeq 7.9\,\text{kW}$

SOL 9.3 Option (D) is correct.

From the given curve, we easily see that relative humidity of air decreases, when temperature of moist air in an airtight vessel increases. So, option (C) is correct. Specific humidity remain constant with temperature increase, so option a & b are incorrect.





SOL 9.4 Option (B) is correct.

Given : $t_{DBT} = 30^\circ$ C, $W = 11.5$ g water vapour/kg dry air

$p_s = 4.24$ kPa, $p = 90$ kPa

Specific humidity, $$W = 0.622\left(\frac{p_v}{p - p_v}\right)$$

Substitute the values, we get

$$11.5 \times 10^{-3} = 0.622\left(\frac{p_v}{90 - p_v}\right)$$

$$18.489 \times 10^{-3} = \frac{p_v}{90 - p_v}$$

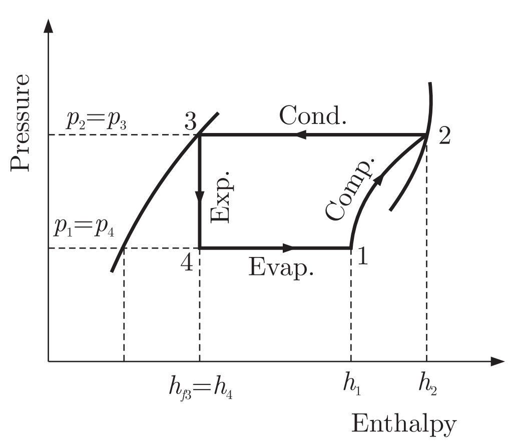
$$(90 \times 18.489 - 18.489 p_v) \times 10^{-3} = p_v \Rightarrow \quad p_v = 1.634 \text{ kPa}$$

Relative humidity $$\phi = \frac{p_v}{p_s} = \frac{1.634}{4.24}$$

$$\phi = 0.3853 = 38.53\% \simeq 38.5\%$$

SOL 9.5 Option (A) is correct.

$p - h$ curve for vapour compression refrigeration cycle is as follows



The given specific enthalpies are

Inlet of condenser $h_2 = 283\,\text{kJ/kg}$

Exit of condenser $h_3 = 116\,\text{kJ/kg} = h_4$ From p-h curve

Exit of evaporator $h_1 = 232\,\text{kJ/kg}$

Now, $$COP = \frac{\text{Refrigerating effect}}{\text{Work done}} = \frac{h_1 - h_4}{h_2 - h_1}$$

Substitute the values, we get

$$COP = \frac{232 - 116}{283 - 232} = \frac{116}{51} = 2.27$$

SOL 9.6 Option (B) is correct.

Given : $p_1 = 100\,\text{kPa}$, $p_2 = 500\,\text{kPa}$, $p_{v1} = ?$

$p_{v2} = 5.628\,\text{kPa}$ (Saturated pressure at 35°C)

We know that,

Specific humidity $$W = 0.622\left(\frac{p_v}{p - p_v}\right)$$

For case II :

$$W = 0.622\left(\frac{5.628}{500 - 5.628}\right) = 7.08 \times 10^{-3}\ \text{kg/kg of dry air}$$

For saturated air specific humidity remains same. So, for case (I) :

$$W = 0.622\left(\frac{p_{v1}}{p_1 - p_{v1}}\right)$$

On substituting the values, we get

$$7.08 \times 10^{-3} = 0.622\left(\frac{p_{v1}}{100 - p_{v1}}\right)$$

$$11.38 \times 10^{-3}(100 - p_{v1}) = p_{v1}$$

$$1.138 = 1.01138 p_{v1}$$

$$p_{v1} = 1.125\,\text{kPa} \simeq 1.13\,\text{kPa}$$

SOL 9.7 Option (C) is correct.

Given : At inlet $t_{DBT} = 40^\circ\text{C}$, $t_{WBT} = 20^\circ\text{C}$

We know that, wet bulb depression $= t_{DBT} - t_{WBT} = 40 - 20 = 20^\circ\text{C}$

And given wet bulb depression at the exit = 25% of wet bulb depression at inlet

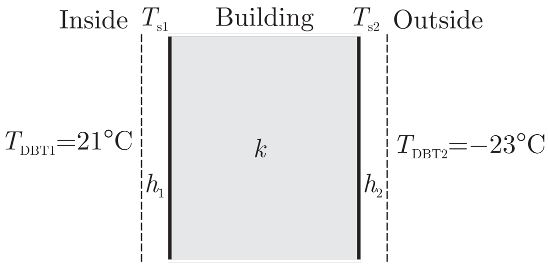
This process becomes adiabatic saturation and for this process,

$$t_{WBT(\text{inlet})} = t_{WBT(\text{outlet})}$$

So, $$t_{DBT(\text{exit})} - 20 = 0.25 \times 20$$

$$t_{DBT(\text{exit})} = 20 + 5 = 25^\circ\text{C}$$

SOL 9.8 Option (B) is correct.



Let h_1 & h_2 be the internal and external surface heat transfer coefficients respectively and building wall has thermal conductivity k.

Given : $h_1 = 8\ \text{W/m}^2\ \text{K}$, $h_2 = 23\ \text{W/m}^2\ \text{K}$, $k = 1.2\ \text{W/m K}$, $T_{DPT} = 10.17^\circ\ \text{C}$

Now to prevent condensation, temperature of inner wall should be more than or equal to the dew point temperature. It is the limiting condition to prevent condensation

So, $T_{s1} = 10.17^\circ\ \text{C}$

Here T_{s1} & T_{s2} are internal & external wall surface temperature of building. Hence, heat flux per unit area inside the building,

$$q_i = \frac{Q}{A} = h_1(T_{DBT1} - T_{s1})$$

$$q_i = 8(21 - 10.17) = 8 \times 10.83 = 86.64\ \text{W/m}^2 \qquad ...(i)$$

& Heat flux per unit area outside the building is

$$q_0 = h_2(T_{s2} - T_{DBT2}) = 23(T_{s2} + 23) \qquad ...(ii)$$

Heat flow will be same at inside & outside the building. So from equation (i) & (ii)

$$q_i = q_0$$

$$86.64 = 23(T_{s2} + 23)$$

$$T_{s2} + 23 = 3.767$$

$$T_{s2} = 3.767 - 23 = -19.23^\circ\ \text{C}$$

For minimum thickness of the wall, use the fourier's law of conduction for the building. Heat flux through wall,

$$q = \frac{k(T_{s1} - T_{s2})}{x} = \frac{1.2 \times (10.17 + 19.23)}{x}$$

Substitute the value of q_i from equation (i), we get

$$86.64 = \frac{1.2 \times 29.4}{x}$$

$$x = \frac{35.28}{86.64} = 0.407\ \text{m}$$

Note :- Same result is obtained with the value of q_o

SOL 9.9 Option (C) is correct.

Given : $\dot{m}_a = 3$ kg/sec,

Using subscript 1 and 2 for the inlet and outlet of the coil respectively.

$h_1 = 85$ kJ/kg of dry air, $W_1 = 19$ grams/kg of dry air $= 19 \times 10^{-3}$ kg/kg of dry air

$h_2 = 43$ kJ/kg of dry air, $W_2 = 8$ grams/kg of dry air $= 8 \times 10^{-3}$ kg/kg of dry air

$h_3 = 67$ kJ/kg

Mass flow rate of water vapour at the inlet of the coil is,

$$\dot{m}_{v1} = W_1 \times \dot{m}_a \qquad W = \frac{\dot{m}_v}{\dot{m}_a}$$

$$\dot{m}_{v1} = 19 \times 10^{-3} \times 3 = 57 \times 10^{-3} \text{ kg/ sec}$$

And mass flow rate of water vapour at the outlet of coil is,

$$\dot{m}_{v2} = W_2 \times \dot{m}_a$$

$$= 8 \times 10^{-3} \times 3 = 24 \times 10^{-3} \text{ kg/ sec}$$

So, mass of water vapour condensed in the coil is,

$$\dot{m}_v = \dot{m}_{v1} - \dot{m}_{v2}$$

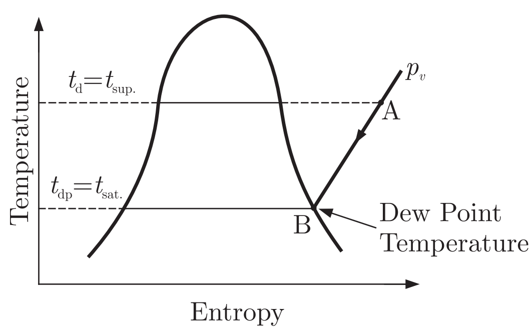
$$= (57 - 24) \times 10^{-3} = 33 \times 10^{-3} \text{ kg/ sec}$$

Therefore, required cooling capacity of the coil = change in enthalpy of dry air + change in enthalpy of condensed water

$$= (85 - 43) \times 3 + 67 \times 33 \times 10^{-3}$$

$$= 128.211 \text{ kW}$$

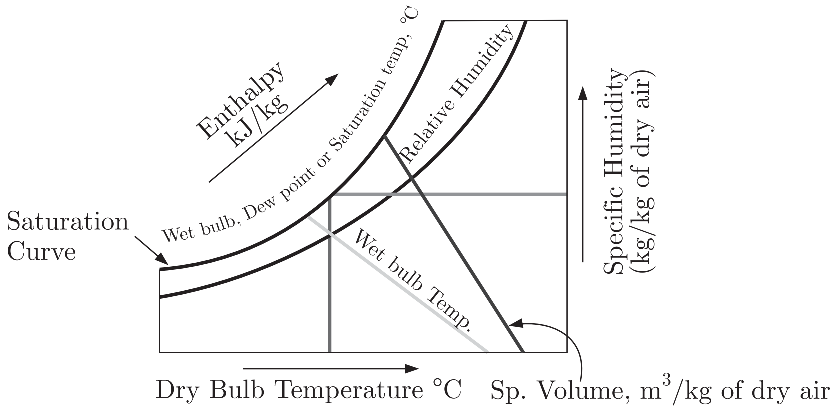
SOL 9.10 Option (C) is correct.



It is the temperature of air recorded by a thermometer, when the moisture (water vapour) present in it begins to condense.

If a sample of unsaturated air, containing superheated water vapour, is cooled at constant pressure, the partial pressure (p_v) of each constituent remains constant until the water vapour reaches the saturated state as shown by point B. At this point B the first drop of dew will be formed and hence the temperature at point B is called dew point temperature.

SOL 9.11 Option (A) is correct.



Hence, the statement 2 & 3 are correct.

SOL 9.12 Option (B) is correct.

From steam table, saturated air pressure corresponding to dry bulb temperature of 35° C is $p_s = 0.05628\,\text{bar}$.

Relative humidity,

$$\phi = \frac{p_v}{p_s} = 0.75$$

$$p_v = 0.75 \times p_s$$

$$= 0.75 \times 0.05628 = 0.04221\,\text{bar}$$

Now the amount of moisture in kg/kg of dry air, (Specific Humidity) is

$$W = 0.622 \times \frac{p_v}{p_b - p_v} \qquad p_b = p_{atm} = 1.01\,\text{bar}$$

$$= 0.622 \times \frac{0.04221}{1.01 - 0.04221}$$

$$= 0.622 \times 0.04362$$

$$= 0.0271\,\text{kg/kg of dry air}$$

SOL 9.13 Option (B) is correct.

Given : $t_{sp} = 42^\circ$ C, $t_{db} = 40^\circ$ C, $t_{wb} = 20^\circ$ C

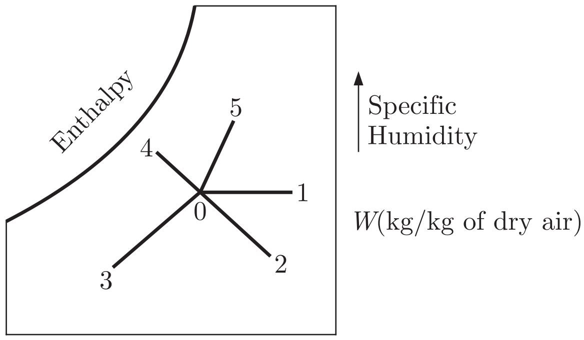
Here we see that $t_{sp} > t_{db}$

Hence air gets heated, Also water is added to it, so it gets humidified.

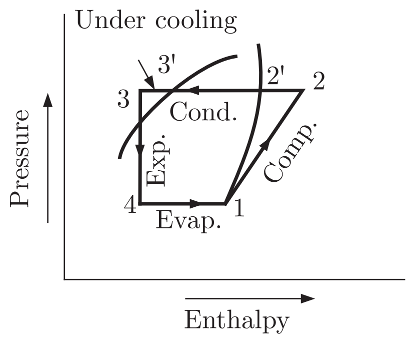
SOL 9.14 Option (A) is correct.

Given curve is the theoretical p-h curve for vapour compression refrigeration cycle.

SOL 9.15 Option (B) is correct.



Process	Process Name	t_{DBT}	W
0-1	Sensible Heating	Increase	Constant
0-2	Chemical dehumidification	Increase	Decrease
0-3	Cooling and dehumidification	Decrease	Decrease
0-4	Humidification with water injection	Decrease	Increase
0-5	Humidification with steam injection	Increase	Increase



Hence, curve given in question is a ideal $p-h$ curve for vapour compression refrigeration cycle.

SOL 9.16 Option (C) is correct.

$$(COP)_{ref.} = \frac{\text{Refrigeration Effect}}{\text{Work done}} = \frac{T_1}{T_2 - T_1}$$

$$\frac{100}{W} = \frac{250}{300 - 250}$$

$$W = \frac{100}{250} \times 50 = 20 \text{ Watt}$$

For supply this work, heat is taken from reservoir 3 & rejected to sink 2. So efficiency,

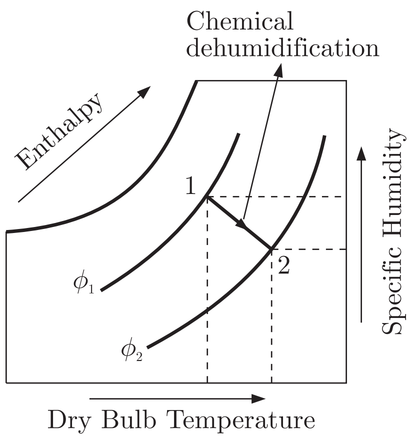
$$\eta = \frac{W}{Q_3} = \frac{T_3 - T_2}{T_3}$$ It works as a heat engine.

$$\frac{20}{Q_3} = \frac{400 - 300}{400} \qquad \Rightarrow Q_3 = 80 \text{ Watt}$$

SOL 9.17 Option (A) is correct.

Air conditioner mounted in a window or through the wall are self-contained units of small capacity of 1 TR to 3 TR. The capillary tube is used as an expansion device in small capacity refrigeration units.

SOL 9.18 Option (B) is correct.



In the process of chemical dehumidification of air , the air is passed over chemicals which have an affinity for moisture and the moisture of air gets condensed out and gives up its latent heat. Due to the condensation, the specific humidity decreases and the heat of condensation supplies sensible heat for heating the air and thus increasing its dry bulb temperature.

So chemical dehumidification increase dry bulb temperature & decreases specific humidity.

SOL 9.19 Option (D) is correct.

If a refrigerant is written in the from of $Rabc$.

The first digit on the right (c) is the number of fluorine (F) atoms, the second digit from the right (b) is one more than the number of hydrogen (H) atoms required & third digit from the right (a) is one less than the Number of carbon (C) atoms in the refrigerant. So, For R134

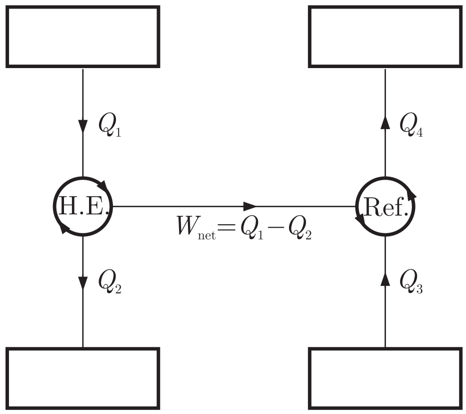
$$\text{First digit from the Right} = 4 = \text{Number of Fluorine atoms}$$

$$\text{Second digit from the right} = 3 - 1 = 2 = \text{Number of hydrogen atoms}$$

$$\text{Third digit from the right} = 1 + 1 = 2 = \text{Number of carbon atoms}$$

Hence, Chemical formula is $C_2H_2F_4$

SOL 9.20 Option (C) is correct.



Given : $(COP)_{refrigerator} = 5$, $(\eta)_{H.E} = 70\% = 0.7$

$$(COP)_{ref.} = \frac{Q_3}{W} = 5 \qquad ...(i)$$

$$(\eta)_{H.E.} = \frac{W}{Q_1} = 0.7 \qquad ...(ii)$$

By multiplying equation (i) & (ii),

$$\frac{Q_3}{W} \times \frac{W}{Q_1} = 5 \times 0.7 \qquad \Rightarrow \frac{Q_3}{Q_1} = 3.5$$

Hence, Energy absorbed (Q_3) from low temperature reservoir by the refrigerator for each kJ of energy absorbed (Q_1) from high temperature source by the engine $= 3.5\,\text{kJ}$

SOL 9.21 Option (B) is correct.

Given : $t_{dp} = 18^\circ\,\text{C} = (273 + 18)\,\text{K} = 291\,\text{K}$, $p = p_{atm} = 1.013\,\text{bar}$

$t_{db} = 30^\circ\,\text{C} = (273 + 30)\,\text{K} = 303\,\text{K}$

$p_v = 0.02062\,\text{bar}$ (for water vapour at dew point).

$c_{air} = 1.005\,\text{kJ/kg K}$, $c_{water} = 1.88\,\text{kJ/kg K}$

Latent heat of vaporization of water at $0^\circ\,\text{C}$.

$$h_{fgdp} = 2500\,\text{kJ/kg}$$

Specific humidity, $$W = \frac{0.622 \times p_v}{p - p_v} = \frac{0.622 \times 0.02062}{1.013 - 0.02062}$$

$$= \frac{0.01282}{0.99238} = 0.01291\,\text{kg/kg of dry air}$$

Enthalpy of moist air is given by,

$$h = 1.022 t_{db} + W(h_{fgdp} + 2.3 t_{dp})\,\text{kJ/kg}$$

$$= 1.022 \times 30 + 0.01291[2500 + 2.3 \times 18]$$

$$= 30.66 + 0.01291 \times 2541.4 = 63.46\,\text{kJ/kg} \simeq 63.15\,\text{kJ/kg}$$

SOL 9.22 Option (A) is correct.

Given : $C = 0.03$, $n = 1.15$, Specific volume at suction $= 0.1089\ \text{m}^3/\text{kg}$

$$\text{Net refrigeration effect} = 2\ \text{ton} \qquad 1\ \text{TR} = 1000 \times 335\ \text{kJ in 24 hr}$$

$$= \frac{2 \times 1000 \times 335}{24 \times 60 \times 60} = 7.75\ \text{kJ/sec}$$

$$\text{Let net mass flow rate} = \dot{m}$$

$$\text{Net refrigeration effect} = \dot{m}(h_1 - h_4)$$

Substitute the values from equation (i), and from the p-h curve,

$$7.75 = \dot{m}(176 - 65)$$

$$m = \frac{7.75}{111} = 0.06981\ \text{kg/sec}$$

Specific volume, $\frac{\nu}{\dot{m}} = 0.1089$

$$\nu = 0.1089 \times 0.06981 = 0.00760 = 7.60 \times 10^{-3}\ \text{m}^3/\text{sec}$$

We know that volumetric efficiency,

$$\eta_v = 1 + C - C\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

Where, p_1 is the suction pressure and p_2 is the discharge pressure.

$$= 1 + 0.03 - 0.03 \times \left(\frac{7.45}{1.50}\right)^{\frac{1}{1.15}}$$

$$= 1.03 - 0.12089 = 0.909$$

Now actual volume displacement rate is,

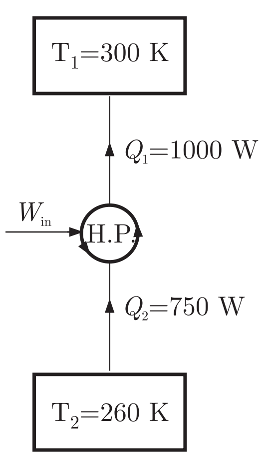
$$\nu_{actual} = \nu \times \eta_v = 7.60 \times 10^{-3} \times 0.909$$

$$= 6.90 \times 10^{-3} \simeq 6.35 \times 10^{-3}\ \text{m}^3/\text{sec}$$

SOL 9.23 Option (C) is correct.

Given : $T_1 = 27^\circ\text{C} = (27 + 273)\ \text{K} = 300\ \text{K}$,

$T_2 = -13^\circ\text{C} = (-13 + 273)\ \text{K} = 260\ \text{K}$, $Q_1 = 1000\ \text{W}$, $Q_2 = 750\ \text{W}$



So, $(COP)_{H.P.} = \frac{Q_1}{Q_1 - Q_2} = \frac{1000}{1000 - 750} = 4$

Alternate Method :

From energy balance

$$W_{in} + Q_2 = Q_1$$

$$W_{in} = Q_1 - Q_2 = 1000 - 750 = 250 \text{ W}$$

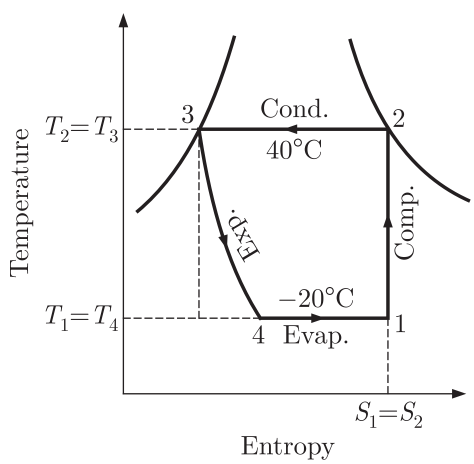
And $(COP)_{H.P.} = \frac{\text{Desired effect}}{W_{in}} = \frac{Q_1}{W_{in}} = \frac{1000}{250} = 4$

SOL 9.24 Option (B) is correct.

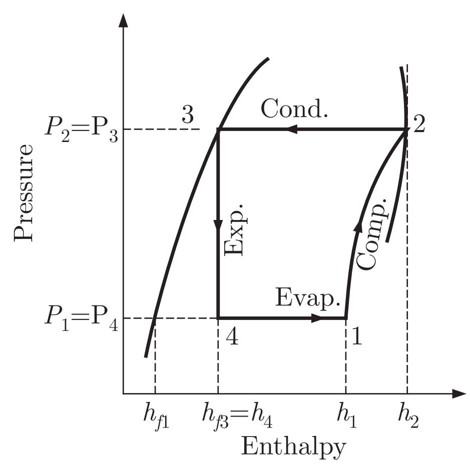
We know that for saturated air, the relative humidity is 100% and the dry bulb temperature, wet bulb temperature and dew point temperature is same. But when air is not saturated, dew point temperature is always less than the wet bulb temperature.

$$\text{DPT} < \text{WBT}$$

SOL 9.25 Option (A) is correct.



(A) *T-S* Diagram



(B) *p-h* Diagram

Given : $T_1 = T_4 = -20^\circ \text{C} = (-20 + 273) \text{ K} = 253 \text{ K}, \ \dot{m} = 0.025 \text{ kg/sec}$

$T_2 - T_3 = 40^\circ \text{C} = (40 + 273) \text{ K} = 313 \text{ K}$

From the given table,

At, $T_2 = 40^\circ \text{C}, \ h_2 = 200 \text{ kJ/kg}$

And $h_3 = h_4 = 80 \text{ kJ/kg}$

From the given T-s curve

$$s_1 = s_2$$

$$s_2 = s_f + x s_{fg} \qquad x = \text{Dryness fraction}$$

{s_2 is taken 0.67 because s_2 at the temperature 40° C & at 2 high temperature and pressure vapour refrigerant exist.}

$$0.67 = 0.07 + x(0.7366 - 0.07) \qquad s_{fg} = s_g - s_f$$

$$0.67 - 0.07 = x \times 0.6666$$

$$0.6 = x \times 0.6666$$

$$x = \frac{0.6}{0.6666} = 0.90$$

And Enthalpy at point 1 is,

$$h_1 = h_f + xh_{fg} = h_f + x(h_g - h_f)$$

$$= 20 + 0.90(180 - 20) = 164\,\text{kJ/kg}$$

Now refrigeration effect is produce in the evaporator.

Heat extracted from the evaporator or refrigerating effect,

$$R_E = \dot{m}(h_1 - h_4) = 0.025(164 - 80) = 2.1\,\text{kW}$$

SOL 9.26 Option (B) is correct.

$$(COP)_{refrigerator} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{\text{Refrigerating effect}}{\text{Work done}}$$

$$= \frac{164 - 80}{200 - 164} = \frac{84}{36} = 2.33$$

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CHAPTER 10

MANUFACTURING ENGINEERING

YEAR 2012 ONE MARK

MCQ 10.1 In abrasive jet machining, as the distance between the nozzle tip and the work surface increases, the material removal rate

(A) increases continuously.

(B) decreases continuously.

(C) decreases, becomes stable and then increases.

(D) increases, becomes stable and then decreases.

MCQ 10.2 Match the following metal forming processes with their associated stresses in the workpiece.

	Metal forming process		Types of stress
1.	Coining	P.	Tensile
2.	Wire Drawing	Q.	Shear
3.	Blanking	R.	Tensile and compressive
4.	Deep Drawing	S.	Compressive

(A) 1-S, 2-P, 3-Q, 4-R

(B) 1-S, 2-P, 3-R, 4-Q

(C) 1-P, 2-Q, 3-S, 4-R

(D) 1-P, 2-R, 3-Q, 4-S

MCQ 10.3 In an interchangeable assembly, shafts of size $25.000^{+0.040}_{-0.010}$ mm mate with holes of size $25.000^{+0.030}_{+0.020}$ mm. The maximum interference (in microns) in the assembly is

(A) 40

(B) 30

(C) 20

(D) 10

MCQ 10.4 During normalizing process of steel, the specimen is heated

(A) between the upper and lower critical temperature and cooled in still air.

(B) above the upper critical temperature and cooled in furnace.

(C) above the upper critical temperature and cooled in still air.

(D) between the upper and lower critical temperature and cooled in furnace

MCQ 10.5 A CNC vertical milling machine has to cut a straight slot of 10 mm width and 2 mm depth by a cutter of 10 mm diameter between points (0,0) and (100,100) on the XY plane (dimensions in mm). The feed rate used for milling is 50 mm/min. Milling time for the slot (in seconds) is

(A) 120 (B) 170

(C) 180 (D) 240

MCQ 10.6 A solid cylinder of diameter 100 mm and height 50 mm is forged between two frictionless flat dies to a height of 25 mm. The percentage change in diameter is

(A) 0 (B) 2.07

(C) 20.7 (D) 41.4

YEAR 2012 TWO MARKS

MCQ 10.7 Detail pertaining to an orthogonal metal cutting process are given below

Chip thickness ratio	0.4
Undeformed thickness	0.6 mm
Rake angle	$+10^\circ$
Cutting speed	2.5 m/s
Mean thickness of primary shear zone	25 microns

The shear strain rate in s^{-1} during the process is

(A) 0.1781×10^5 (B) 0.7754×10^5

(C) 1.0104×10^5 (D) 4.397×10^5

MCQ 10.8 In a single pass drilling operation, a through hole of 15 mm diameter is to be drilled in a steel plate of 50 mm thickness. Drill spindle speed is 500 rpm, feed is 0.2 mm/rev and drill point angle is 118°. Assuming 2 mm clearance at approach and exit, the total drill time (in seconds) is

(A) 35.1 (B) 32.4

(C) 31.2 (D) 30.1

MCQ 10.9 Calculate the punch size in mm, for a circular blanking operation for which details are given below.

Size of the blank	25 mm
Thickness of the sheet	2 mm
Radial clearance between punch and die	0.06 mm
Die allowance	0.05 mm

(A) 24.83 (B) 24.89
(C) 25.01 (D) 25.17

MCQ 10.10 In a single pass rolling process using 410 mm diameter steel rollers, a strip of width 140 mm and thickness 8 mm undergoes 10% reduction of thickness. The angle of bite in radians is
(A) 0.006 (B) 0.031
(C) 0.062 (D) 0.600

MCQ 10.11 In a DC are welding operation, the voltage-arc length characteristic was obtained as $V_{arc} = 20 + 5l$ where the arc length l was varied between 5 mm and 7 mm. Here V_{arc} denotes the arc voltage in Volts. The arc current was varied from 400 A to 500 A. Assuming linear power source characteristic, the open circuit voltage and short circuit current for the welding operation are
(A) $45\text{ V}, 450\text{ A}$ (B) $75\text{ V}, 750\text{ A}$
(C) $95\text{ V}, 950\text{ A}$ (D) $150\text{ V}, 1500\text{ A}$

YEAR 2011 ONE MARK

MCQ 10.12 The maximum possible draft in cold rolling of sheet increases with the
(A) increase in coefficient of friction
(B) decrease in coefficient of friction
(C) decrease in roll radius
(D) increase in roll velocity

MCQ 10.13 The operation in which oil is permeated into the pores of a powder metallurgy product is known as
(A) mixing (B) sintering
(C) impregnation (D) infiltration

MCQ 10.14 A hole is of dimension $\phi 9^{+0.015}_{+0}$ mm. The corresponding shaft is of dimension $\phi 9^{+0.010}_{+0.001}$ mm. The resulting assembly has
(A) loose running fit (B) close running fit
(C) transition fit (D) interference fit

MCQ 10.15 Green sand mould indicates that
(A) polymeric mould has been cured
(B) mould has been totally dried
(C) mould is green in color
(D) mould contains moisture

MCQ 10.16 Which one among the following welding processes uses non-consumable electrode ?

(A) Gas metal arc welding (B) Submerged arc welding

(C) Gas tungsten arc welding (D) Flux coated arc welding

MCQ 10.17 The crystal structure of austenite is

(A) body centered cubic (B) face centered cubic

(C) hexagonal closed packed (D) body centered tetragonal

YEAR 2011 TWO MARKS

MCQ 10.18 A single-point cutting tool with 12° rake angle is used to machine a steel work-piece. The depth of cut, i.e., uncut thickness is 0.81 mm. The chip thickness under orthogonal machining condition is 1.8 mm. The shear angle is approximately

(A) 22° (B) 26°

(C) 56° (D) 76°

MCQ 10.19 Match the following non-traditional machining processes with the corresponding material removal mechanisms :

	Machining process		Mechanism of material removal
P.	Chemical machining	1.	Erosion
Q.	Electro-chemical machining	2.	Corrosive reaction
R.	Electro-discharge machining	3.	Ion displacement
S.	Ultrasonic machining	4.	Fusion and vaporization

(A) P-2, Q-3, R-4, S-1 (B) P-2, Q-4, R-3, S-1

(C) P-3, Q-2, R-4, S-1 (D) P-2, Q-3, R-1, S-4

MCQ 10.20 A cubic casting of 50 mm side undergoes volumetric solidification shrinkage and volumetric solid contraction of 4% and 6% respectively. No riser is used. Assume uniform cooling in all directions. The side of the cube after solidification and contraction is

(A) 48.32 mm (B) 49.90 mm

(C) 49.94 mm (D) 49.96 mm

MCQ 10.21 The shear strength of a sheet metal is 300 MPa. The blanking force required to produce a blank of 100 mm diameter from a 1.5 mm thick sheet is close to

(A) 45 kN (B) 70 kN

(C) 141 kN (D) 3500 kN

YEAR 2010 ONE MARK

MCQ 10.22 The material property which depends only on the basic crystal structure is

(A) fatigue strength (B) work hardening

(C) fracture strength (D) elastic constant

MCQ 10.23 In a gating system, the ratio 1 : 2 : 4 represents

(A)	sprue base area	: runner area	: ingate area
(B)	pouring basin area	: ingate area	: runner area
(C)	sprue base area	: ingate area	: casting area
(D)	runner area	: ingate area	: casting area

MCQ 10.24 A shaft has a dimension, $\phi 35^{-0.009}_{-0.025}$. The respective values of fundamental deviation and tolerance are

(A) $-0.025, \pm 0.008$ (B) $-0.025, 0.016$

(C) $-0.009, \pm 0.008$ (D) $-0.009, 0.016$

MCQ 10.25 In a CNC program block, N002 GO2 G91 X40 Z40......,GO2 and G91 refer to

(A) circular interpolation in counterclockwise direction and incremental dimension

(B) circular interpolation in counterclockwise direction and absolute dimension

(C) circular interpolation in clockwise direction and incremental dimension

(D) circular interpolation in clockwise direction and absolute dimension

YEAR 2010 TWO MARKS

MCQ 10.26 For tool A, Taylor's tool life exponent (n) is 0.45 and constant (K) is 90. Similarly for tool B, $n = 0.3$ and $K = 60$. The cutting speed (in m/min) above which tool A will have a higher tool life than tool B is

(A) 26.7 (B) 42.5

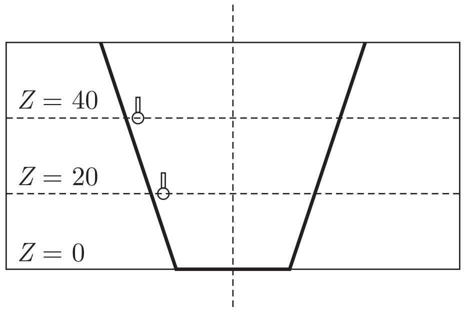
(C) 80.7 (D) 142.9

MCQ 10.27 Two pipes of inner diameter 100 mm and outer diameter 110 mm each are joined by flash-butt welding using 30 V power supply. At the interference, 1 mm of material melts from each pipe which has a resistance of $42.4\,\Omega$. If the unit melt energy is $64.4\,MJm^{-3}$, then time required for welding (in s) is

(A) 1 (B) 5

(C) 10 (D) 20

MCQ 10.28 A taper hole is inspected using a CMM, with a probe of 2 mm diameter. At a height, $Z = 10\,\text{mm}$ from the bottom, 5 points are touched and a diameter of circle (not compensated for probe size) is obtained as 20 mm. Similarly, a 40 mm diameter is obtained at a height $Z = 40\,\text{mm}$. The smaller diameter (in mm) of hole at $Z = 0$ is

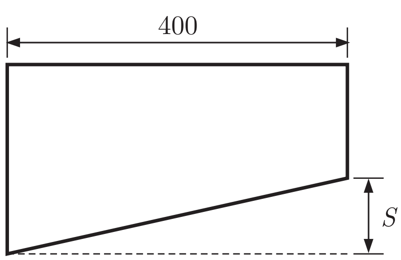


(A) 13.334 (B) 15.334

(C) 15.442 (D) 15.542

• Common Data For Q.28 and Q.29

In shear cutting operation, a sheet of 5 mm thickness is cut along a length of 200 mm. The cutting blade is 400 mm long (see fig.) and zero-shear ($S = 0$) is provided on the edge. The ultimate shear strength of the sheet is 100 MPa and penetration to thickness ratio is 0.2. Neglect friction.



MCQ 10.29 Assuming force vs displacement curve to be rectangular, the work done (in J) is

(A) 100 (B) 200

(C) 250 (D) 300

MCQ 10.30 A shear of $20\,\text{mm}$ ($S = 0\,\text{mm}$) is now provided on the blade. Assuming force vs displacement curve to be trapezoidal, the maximum force (in kN) exerted is

(A) 5 (B) 10

(C) 20 (D) 40

YEAR 2009 ONE MARK

MCQ 10.31 Friction at the tool-chip interface can be reduced by

(A) decreasing the rake angle
(B) increasing the depth of cut
(C) decreasing the cutting speed
(D) increasing the cutting speed

MCQ 10.32 Two streams of liquid metal which are not hot enough to fuse properly result into a casting defect known as

(A) cold shut
(B) swell
(C) sand wash
(D) scab

MCQ 10.33 The effective number of lattice points in the unit cell of simple cubic, body centered cubic, and face centered cubic space lattices, respectively, are

(A) 1, 2, 2
(B) 1, 2, 4
(C) 2, 3, 4
(D) 2, 4, 4

MCQ 10.34 Which of the following is the correct data structure for solid models ?

(A) solid part → faces → edges → vertices
(B) solid part → edges → faces → vertices
(C) vertices → edges → faces → solid parts
(D) vertices → faces → edges → solid parts

YEAR 2009 TWO MARKS

MCQ 10.35 Minimum shear strain in orthogonal turning with a cutting tool of zero rake angle is

(A) 0.0
(B) 0.5
(C) 1.0
(D) 2.0

MCQ 10.36 Electrochemical machining is performed to remove material from an iron surface of 20 mm $\times$ 20 mm under the following conditions :

Inter electrode gap = 0.2 mm
Supply voltage (DC) = 12 V
Specific resistance of electrolyte = 2 Ω cm
Atomic weight of Iron = 55.85
Valency of Iron = 2
Faraday's constant = 96540 Coulombs

The material removal rate (in g/s) is

(A) 0.3471
(B) 3.471
(C) 34.71
(D) 347.1

MCQ 10.37 Match the following:

	NC code		Definition
P.	M05	1.	Absolute coordinate system
Q.	G01	2.	Dwell
R.	G04	3.	Spindle stop
S.	G09	4.	Linear interpolation

(A) P-2, Q-3, R-4, S-1

(B) P-3, Q-4, R-1, S-2

(C) P-3, Q-4, R-2, S-1

(D) P-4, Q-3, R-2, S-1

MCQ 10.38 What are the upper and lower limits of the shaft represented by 60 f_8 ?
Use the following data :
Diameter 60 lies in the diameter step of 50-80 mm.
Fundamental tolerance unit, i in $\mu m = 0.45D^{1/3} + 0.001D$
Where D is the representative size in mm;
Tolerance value for $IT8 = 25i$,
Fundamental deviation for 'f' shaft $= - 5.5D^{0.41}$

(A) Lower limit = 59.924 mm, Upper limit = 59.970 mm

(B) Lower limit = 59.954 mm, Upper limit = 60.000 mm

(C) Lower limit = 59.970 mm, Upper limit = 60.016 mm

(D) Lower limit = 60.000 mm, Upper limit = 60.046 mm

MCQ 10.39 Match the items in Column I and Column II.

	Column I		Column II
P.	Metallic Chills	1.	Support for the core
Q.	Metallic Chaplets	2.	Reservoir of the molten metal
R.	Riser	3.	Control cooling of critical sections
S.	Exothermic Padding	4.	Progressive solidification

(A) P-1, Q-3, R-2, S-4

(B) P-1, Q-4, R-2, S-3

(C) P-3, Q-4, R-2, S-1

(D) P-4, Q-1, R-2, S-3

MCQ 10.40 The exponent (n) and constant (K) of the Taylor's tool life equation are

(A) $n = 0.5$ and $K = 540$

(B) $n = 1$ and $K = 4860$

(C) $n = -1$ and $K = 0.74$

(D) $n = -0.5$ and $K = 1.155$

MCQ 10.41 What is the percentage increase in tool life when the cutting speed is halved ?
(A) 50% (B) 200%
(C) 300% (D) 400%

YEAR 2008 ONE MARK

MCQ 10.42 For generating a Coon's surface we require
(A) a set of grid points on the surface
(B) a set of grid control points
(C) four bounding curves defining the surface
(D) two bounding curves and a set of grid control points

MCQ 10.43 Internal gear cutting operation can be performed by
(A) milling (B) shaping with rack cutter
(C) shaping with pinion cutter (D) hobbing

YEAR 2008 TWO MARKS

MCQ 10.44 While cooling, a cubical casting of side 40 mm undergoes 3%, 4% and 5% volume shrinkage during the liquid state, phase transition and solid state, respectively. The volume of metal compensated from the riser is
(A) 2% (B) 7%
(C) 8% (D) 9%

MCQ 10.45 In a single point turning tool, the side rake angle and orthogonal rake angle are equal. φ is the principal cutting edge angle and its range is $0^\circ \le \varphi \le 90^\circ$. The chip flows in the orthogonal plane. The value of φ is closest to
(A) 0° (B) 45°
(C) 60° (D) 90°

MCQ 10.46 A researcher conducts electrochemical machining (ECM) on a binary alloy (density 6000 kg/m^3) of iron (atomic weight 56, valency 2) and metal (atomic weight 24, valency 4). Faraday's constant = 96500 coulomb/mole. Volumetric material removal rate of the alloy is 50 mm^3/s at a current of 2000 A. The percentage of the metal P in the alloy is closest to
(A) 40 (B) 25
(C) 15 (D) 79

MCQ 10.47 In a single pass rolling operation, a 20 mm thick plate with plate width of 100 mm, is reduced to 18 mm. The roller radius is 250 mm and rotational speed is 10 rpm. The average flow stress for the plate material is 300 MPa.

The power required for the rolling operation in kW is closest to

(A) 15.2 (B) 18.2

(C) 30.4 (D) 45.6

MCQ 10.48 In arc welding of a butt joint, the welding speed is to be selected such that highest cooling rate is achieved. Melting efficiency and heat transfer efficiency are 0.5 and 0.7, respectively. The area of the weld cross section is $5\,mm^2$ and the unit energy required to melt the metal is $10\,J/mm^3$. If the welding power is $2\,kW$, the welding speed in mm/s is closest to

(A) 4 (B) 14

(C) 24 (D) 34

MCQ 10.49 In the deep drawing of cups, blanks show a tendency to wrinkle up around the periphery (flange). The most likely cause and remedy of the phenomenon are, respectively,

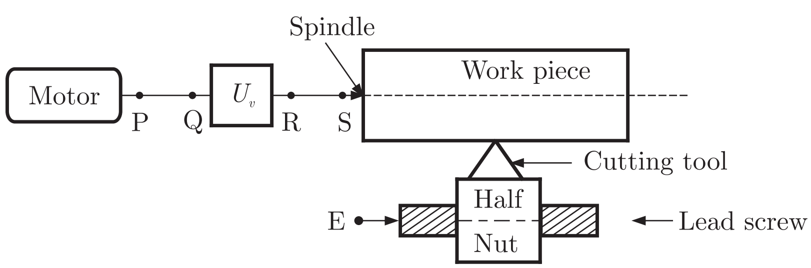
(A) Buckling due to circumferential compression; Increase blank holder pressure

(B) High blank holder pressure and high friction; Reduce blank holder pressure and apply lubricant

(C) High temperature causing increase in circumferential length; Apply coolant to blank

(D) Buckling due to circumferential compression; decrease blank holder pressure

MCQ 10.50 The figure shows an incomplete schematic of a conventional lathe to be used for cutting threads with different pitches. The speed gear box U_v is shown and the feed gear box U_s is to be placed. P, Q, R and S denote locations and have no other significance. Changes in U_v should NOT affect the pitch of the thread being cut and changes in U_s should NOT affect the cutting speed.



The correct connections and the correct placement of U_s are given by

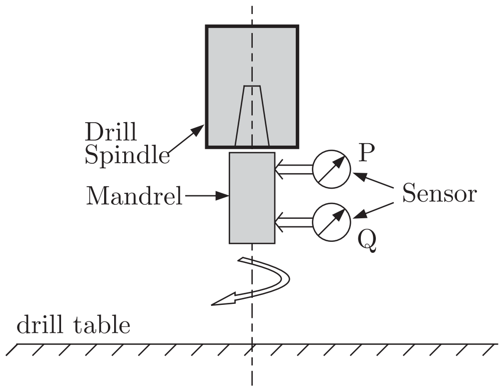
(A) Q and E are connected. U_s is placed between P and Q.

(B) S and E are connected. U_s is placed between R and S

(C) Q and E are connected. U_s is placed between Q and E

(D) S and E are connected. U_s is placed between S and E

MCQ 10.51 A displacement sensor (a dial indicator) measure the lateral displacement of a mandrel mounted on the taper hole inside a drill spindle. The mandrel axis is an extension of the drill spindle taper hole axis and the protruding portion of the mandrel surface is perfectly cylindrical measurements are taken with the sensor placed at two positions P and Q as shown in the figure. The reading are recorded as $R_x =$ maximum deflection minus minimum deflection, corresponding to sensor position at X, over one rotation.



If $R_P = R_Q > 0$, which one of the following would be consistent with the observation ?

(A) The drill spindle rotational axis is coincident with the drill spindle taper hole axis

(B) The drill spindle rotational axis intersects the drill spindle taper hole axis at point P

(C) The drill spindle rotational axis is parallel to the drill spindle taper hole axis

(D) The drill spindle rotational axis intersects the drill spindle taper hole axis at point Q

• Common Data For Q.52 and Q.53

Orthogonal turning is performed on a cylindrical workpiece with the shear strength of 250 MPa. The following conditions are used: cutting velocity is 180 m/min, feed is 0.20 mm/rev, depth of cut is 3 mm, chip thickness ratio $= 0.5$. The orthogonal rake angle is 7°. Apply Merchant's theory for analysis.

MCQ 10.52 The shear plane angle (in degree) and the shear force respectively are

(A) 52, 320 N (B) 52, 400 N

(C) 28, 400 N (D) 28, 320 N

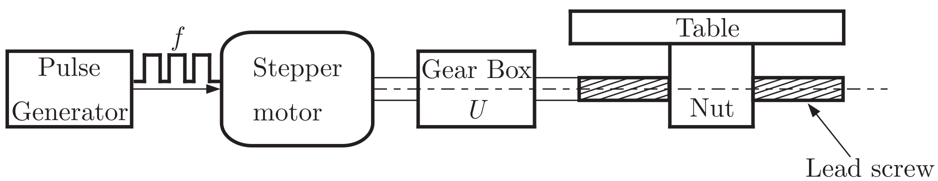
MCQ 10.53 The cutting and frictional forces, respectively, are

(A) 568 N, 387 N (B) 565 N, 381 N

(C) 440 N, 342 N (D) 480 N, 356 N

• Common Data For Q.54 and Q.55

In the feed drive of a Point-to-Point open loop CNC drive, a stepper motor rotating at 200 steps/rev drives a table through a gear box and lead screw-nut mechanism (pitch=4 mm, number of starts=1). The gear ratio $=\left(\frac{\text{Output rotational speed}}{\text{Input rotational speed}}\right)$ is given by $U = \frac{1}{4}$. The stepper motor (driven by voltage pulses from a pulse generator) executes 1 step/pulse of the pulse generator. The frequency of the pulse train from the pulse generator is $f = 10,000$ pulses per minute.



MCQ 10.54 The basic Length Unit (BLU), i.e, the table movement corresponding to 1 pulse of the pulse generator, is

(A) 0.5 microns (B) 5 microns

(C) 50 microns (D) 500 microns

MCQ 10.55 A customer insists on a modification to change the BLU of the CNC drive to 10 microns without changing the table speed. The modification can be accomplished by

(A) changing U to $\frac{1}{2}$ and reducing f to $\frac{f}{2}$

(B) changing U to $\frac{1}{8}$ and increasing f to $2f$

(C) changing U to $\frac{1}{2}$ and keeping f unchanged

(D) keeping U unchanged and increasing f to $2f$

YEAR 2007 ONE MARK

MCQ 10.56 If a particular Fe-C alloy contains less than 0.83% carbon, it is called

(A) high speed steel (B) hypoeutectoid steel

(C) hypereutectoid steel (D) cast iron

MCQ 10.57 Which of the following engineering materials is the most suitable candidate for hot chamber die casting ?

(A) low carbon steel (B) titanium

(C) copper (D) tin

MCQ 10.58 Which one of the following is a solid state joining process ?

(A) gas tungsten arc welding (B) resistance spot welding

(C) friction welding (D) submerged arc welding

MCQ 10.59 In orthogonal turning of a low carbon steel bar of diameter 150 mm with uncoated carbide tool, the cutting velocity is 90 m/min. The feed is 0.24 mm/rev and the depth of cut is 2 mm. The chip thickness obtained is 0.48 mm. If the orthogonal rake angle is zero and the principle cutting edge angle is 90°, the shear angle in degree is

(A) 20.56 (B) 26.56

(C) 30.56 (D) 36.56

MCQ 10.60 Which type of motor is NOT used in axis or spindle drives of CNC machine tools ?

(A) induction motor (B) dc servo motor

(C) stepper motor (D) linear servo motor

MCQ 10.61 Volume of a cube of side 'l' and volume of a sphere of radius 'r' are equal. Both the cube and the sphere are solid and of same material. They are being cast. The ratio of the solidification time of the cube to the same of the sphere is

(A) $\left(\frac{4\pi}{6}\right)^3\left(\frac{r}{l}\right)^6$ (B) $\left(\frac{4\pi}{6}\right)\left(\frac{r}{l}\right)^2$

(C) $\left(\frac{4\pi}{6}\right)^2\left(\frac{r}{l}\right)^3$ (D) $\left(\frac{4\pi}{6}\right)^2\left(\frac{r}{l}\right)^4$

YEAR 2007 TWO MARKS

MCQ 10.62 In electrodischarge machining (EDM), if the thermal conductivity of tool is high and the specific heat of work piece is low, then the tool wear rate and material removal rate are expected to be respectively

(A) high and high (B) low and low

(C) high and low (D) low and high

MCQ 10.63 In orthogonal turning of medium carbon steel, the specific machining energy is 2.0 J/mm^3. The cutting velocity, feed and depth of cut are 120 m/min, 0.2 mm/rev. and 2 mm respectively. The main cutting force in N is

(A) 40 (B) 80

(C) 400 (D) 800

MCQ 10.64 A direct current welding machine with a linear power source characteristic provides open circuit voltage of $80\,\text{V}$ and short circuit current of $800\,\text{A}$. During welding with the machine, the measured arc current is $500\,\text{A}$ corresponding to an arc length of $5.0\,\text{mm}$ and the measured arc current is $460\,\text{A}$ corresponding to an arc length of $7.0\,\text{mm}$. The linear voltage (E) arc length (L) characteristic of the welding arc can be given as (where E is in volt and L in in mm)

(A) $E = 20 + 2L$ (B) $E = 20 + 8L$

(C) $E = 80 + 2L$ (D) $E = 80 + 8L$

MCQ 10.65 A hole is specified as $40^{0.050}_{0.000}$ mm. The mating shaft has a clearance fit with minimum clearance of $0.01\,\text{mm}$. The tolerance on the shaft is $0.04\,\text{mm}$. The maximum clearance in mm between the hole and the shaft is

(A) 0.04 (B) 0.05

(C) 0.10 (D) 0.11

MCQ 10.66 In orthogonal turning of low carbon steel pipe with principal cutting edge angle of 90°, the main cutting force is $1000\,\text{N}$ and the feed force is $800\,\text{N}$. The shear angle is 25° and orthogonal rake angle is zero. Employing Merchant's theory, the ratio of friction force to normal force acting on the cutting tool is

(A) 1.56 (B) 1.25

(C) 0.80 (D) 0.64

MCQ 10.67 Two metallic sheets, each of 2.0 mm thickness, are welded in a lap joint configuration by resistance spot welding at a welding current of $10\,\text{kA}$ and welding time of 10 millisecond. A spherical fusion zone extending up to full thickness of each sheet is formed. The properties of the metallic sheets are given as :

Ambient temperature $= 293$ K

Melting temperature $= 1793$ K

Density $= 7000\,\text{kg/m}^3$

Latent heat of fusion $= 300\,\text{kJ/kg}$

Specific heat $= 800\,\text{J/kgK}$

Assume :

(i) contact resistance along sheet interface is 500 micro-ohm and along electrode-sheet interface is zero;

(ii) no conductive heat loss through the bulk sheet materials ; and

(iii) the complete weld fusion zone is at the melting temperature.

The melting efficiency (in %) of the process is

(A) 50.37 (B) 60.37

(C) 70.37 (D) 80.37

MCQ 10.68 In open-die forging, disc of diameter 200 mm and height 60 mm is compressed without any barreling effect. The final diameter of the disc is 400 mm. The true strain is

(A) 1.986 (B) 1.686

(C) 1.386 (D) 0.602

MCQ 10.69 The thickness of a metallic sheet is reduced from an initial value of 16 mm to a final value of 10 mm in one single pass rolling with a pair of cylindrical rollers each of diameter of 400 mm. The bite angle in degree will be.

(A) 5.936 (B) 7.936

(C) 8.936 (D) 9.936

MCQ 10.70 Match the correct combination for following metal working processes.

	Processes		Associated state of stress
P:	Blanking	1.	Tension
Q:	Stretch Forming	2.	Compression
R:	Coining	3.	Shear
S:	Deep Drawing	4.	Tension and Compression
		5.	Tension and Shear

(A) P - 2, Q - 1, R - 3, S - 4

(B) P - 3, Q - 4, R - 1, S - 5

(C) P - 5, Q - 4, R - 3, S - 1

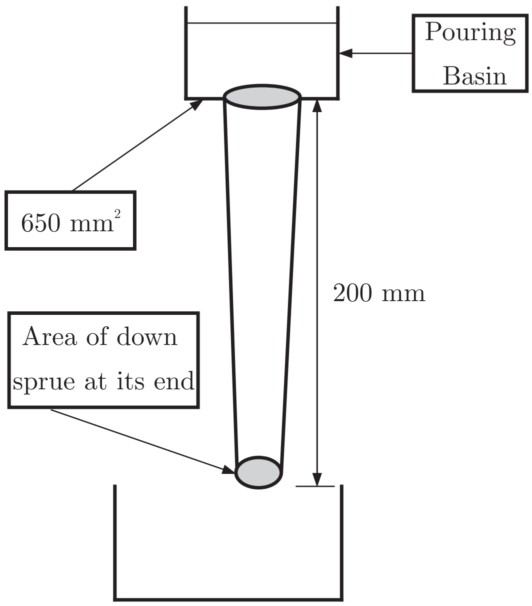
(D) P - 3, Q - 1, R - 2, S - 4

MCQ 10.71 The force requirement in a blanking operation of low carbon steel sheet is 5.0 kN. The thickness of the sheet is 't' and diameter of the blanked part is 'd'. For the same work material, if the diameter of the blanked part is increased to $1.5d$ and thickness is reduced to $0.4t$, the new blanking force in kN is

(A) 3.0 (B) 4.5

(C) 5.0 (D) 8.0

MCQ 10.72 A 200 mm long down sprue has an area of cross-section of 650 mm^2 where the pouring basin meets the down sprue (i.e at the beginning of the down sprue). A constant head of molten metal is maintained by the pouring basin. The molten metal flow rate is 6.5×10^5 mm^3/s. Considering the end of down sprue to be open to atmosphere and an acceleration due to gravity of 10^4 mm/s^2, the area of the down sprue in mm^2 at its end (avoiding aspiration effect) should be



(A) 650.0 (B) 350.0

(C) 290.7 (D) 190.0

MCQ 10.73 Match the most suitable manufacturing processes for the following parts.

	Parts		Manufacturing Process
P.	Computer chip	1.	Electrochemical Machining
Q.	Metal forming dies and molds	2.	Ultrasonic Machining
R.	Turbine blade	3.	Electrodischarge Machining
S.	Glass	4.	Photochemical Machining

(A) P - 4, Q - 3, R - 1, S - 2

(B) P - 4, Q - 3, R - 2, S - 1

(C) P - 3, Q - 1, R - 4, S - 2

(D) P - 1, Q - 2, R - 4, S - 3

• Common Data For Q. 74 and Q.75

A low carbon steel bar of 147 mm diameter with a length of 630 mm is being turned with uncoated carbide insert. The observed tool lives are 24 min and 12 min for cutting velocities of 90 m/min and 120 m/ min. respectively. The feed and depth of cut are 0.2 mm/rev and 2 mm respectively. Use the unmachined diameter to calculate the cutting velocity.

MCQ 10.74 When tool life is 20 min, the cutting velocity in m/min is

(A) 87 (B) 97

(C) 107 (D) 114

MCQ 10.75 Neglect over-travel or approach of the tool. When tool life is 20 min., the machining time in min for a single pass is

(A) 5 (B) 10

(C) 15 (D) 20

YEAR 2006 ONE MARK

MCQ 10.76 An expendable pattern is used in

(A) slush casting (B) squeeze casting

(C) centrifugal casting (D) investment casting

MCQ 10.77 The main purpose of spheroidising treatment is to improve

(A) hardenability of low carbon steels

(B) machinability of low carbon steels

(C) hardenability of high carbon steels

(D) machinability of high carbon steels

MCQ 10.78 NC contouring is an example of

(A) continuous path positioning (B) point-to-point positioning

(C) absolute positioning (D) incremental positioning

MCQ 10.79 A ring gauge is used to measure

(A) outside diameter but not roundness

(B) roundness but not outside diameter

(C) both outside diameter and roundness

(D) only external threads

YEAR 2006 TWO MARKS

MCQ 10.80 The ultimate tensile strength of a material is 400 MPa and the elongation up to maximum load is 35%. If the material obeys power law of hardening, then the true stress-true strain relation (stress in MPa) in the plastic deformation range is

(A) $\sigma = 540\varepsilon^{0.30}$ (B) $\sigma = 775\varepsilon^{0.30}$

(C) $\sigma = 540\varepsilon^{0.35}$ (D) $\sigma = 775\varepsilon^{0.35}$

MCQ 10.81 In a sand casting operation, the total liquid head is maintained constant such that it is equal to the mould height. The time taken to fill the mould with a top gate is t_A. If the same mould is filled with a bottom gate, then the time taken is t_B. Ignore the time required to fill the runner and frictional effects. Assume atmospheric pressure at the top molten metal surfaces. The relation between t_A and t_B is

(A) $t_B = \sqrt{2}\, t_A$ (B) $t_B = 2t_A$

(C) $t_B = \frac{t_A}{\sqrt{2}}$ (D) $t_B = 2\sqrt{2}\, t_A$

MCQ 10.82 A 4 mm thick sheet is rolled with 300 mm diameter roll to reduce thickness without any change in its width. The friction coefficient at the work-roll interface is 0.1. The minimum possible thickness of the sheet that can be produced in a single pass is

(A) 1.0 mm (B) 1.5 mm

(C) 2.5 mm (D) 3.7 mm

MCQ 10.83 In a wire drawing operation, diameter of a steel wire is reduced from 10 mm to 8 mm. The mean flow stress of the material is 400 MPa. The ideal force required for drawing (ignoring friction and redundant work) is

(A) 4.48 kN (B) 8.97 kN

(C) 20.11 kN (D) 31.41 kN

MCQ 10.84 Match the item in columns I and II

	Column I		Column II
P.	Wrinkling	1.	Yield point elongation
Q.	Orange peel	2.	Anisotropy
R.	Stretcher strains	3.	Large grain size
S.	Earing	4.	Insufficient blank holding force
		5.	Fine grain size
		6.	Excessive blank holding force

(A) P-6, Q-3, R-1, S-2

(B) P-4, Q-5, R-6, S-1

(C) P-2, Q-5, R-3, S-1

(D) P-4, Q-3, R-1, S-2

MCQ 10.85 In an arc welding process, the voltage and current are 25 V and 300 A respectively. The arc heat transfer efficiency is 0.85 and welding speed is 8 mm/sec. The net heat input (in J/mm) is

(A) 64 (B) 797

(C) 1103 (D) 79700

MCQ 10.86 If each abrasive grain is viewed as a cutting tool, then which of the following represents the cutting parameters in common grinding operations ?

(A) Large negative rake angle, low shear angle and high cutting speed

(B) Large positive rake angle, low shear angle and high cutting speed

(C) Large negative rake angle, high shear angle and low cutting speed

(D) Zero rake angle, high shear angle and high cutting speed

MCQ 10.87 Arrange the processes in the increasing order of their maximum material removal rate.

Electrochemical Machining (ECM)
Ultrasonic Machining (USM)
Electron Beam Machining (EBM)
Laser Beam Machining (LBM) and
Electric Discharge Machining (EDM)

(A) USM, LBM, EBM, EDM, ECM

(B) EBM, LBM, USM, ECM, EDM

(C) LBM, EBM, USM, ECM, EDM

(D) LBM, EBM, USM, EDM, ECM

MCQ 10.88 Match the items in columns I and II.

	Column I		Column II
P.	Charpy test	1.	Fluidity
Q.	Knoop test	2.	Microhardness
R.	Spiral test	3.	Formability
S.	Cupping test	4.	Toughness
		5.	Permeability

(A) P-4, Q-5, R-3, S-2

(B) P-3, Q-5, R-1, S-4

(C) P-2, Q-4, R-3, S-5

(D) P-4, Q-2, R-1, S-3

• Common Data For Q.89, 90 and Q.91

In an orthogonal machining operation :

Uncut thickness $= 0.5$ mm

Cutting speed $= 20$ m/min

Rake angel $= 15^\circ$

Width of cut $= 5$ mm   Chip thickness $= 0.7$ mm

Thrust force = 200 N Cutting force = 1200 N
Assume Merchant's theory.

MCQ 10.89 The values of shear angle and shear strain, respectively, are

(A) 30.3° and 1.98 (B) 30.3° and 4.23

(C) 40.2° and 2.97 (D) 40.2° and 1.65

MCQ 10.90 The coefficient of friction at the tool-chip interface is

(A) 0.23 (B) 0.46

(C) 0.85 (D) 0.95

MCQ 10.91 The percentage of total energy dissipated due to friction at the tool-chip interface is

(A) 30% (B) 42%

(C) 58% (D) 70%

YEAR 2005 ONE MARK

MCQ 10.92 Match the items of List-I (Equipment) with the items of List-II (Process) and select the correct answer using the given codes.

	List-I (Equipment)		List-II (Process)
P.	Hot Chamber Machine	1.	Cleaning
Q.	Muller	2.	Core making
R.	Dielectric Baker	3.	Die casting
S.	Sand Blaster	4.	Annealing
		5.	Sand mixing

(A) P-2, Q-1, R-4, S-5

(B) P-4, Q-2, R-3, S-5

(C) P-4, Q-5, R-1, S-2

(D) P-3, Q-5, R-2, S-1

MCQ 10.93 When the temperature of a solid metal increases,

(A) strength of the metal decreases but ductility increases

(B) both strength and ductility of the metal decreases

(C) both strength and ductility of the metal increases

(D) strength of the metal increases but ductility decreases

MCQ 10.94 The strength of a brazed joint

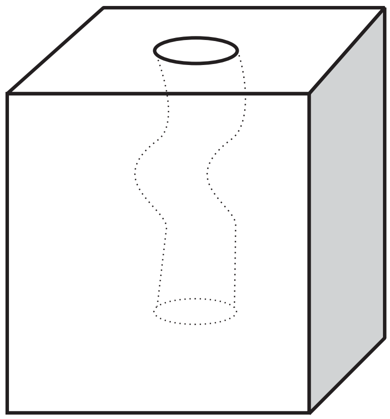
(A) decreases with increase in gap between the two joining surfaces

(B) increases with increase in gap between the two joining surfaces

(C) decreases up to certain gap between the two joining surfaces beyond which it increases

(D) increases up to certain gap between the two joining surfaces beyond which it decreases

MCQ 10.95 A zigzag cavity in a block of high strength alloy is to be finish machined. This can be carried out by using.



(A) electric discharge machining

(B) electric-chemical machining

(C) laser beam machining

(D) abrasive flow machining

MCQ 10.96 In order to have interference fit, it is essential that the lower limit of the shaft should be

(A) greater than the upper limit of the hole

(B) lesser than the upper limit of the hole

(C) greater than the lower limit of the hole

(D) lesser than the lower limit of the hole

MCQ 10.97 When 3-2-1 principle is used to support and locate a three dimensional work-piece during machining, the number of degrees of freedom that are restricted is

(A) 7

(B) 8

(C) 9

(D) 10

MCQ 10.98 Which among the NC operations given below are continuous path operations ?

Arc Welding (AW)

Drilling (D)

Laser Cutting of Sheet Metal (LC)

Milling (M)

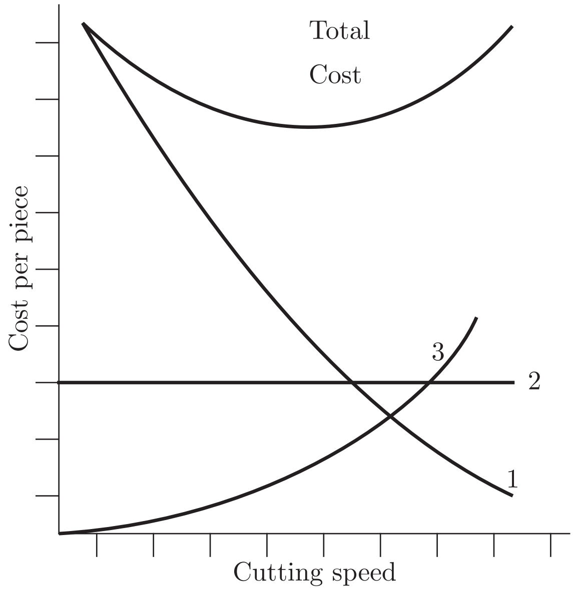
Punching in Sheet Metal (P)

Spot Welding (SW)

(A) AW, LC and M (B) AW, D, LC and M

(C) D, LC, P and SW (D) D, LC, and SW

MCQ 10.99 The figure below shows a graph which qualitatively relates cutting speed and cost per piece produced.



The three curves 1, 2 and 3 respectively represent

(A) machining cost, non-productive cost, tool changing cost

(B) non-productive cost, machining cost, tool changing cost

(C) tool changing cost, machining cost, non-productive cost

(D) tool changing cost, non-productive cost, machining cost

YEAR 2005 TWO MARKS

MCQ 10.100 A mould has a downsprue whose length is 20 cm and the cross sectional area at the base of the downsprue is $1\,\text{cm}^2$. The downsprue feeds a horizontal runner leading into the mould cavity of volume $1000\,\text{cm}^3$. The time required to fill the mould cavity will be

(A) 4.05 s (B) 5.05 s

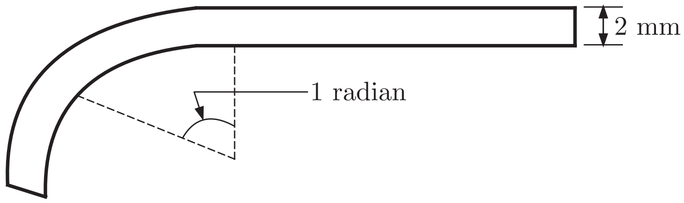
(C) 6.05 s (D) 7.25 s

MCQ 10.101 Spot welding of two 1 mm thick sheets of steel (density $= 8000\,\text{kg/m}^3$) is carried out successfully by passing a certain amount of current for 0.1 second through the electrodes. The resultant weld nugget formed is 5 mm in diameter and 1.5 mm thick. If the latent heat of fusion of steel is 1400 kJ/kg and the effective resistance in the welding operation is $200\,\mu\Omega$, the current passing through the electrodes is approximately

(A) 1480 A (B) 3300 A

(C) 4060 A (D) 9400 A

MCQ 10.102 A 2 mm thick metal sheet is to be bent at an angle of one radian with a bend radius of 100 mm. If the stretch factor is 0.5, the bend allowance is



(A) 99 mm (B) 100 mm

(C) 101 mm (D) 102 mm

MCQ 10.103 A $600\,\text{mm} \times 30\,\text{mm}$ flat surface of a plate is to be finish machined on a shaper. The plate has been fixed with the $600\,\text{mm}$ side along the tool travel direction. If the tool over-travel at each end of the plate is $20\,\text{mm}$, average cutting speed is $8\,\text{m/min.}$, feed rate is $0.3\,\text{mm/}$ stroke and the ratio of return time to cutting time of the tool is 1:2, the time required for machining will be

(A) 8 minutes (B) 12 minutes

(C) 16 minutes (D) 20 minutes

MCQ 10.104 The tool of an NC machine has to move along a circular arc from (5, 5) to (10, 10) while performing an operation. The centre of the arc is at (10, 5). Which one of the following NC tool path command performs the above mentioned operation ?

(A) N010 GO2 X10 Y10 X5 Y5 R5

(B) N010 GO3 X10 Y10 X5 Y5 R5

(C) N010 GO1 X5 Y5 X10 Y10 R5

(D) N010 GO2 X5 Y5 X10 Y10 R5

MCQ 10.105 Two tools P and Q have signatures 5°-5°-6°-6°-8°-30°-0 and 5°-5°-7°-7°-8°-15°-0 (both ASA) respectively. They are used to turn components under the same machining conditions. If h_P and h_Q denote the peak-to-valley heights of surfaces produced by the tools P and Q, the ratio h_P/h_Q will be

(A) $\frac{\tan 8^\circ + \cot 15^\circ}{\tan 8^\circ + \cot 30^\circ}$ (B) $\frac{\tan 15^\circ + \cot 8^\circ}{\tan 30^\circ + \cot 8^\circ}$

(C) $\frac{\tan 15^\circ + \cot 7^\circ}{\tan 30^\circ + \cot 7^\circ}$ (D) $\frac{\tan 7^\circ + \cot 15^\circ}{\tan 7^\circ + \cot 30^\circ}$

YEAR 2004 ONE MARK

MCQ 10.106 In an interchangeable assembly, shafts of size $25.000^{+0.040}_{-0.0100}$ mm mate with holes of size $25.000^{+0.020}_{-0.000}$ mm. The maximum possible clearance in the assembly will be

(A) 10 microns
(B) 20 microns
(C) 30 microns
(D) 60 microns

MCQ 10.107 During the execution of a CNC part program block
$NO20\ GO2\ X45.0\ Y25.0\ R5.0$ the type of tool motion will be

(A) circular Interpolation – clockwise
(B) circular Interpolation – counterclockwise
(C) linear Interpolation
(D) rapid feed

MCQ 10.108 The mechanism of material removal in EDM process is

(A) Melting and Evaporation
(B) Melting and Corrosion
(C) Erosion and Cavitation
(D) Cavitation and Evaporation

MCQ 10.109 Two 1 mm thick steel sheets are to be spot welded at a current of $5000\,\text{A}$. Assuming effective resistance to be $200\,\mu\text{m}$ and current flow time of 0.2 second, heat generated during the process will be

(A) 0.2 Joule
(B) 1 Joule
(C) 5 Joule
(D) 1000 Joule

MCQ 10.110 Misrun is a casting defect which occurs due to

(A) very high pouring temperature of the metal
(B) insufficient fluidity of the molten metal
(C) absorption of gases by the liquid metal
(D) improper alignment of the mould flasks

MCQ 10.111 The percentage of carbon in gray cast iron is in the range of

(A) 0.25 to 0.75 percent
(B) 1.25 to 1.75 percent
(C) 3 to 4 percent
(D) 8 to 10 percent

YEAR 2004 TWO MARKS

MCQ 10.112 GO and NO-GO plug gauges are to be designed for a hole $20.000^{+0.050}_{+0.010}$ mm. Gauge tolerances can be taken as 10% of the hole tolerance. Following ISO system of gauge design, sizes of GO and NO-GO gauge will be respectively

(A) 20.010 mm and 20.050 mm
(B) 20.014 mm and 20.046 mm
(C) 20.006 mm and 20.054 mm
(D) 20.014 mm and 20.054 mm

MCQ 10.113 10 mm diameter holes are to be punched in a steel sheet of 3 mm thickness. Shear strength of the material is $400\,N/mm^2$ and penetration is 40%. Shear provided on the punch is 2 mm. The blanking force during the operation will be

(A) 22.6 kN (B) 37.7 kN

(C) 61.6 kN (D) 94. 3 kN

MCQ 10.114 Through holes of 10 mm diameter are to be drilled in a steel plate of 20 mm thickness. Drill spindle speed is 300 rpm, feed 0.2 mm/rev and drill point angle is 120°. Assuming drill overtravel of 2 mm, the time for producing a hole will be

(A) 4 seconds (B) 25 seconds

(C) 100 seconds (D) 110 seconds

MCQ 10.115 Gray cast iron blocks $200 \times 100 \times 10$ mm are to be cast in sand moulds. Shrinkage allowance for pattern making is 1%. The ratio of the volume of pattern to that of the casting will be

(A) 0.97 (B) 0.99

(C) 1.01 (D) 1.03

MCQ 10.116 In a 2-D CAD package, clockwise circular arc of radius 5, specified from $P_1(15,10)$ to $P_2(10,15)$ will have its centre at

(A) (10, 10) (B) (15, 10)

(C) (15, 15) (D) (10, 15)

MCQ 10.117 In an orthogonal cutting test on mild steel, the following data were obtained

Cutting speed	:	40 m/min
Depth of cut	:	0.3 mm
Tool rake angle	:	$+5^\circ$
Chip thickness	:	1.5 mm
Cutting force	:	900 N
Thrust force	:	450 N

Using Merchant's analysis, the friction angle during the machining will be

(A) 26.6° (B) 31.5°

(C) 45° (D) 63.4°

MCQ 10.118 In a rolling process, sheet of 25 mm thickness is rolled to 20 mm thickness. Roll is of diameter 600 mm and it rotates at 100 rpm. The roll strip contact length will be

(A) 5 mm (B) 39 mm

(C) 78 mm (D) 120 mm

MCQ 10.119 In a machining operation, doubling the cutting speed reduces the tool life to $\frac{1}{8}$th of the original value. The exponent n in Taylor's tool life equation $VT^n = C$, is

(A) $\frac{1}{8}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

MCQ 10.120 Match the following

	Feature to be inspected		Instrument
P.	Pitch and Angle errors of screw thread	1.	Auto Collimator
Q.	Flatness error of a surface	2.	Optical Interferometer
R.	Alignment error of a machine slideway	3.	Dividing Head and Dial Gauge
S.	Profile of a cam	4.	Spirit Level
		5.	Sine bar
		6.	Tool maker's Microscope

(A) P-6 Q-2 R-4 S-6
(B) P-5 Q-2 R-1 S-6
(C) P-6 Q-4 R-1 S-3
(D) P-1 Q-4 R-5 S-2

MCQ 10.121 Match the following

	Product		Process
P.	Molded luggage	1.	Injection molding
Q.	Packaging containers for Liquid	2.	Hot rolling
R.	Long structural shapes	3.	Impact extrusion
S.	Collapsible tubes	4.	Transfer molding
		5.	Blow molding
		6.	Coining

(A) P-1 Q-4 R-6 S-3
(B) P-4 Q-5 R-2 S-3
(C) P-1 Q-5 R-3 S-2
(D) P-5 Q-1 R-2 S-4

MCQ 10.122 Typical machining operations are to be performed on hard-to-machine materials by using the processes listed below. Choose the best set of Operation-Process combinations

	Operation		Process
P.	Deburring (internal surface)	1.	Plasma Arc Machining
Q.	Die sinking	2.	Abrasive Flow Machining
R.	Fine hole drilling in thin sheets	3.	Electric Discharge Machining
S.	Tool sharpening	4.	Ultrasonic Machining
		5.	Laser beam Machining
		6.	Electrochemical Grinding

(A) P-1 Q-5 R-3 S-4
(B) P-1 Q-4 R-1 S-2
(C) P-5 Q-1 R-2 S-6
(D) P-2 Q-3 R-5 S-6

MCQ 10.123 From the lists given below choose the most appropriate set of heat treatment process and the corresponding process characteristics

	Process		Characteristics
P.	Tempering	1.	Austenite is converted into bainite
Q.	Austempering	2.	Austenite is converted into martensite
R.	Martempering	3.	Cementite is converted into globular structure
		4.	Both hardness and brittleness are reduced
		5.	Carbon is absorbed into the metal

(A) P-3 Q-1 R-5 (B) P-4 Q-3 R-2
(C) P-4 Q-1 R-2 (D) P-1 Q-5 R-4

YEAR 2003 ONE MARK

MCQ 10.124 During heat treatment of steel, the hardness of various structures in increasing order is
(A) martensite, fine pearlite, coarse pearlite, spherodite
(B) fine pearlite, Martensite, spherodite, coarse pearlite
(C) martensite, coarse pearlite, fine pearlite, spherodite
(D) spherodite, coarse pearlite, fine pearlite, martensite

MCQ 10.125 Hardness of green sand mould increases with
(A) increase in moisture content beyond 6 percent
(B) increase in permeability
(C) decrease in permeability
(D) increase in both moisture content and permeability

MCQ 10.126 In Oxyacetylene gas welding, temperature at the inner cone of the flame is around

(A) 3500° C (B) 3200° C

(C) 2900° C (D) 2550° C

MCQ 10.127 Cold working of steel is defined as working

(A) at its recrystallisation temperature

(B) above its recrystallisation temperature

(C) below its recrystallisation temperature

(D) at two thirds of the melting temperature of the metal

MCQ 10.128 Quality screw threads are produced by

(A) thread milling

(B) thread chasing

(C) thread cutting with single point tool

(D) thread casting

MCQ 10.129 As tool and work are not in contact in EDM process

(A) no relative motion occurs between them

(B) no wear of tool occurs

(C) no power is consumed during metal cutting

(D) no force between tool and work occurs

MCQ 10.130 The dimensional limits on a shaft of $25h7$ are

(A) 25.000, 25.021 mm

(B) 25.000, 24.979 mm

(C) 25.000, 25.007 mm

(D) 25.000, 24.993 mm

YEAR 2003 TWO MARKS

MCQ 10.131 Hardness of steel greatly improves with

(A) annealing (B) cyaniding

(C) normalizing (D) tempering

MCQ 10.132 With a solidification factor of $0.97 \times 10^6\ \text{s/m}^2$, the solidification time (in seconds) for a spherical casting of 200 mm diameter is

(A) 539 (B) 1078

(C) 4311 (D) 3233

MCQ 10.133 A shell of 100 mm diameter and 100 mm height with the corner radius of 0.4 mm is to be produced by cup drawing. The required blank diameter is

(A) 118 mm (B) 161 mm

(C) 224 mm (D) 312 mm

MCQ 10.134 A brass billet is to be extruded from its initial diameter of 100 mm to a final diameter of 50 mm. The working temperature of 700°C and the extrusion constant is 250 MPa. The force required for extrusion is

(A) 5.44 MN (B) 2.72 MN

(C) 1.36 MN (D) 0.36 MN

MCQ 10.135 A metal disc of 20 mm diameter is to be punched from a sheet of 2 mm thickness. The punch and the die clearance is 3%. The required punch diameter is

(A) 19.88 mm (B) 19.84 mm

(C) 20.06 mm (D) 20.12 mm

MCQ 10.136 A batch of 10 cutting tools could produce 500 components while working at 50 rpm with a tool feed of 0.25 mm/rev and depth of cut of 1mm. A similar batch of 10 tools of the same specification could produce 122 components while working at 80 rpm with a feed of 0.25 mm/rev and 1 mm depth of cut. How many components can be produced with one cutting tool at 60 rpm ?

(A) 29 (B) 31

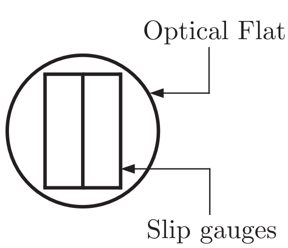
(C) 37 (D) 42

MCQ 10.137 A thread nut of $M16$ ISO metric type, having 2 mm pitch with a pitch diameter of 14.701 mm is to be checked for its pitch diameter using two or three number of balls or rollers of the following sizes

(A) Rollers of 2 mm φ (B) Rollers of 1.155 mm φ

(C) Balls of 2 mm φ (D) Balls of 1.155 mm φ

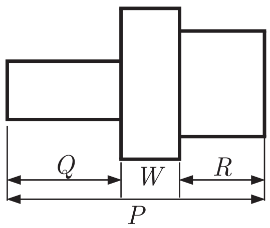
MCQ 10.138 Two slip gauges of 10 mm width measuring 1.000 mm and 1.002 mm are kept side by side in contact with each other lengthwise. An optical flat is kept resting on the slip gauges as shown in the figure. Monochromatic light of wavelength 0.0058928 mm is used in the inspection. The total number of straight fringes that can be observed on both slip gauges is



(A) 2 (B) 6

(C) 8 (D) 13

MCQ 10.139 A part shown in the figure is machined to the sizes given below



$P = 35.00 \pm 0.08$ mm, $Q = 12.00 \pm 0.02$ mm, $R = 13.00^{+0.04}_{-0.02}$ mm

With 100% confidence, the resultant dimension W will have the specification

(A) 9.99 ± 0.03 mm (B) 9.99 ± 0.13 mm

(C) 10.00 ± 0.03 mm (D) 10.00 ± 0.13 mm

MCQ 10.140 Match the following

	Working material		Type of Joining
P.	Aluminium	1.	Submerged Arc Welding
Q.	Die steel	2.	Soldering
R.	Copper wire	3.	Thermit Welding
S.	Titanium sheet	4.	Atomic Hydrogen Welding
		5.	Gas Tungsten Arc Welding
		6.	Laser Beam Welding

(A)	P-2	Q-5	R-1	S-3
(B)	P-6	Q-3	R-4	S-1
(C)	P-4	Q-1	R-6	S-2
(D)	P-5	Q-4	R-2	S-6

• Common Data For Q.141 and Q.142

A cylinder is turned on a lathe with orthogonal machining principle. Spindle rotates at 200 rpm. The axial feed rate is 0.25 mm per revolution. Depth of cut is 0.4 mm. The rake angle is 10°. In the analysis it is found that the shear angle is 27.75°.

MCQ 10.141 The thickness of the produced chip is

(A) 0.511 mm (B) 0.528 mm

(C) 0.818 mm (D) 0.846 mm

MCQ 10.142 In the above problem, the coefficient of friction at the chip tool interface obtained using Earnest and Merchant theory is

(A) 0.18 (B) 0.36

(C) 0.71 (D) 0.98

YEAR 2002 ONE MARK

MCQ 10.143 A lead-screw with half nuts in a lathe, free to rotate in both directions has

(A) V-threads (B) Whitworth threads

(C) Buttress threads (D) Acme threads

MCQ 10.144 The primary purpose of a sprue in a casting mould is to

(A) feed the casting at a rate consistent with the rate of solidification.

(B) act as a reservoir for molten metal

(C) feed molten metal from the pouring basin to the gate

(D) help feed the casting until all solidification takes place

MCQ 10.145 Hot rolling of mild steel is carried out

(A) at re-crystallization temperature

(B) between 100°C to 150°C

(C) between re-crystallization temperature

(D) above re-crystallization temperature

MCQ 10.146 Which of the following arc welding processes does not use consumable electrodes ?

(A) GMAW (B) GTAW

(C) Submerged Arc Welding (D) None of these

MCQ 10.147 Trepanning is performed for

(A) finishing a drilled hole

(B) producing a large hole without drilling

(C) truing a hole for alignment

(D) enlarging a drilled hole

MCQ 10.148 The hardness of a grinding wheel is determined by the

(A) hardness of abrasive grains

(B) ability of the bond to retain abrasives

(C) hardness of the bond

(D) ability of the grinding wheel to penetrate the work piece

YEAR 2002 TWO MARKS

MCQ 10.149 In centrifugal casting, the impurities are
(A) uniformly distributed
(B) forced towards the outer surface
(C) trapped near the mean radius of the casting
(D) collected at the centre of the casting

MCQ 10.150 The ductility of a material with work hardening
(A) increases
(B) decreases
(C) remains unaffected
(D) unpredictable

MCQ 10.151 The temperature of a carburising flame in gas welding is ......that of a neutral or an oxidising flame.
(A) lower than
(B) higher than
(C) equal to
(D) unrelated to

MCQ 10.152 In a blanking operation, the clearance is provided on
(A) the die
(B) both the die and the punch equally
(C) the punch
(D) neither the punch nor the die

MCQ 10.153 A built-up-edge is formed while machining
(A) ductile materials at high speed
(B) ductile materials at low speed
(C) brittle materials at high speed
(D) brittle materials at low speed

MCQ 10.154 The time taken to drill a hole through a 25 mm thick plate with the drill rotating at 300 rpm and moving at a feed rate of 0.25 mm/rev is
(A) 10 s
(B) 20 s
(C) 60 s
(D) 100 s

YEAR 2001 ONE MARK

MCQ 10.155 Shrinkage allowance on pattern is provided to compensate for shrinkage when
(A) the temperature of liquid metal drops from pouring to freezing temperature.
(B) the metal changes from liquid to solid state at freezing temperature
(C) the temperature of solid phase drops from freezing to room temperature
(D) the temperature of metal drops from pouring to room temperature

MCQ 10.156 The cutting force in punching and blanking operations mainly depends on

(A) the modulus of elasticity of metal (B) the shear strength of metal

(C) the bulk modulus of metal (D) the yield strength of metal

MCQ 10.157 In ECM, the material removal is due to

(A) corrosion (B) erosion

(C) fusion (D) ion displacement

MCQ 10.158 Two plates of the same metal having equal thickness are to be butt welded with electric arc. When the plate thickness changes, welding is achieved by

(A) adjusting the current

(B) adjusting the duration of current

(C) changing the electrode size

(D) changing the electrode coating

MCQ 10.159 Allowance in limits and fits refers to

(A) maximum clearance between shaft and hole

(B) minimum clearance between shaft and hole

(C) difference between maximum and minimum sizes of hole

(D) difference between maximum and minimum sizes of shaft.

YEAR 2001 TWO MARKS

MCQ 10.160 The height of the downsprue is 175 mm and its cross-sectional area at the base is 200 mm^2. The cross-sectional area of the horizontal runner is also 200 mm^2, assuming no losses, indicate the correct choice for the time (in sec) required to fill a mold cavity of volume 10^6 mm^3. (Use $g = 10\text{ m/s}^2$).

(A) 2.67 (B) 8.45

(C) 26.72 (D) 84.50

MCQ 10.161 For rigid perfectly plastic work material, negligible interface friction and no redundant work, the theoretically maximum possible reduction in the wire drawing operation is

(A) 0.36 (B) 0.63

(C) 1.00 (D) 2.72

MCQ 10.162 During orthogonal cutting of mild steel with a 10° rake angle, the chip thickness ratio was obtained as 0.4. The shear angle (in degree) evaluated from this data is

(A) 6.53 (B) 20.22

(C) 22.94 (D) 50.00

MCQ 10.163 Resistance spot welding is performed on two plates of 1.5 mm thickness with 6 mm diameter electrode, using 15000 A current for a time duration of 0.25 s. Assuming the interface resistance to be $0.0001\,\Omega$, the heat generated to form the weld is

(A) 5625 W-s (B) 8437 W-s

(C) 22500 W-s (D) 33750 W-s

MCQ 10.164 3-2-1 method of location in a jig or fixture would collectively restrict the work piece in n degrees of freedom, where the value of n is

(A) 6 (B) 8

(C) 9 (D) 12

MCQ 10.165 In an NC machining operation, the tool has to be moved from point (5, 4) to point (7, 2) along a circular path with centre at (5, 2). Before starting the operation, the tool is at (5, 4). The correct G and N codes for this motion are

(A) $N010\ GO3\ X7.0\ Y2.0\ I5.0\ J2.0$

(B) $N010\ GO2\ X7.0\ Y2.0\ I5.0\ J2.0$

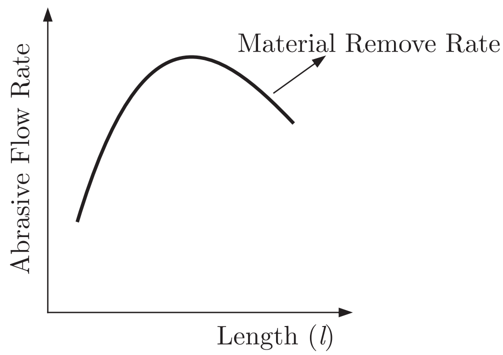
(C) $N010\ GO1\ X7.0\ Y2.0\ I5.0\ J2.0$

(D) $N010\ GOO\ X7.0\ Y2.0\ I5.0\ J2.0$

****

SOLUTION

SOL 10.1 Option (D) is correct.



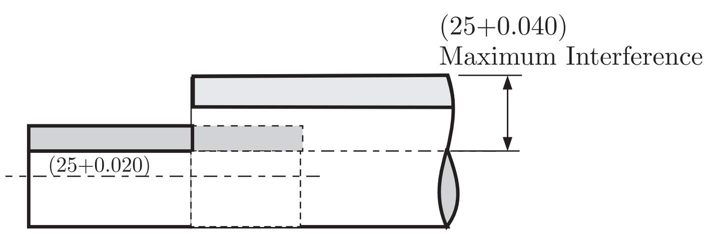
Graph for abrasive jet machining for the distance between the nozzle tip and work surface (l) and abrasive flow rate is given in figure.
It is clear from the graph that the material removal rate is first increases because of area of jet increase than becomes stable and then decreases due to decrease in jet velocity.

SOL 10.2 Option (A) is correct.

	Metal forming process		Types of stress
1.	Coining	S.	Compressive
2.	Wire Drawing	P.	Tensile
3.	Blanking	Q.	Shear
4.	Deep Drawing	R.	Tensile and compressive

Hence, correct match list is, 1-S, 2-P, 3-Q, 4-R

SOL 10.3 Option (C) is correct.
An interference fit for shaft and hole is as given in figure below.



Maximum Interference

$$= \text{Maximum limit of shat} - \text{Minimum limit of hole}$$

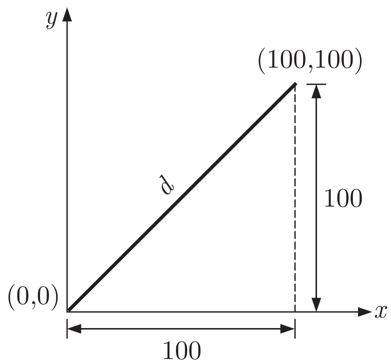
$$= (25 + 0.040) - (25 + 0.020)$$
$$= 0.02 \text{ mm } = 20 \text{ microns}$$

SOL 10.4 Option (C) is correct

Normalising involves prolonged heating just above the critical temperature to produce globular form of carbine and then cooling in air.

SOL 10.5 Option (B) is correct.

Given : width $(b) = 10\,\text{mm}$, depth $= 2\,\text{mm}$



Distance travelled for cut between points $(0,0)$ and $(100,100)$

By Pythagoras theorem

$$d = \sqrt{100^2 + 100^2} = 141.42\,\text{mm}$$

$$\text{Feed rate } f = 50\,\text{mm/ min}$$
$$= \frac{50}{60} = 0.833\,\text{mm/ sec.}$$

Time required to cut distance (d)

$$t = \frac{d}{f} = \frac{141.42}{0.833} = 169.7 \simeq 170\,\text{sec.}$$

SOL 10.6 Option (D) is correct.

Since volume of cylinder remains same. Therefore

$$\text{Volume before forging} = \text{Volume after forging}$$
$$\pi\frac{d_1^2}{4} \times h_1 = \pi\frac{d_2^2}{4} \times h_2$$
$$\pi \times \frac{100^2}{4} \times 50 = \pi \times \frac{d_2^2}{4} \times 25$$
$$d_2^2 = (100)^2 \times 2$$
$$d_2 = 100 \times \sqrt{2} = 141.42$$

Percentage change in diameter

$$= \frac{d_2 - d_1}{d_1} \times 100 = \frac{141.42 - 100}{100} \times 100$$

$$\% \text{ change in } (d) = 41.42\%$$

SOL 10.7 Option (C) is correct.

$$\text{Shear strain rate} = \frac{\cos\alpha}{\cos(\phi - \alpha)} \times \frac{V}{\Delta y}$$

Where
α = Rake angle = 10°
V = cutting speed = 2.5 m/s
Δy = Mean thickness of primary shear zone
= 25 microns = 25×10^{-6} m
ϕ = shear angle

Shear angle, $$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha}$$

where r = chip thickness ratio = 0.4

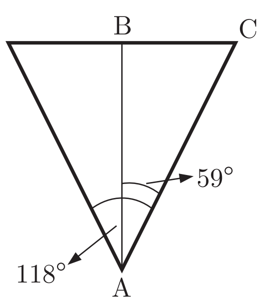
$$\tan\phi = \frac{0.4 \times \cos 10^\circ}{1 - 0.4\sin 10^\circ} = 0.4233$$

$$\phi = \tan^{-1}(0.4233) \cong 23^\circ$$

$$\text{Shear Strain rate} = \frac{\cos 10^\circ}{\cos(23 - 10)} \times \frac{2.5}{25 \times 10^{-6}} = 1.0104 \times 10^5 s^{-1}$$

SOL 10.8 Option (A) is correct.

Drill bit tip is shown as below.



BC = radius of hole or drill bit $(R) = \frac{15}{2} = 7.5$ mm

From ΔABC $$\tan 59^\circ = \frac{BC}{AB} = \frac{7.5}{AB}$$

$$AB = \frac{7.5}{\tan 59^\circ} = 4.506 \text{ mm}$$

Travel distance of drill bit

l = thickness of steel plate (t) + clearance at approach + clearance at exit + AB

$$= 50 \text{ mm} + 2 + 2 + 4.506 = 58.506 \text{ mm}$$

$$\text{Total drill time} = \frac{\text{distance}}{\text{feed rate}}$$

$$f = 0.2\text{ mm/rev} = \frac{0.2 \times \text{rpm}}{60} = \frac{0.2 \times 500}{60} = 1.66\text{ mm/s}$$

Hence drill time, $t = \frac{58.506}{1.60} = 35.1\text{ sec.}$

SOL 10.9 Option (A) is correct.

Punch diameter, $d = D - 2c - a$

where D = Blank diameter = 25 mm

c = Clearance = 0.06 mm

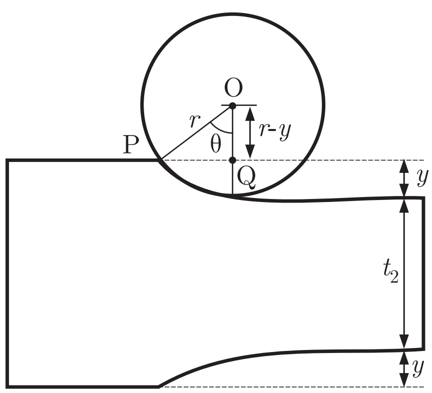
a = Die allowance = 0.05 mm

Hence, $d = 25 - 2 \times 0.06 - 0.05 = 24.83\text{ mm}$

SOL 10.10 Option (C) is correct.

Given : $t_1 = 8\text{ mm},\ d = 410\text{ mm},\ r = 205\text{ mm}$

Reduction of thickness, $\Delta t = 10\%\text{ of } t_1 = \frac{10}{100} \times 8 = 0.8\text{ mm}$



$$y = \frac{\Delta t}{2} = 0.4\text{ mm}$$

From ΔOPQ, $\cos\theta = \left(\frac{r-y}{r}\right)$

$$= \left[\frac{205 - 0.4}{205}\right] = 0.99804$$

$$\theta = \cos^{-1}(0.99804) = 3.58^\circ$$

Angle of bite in radians is

$$\theta = 3.58 \times \frac{\pi}{180}\text{ rad} = 0.062\text{ rad.}$$

Alternate Method :

Angle of bite, $\theta = \tan^{-1}\left[\sqrt{\frac{t_i - t_f}{r}}\right]$

Where, t_i = Initial thickness = 8 mm

$$t_f = \text{Final reduced thickness} = 8 - 8 \times \frac{10}{100} = 7.2 \text{ mm}$$

$$r = \text{radius of roller} = \frac{410}{2} = 205 \text{ mm}$$

$$\theta = \tan^{-1}\left[\sqrt{\frac{8-7.2}{205}}\right] = 3.5798^\circ$$

And in radians, $\theta = 3.5798 \times \frac{\pi}{180} = 0.0624$ rad.

SOL 10.11 Option (C) is correct.

From power source characteristic,

$$\frac{V}{OCV} + \frac{I}{SCC} = 1 \quad \text{...(i)}$$

where, V = Voltage

OCV = Open circuit voltage

SCC = Short circuit current

I = Current.

From voltage arc length characteristic

$$V_{arc} = 20 + 5l$$

For $l_1 = 5$ mm, $V_1 = 20 + 5 \times 5 = 45$ V

For $l_2 = 7$ mm, $V_2 = 20 + 5 \times 7 = 55$ V

and $I_1 = 500$ Amp. and $I_2 = 400$ Amp.

Substituting these value in Eq. (i)

$$\frac{V_1}{OCV} + \frac{I_1}{SCC} = 1$$

$$\frac{45}{OCV} + \frac{500}{SCC} = 1 \quad \text{...(ii)}$$

$$\frac{V_2}{OCV} + \frac{I_2}{SCC} = 1 \quad \Rightarrow \quad \frac{55}{OCV} + \frac{400}{SCC} = 1 \quad \text{...(iii)}$$

Solving Eq. (ii) and (iii), we get

$$OCV = 95 \text{ V}$$

$$SCC = 950 \text{ Amp.}$$

SOL 10.12 Option (A) is correct.

The main objective in rolling is to decrease the thickness of the metal.

The relation for the rolling is given by

$$F = \mu P_r$$

Where ; F = tangential frictional force

μ = Coefficient of friction

P_r = Normal force between the roll and work piece

Now, from the increase in μ, the draft in cold rolling of sheet increases.

SOL 10.13 Option (C) is correct.
If the pores in a sintered compact are filled with an oil, the operation is called as impregnation. The lubricants are added to the porous bearings, gears and pump rotors etc.

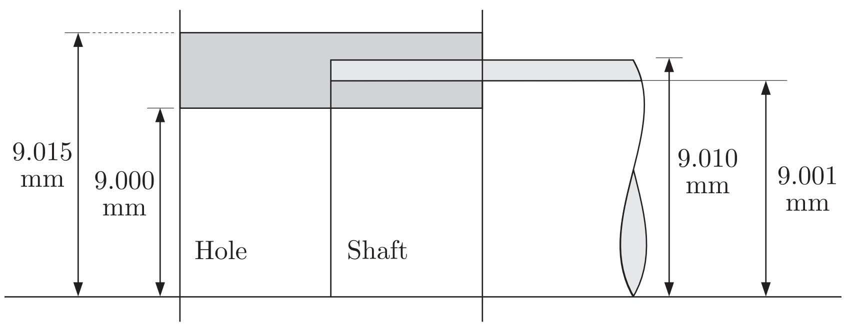
SOL 10.14 Option (C) is correct.
In transition fit, the tolerance zones of holes and shaft overlap.

Upper limit of hole $= 9 + 0.015 = 9.015\,\text{mm}$

Lower limit of hole $= 9 + 0.000 = 9.000\,\text{mm}$

Upper limit of shaft $= 9 + 0.010 = 9.010\,\text{mm}$

Lower limit of shaft $= 9 + 0.001 = 9.001\,\text{mm}$



Now, we can easily see from figure dimensions that it is a transition fit.

SOL 10.15 Option (D) is correct.
A green sand mould is composed of a mixture of sand (silica sand, SiO_2), clay (which acts as binder) and water.
The word green is associated with the condition of wetness or freshness and because the mould is left in the damp condition, hence the name " green sand mould".

SOL 10.16 Option (C) is correct.
GTAW is also called as Tungsten Inert Gas Welding (TIG). The arc is maintained between the work piece and a tungsten electrode by an inert gas. The electrode is non-consumable since its melting point is about 3400°C.

SOL 10.17 Option (B) is correct.
Austenite is a solid solution of carbon in γ-iron. It has F.C.C structure. It has a solid solubility of upto 2% C at 1130°C.

SOL 10.18 Option (B) is correct.

Given : $\alpha = 12^\circ$, $t = 0.81$ mm, $t_c = 1.8$ mm

Shear angle, $$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha} \quad ...(i)$$

Chip thickness ratio, $$r = \frac{t}{t_c} = \frac{0.81}{1.8} = 0.45$$

From equation (i), $$\tan\phi = \frac{0.45\cos 12^\circ}{1 - 0.45\sin 12^\circ}$$

$$\phi = \tan^{-1}(0.486) = 25.91^\circ \simeq 26^\circ$$

SOL 10.19 Option (A) is correct.

	Machining process		Mechanism of material removal
P.	Chemical machining	2.	Corrosive reaction
Q.	Electro-chemical machining	3.	Ion displacement
R.	Electro-discharge machining	4.	Fusion and vaporization
S.	Ultrasonic machining	1.	Erosion

So, correct pairs are, P-2, Q-3, R-4, S-1

SOL 10.20 Option (A) is correct.

Given : $a = 50$ mm, $V = a^3 = (50)^3 = 125000\text{ mm}^3$

Firstly side undergoes volumetric solidification shrinkage of 4%.

So, Volume after shrinkage,

$$V_1 = 125000 - 125000 \times \frac{4}{100} = 120000\text{ mm}^3$$

After this, side undergoes a volumetric solid contraction of 6%.

So, volume after contraction,

$$V_2 = 120000 - 120000 \times \frac{6}{100} = 112800\text{ mm}^3$$

Here V_2 is the combined volume after shrinkage and contraction.

Let at volume V_2, side of cube is b.

So, $$b^3 = 112800 = \sqrt[3]{112800} = 48.32\text{ mm}$$

SOL 10.21 Option (C) is correct.

Given : $\tau = 300$ MPa, $D = 100$ mm, $t = 1.5$ mm

Blanking force $$F_b = \tau \times \text{Area} = \tau \times \pi D t$$

$$F_b = 300 \times 10^6 \times 3.14 \times 100 \times 1.5 \times 10^{-6}$$

$$= 141300\text{ N} = 141.3\text{ kN} \simeq 141\text{ kN}$$

SOL 10.22 Option (C) is correct.

Fracture strength be a material property which depends on the basic crystal structure. Fracture strength depends on the strength of the material.

SOL 10.23 Option (A) is correct.

Gate Ratio is defined as the ratio of sprue base area, followed by the total runner area and the total ingate area. The sprue base area is taken is unity.

So, $1:2:4 = \text{Sprue base area} : \text{Runner area} : \text{Total ingate area}$

SOL 10.24 Option (D) is correct.

We know that, shaft tolerance

$$= \text{Upper limit of shaft} - \text{Lower limit of shaft}$$

$$= (35 - 0.009) - (35 - 0.025) = 34.991 - 34.975 = 0.016$$

Fundamental deviation for basic shaft is lower deviation.

$$= -0.009$$

SOL 10.25 Option (C) is correct.

GO2 represent circular interpolation in clockwise direction.

G91 represent incremental dimension.

SOL 10.26 Option (A) is correct.

For Tool A, $n = 0.45, \text{K} = 90$

For Tool B, $n = 0.3, \text{K} = 60$

Now, From the Taylor's tool life equation ($VT^n = \text{K}$)

For Tool A, $V_A T_A{}^{0.45} = 90$ ...(i)

For Tool B, $V_B T_B{}^{0.3} = 60$ ...(ii)

Dividing equation (i) by equation (ii), we get

$$\left(\frac{V_A}{V_B}\right) \times \frac{T_A^{0.45}}{T_B^{0.3}} = \frac{90}{60} \quad \text{...(iii)}$$

Let V is the speed above which tool A will have a higher life than B. But at V, $T_A = T_B$

Then $V_A = V_B = V(\text{let})$

$T_A = T_B = T(\text{let})$

So, from equation(iii) $\frac{T^{0.45}}{T^{0.3}} = \frac{3}{2} \Rightarrow T^{0.45-0.3} = \frac{3}{2}$

$$T = \left(\frac{3}{2}\right)^{\frac{1}{0.15}} = 14.92 \text{ min}.$$

From equation (i), $V \times T^{0.45} = 90$

$$V \times (14.92)^{0.45} = 90$$

$$V = 26.67 \text{ m/min} \simeq 26.7 \text{ m/min}$$

SOL 10.27 Option (C) is correct.

Given: $d_i = 100$ mm, $d_o = 110$ mm, $V = 30$ Volt, $R = 42.4\,\Omega$, $E_u = 64.4$ MJ/m^3

Each pipe melts 1 mm of material. So, thickness of material melt, $t = 2 \times 1 = 2$ mm

Melting energy in whole volume is given by

$$Q = \text{Area} \times \text{thickness} \times E_u = \frac{\pi}{4}(d_o^2 - d_i^2) \times t \times E_u$$

$$Q = \frac{\pi}{4}[(110)^2 - (100)^2] \times 10^{-6} \times 2 \times 10^{-3} \times 64.4 \times 10^6$$

$$= 212.32 \text{ J} \qquad \text{...(i)}$$

The amount of heat generated at the contacting area of the element to be weld is,

$$Q = I^2 Rt = \frac{V^2}{R} t \qquad I = \frac{V}{R}$$

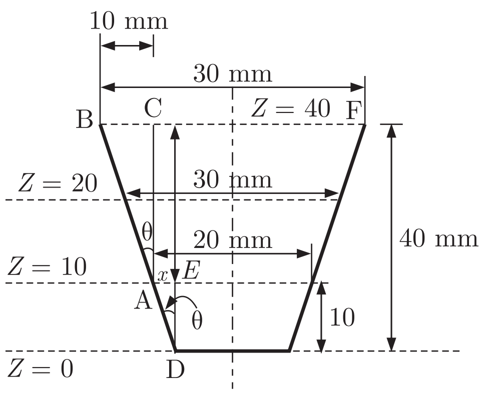
$$t = \frac{Q \times R}{V^2}$$

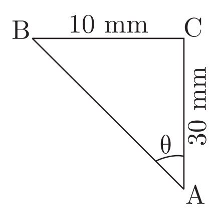
Substitute the values, we get

$$t = \frac{212.32 \times 42.4}{(30)^2} = 10 \text{ sec}$$

SOL 10.28 Option (A) is correct

Draw a perpendicular from the point A on the line BF, which intersect at point C.





Let Angle $\angle BAC = \theta$

$$AE = x$$

Now, take the right angle triangle ΔABC,

$$\tan\theta = \frac{BC}{AC} = \frac{10}{30} = \frac{1}{3} \qquad \text{...(i)}$$

From the same triangle ΔADE,

$$\tan\theta = \frac{x}{DE} = \frac{x}{10}$$

Put the value of $\tan\theta$, from the equation (i),

So, $$\frac{1}{3} = \frac{x}{10} \Rightarrow x = \frac{10}{3}\,\text{mm} = 3.333\,\text{mm}$$

Now, diameter at $Z = 0$ is,

$$d = 20 - 2x = 20 - 2 \times 3.333 = 13.334\,\text{mm}$$

SOL 10.29 Option (B) is correct.

Given : $t = 5\,\text{mm}$, $L = 200\,\text{mm}$, $\tau_s = 100\,\text{MPa}$

Penetration to thickness ratio $\frac{p}{t} = 0.2 = k$

Force vs displacement curve to be rectangle,

So, Shear area, $$A = (200 + 200) \times 5 = 2000\,\text{mm}^2$$

Work done, $$W = \tau \times A \times k \times t$$

Substitute the values, we get

$$W = 100 \times 10^6 \times 2000 \times 10^{-6} \times 0.2 \times 5 \times 10^{-3}$$
$$= 100 \times 2 \times 0.2 \times 5 = 200\ \text{Joule}$$

SOL 10.30 Option (B) is correct.

Given : Shear $S = 20\,\text{mm}$

Now force vs displacement curve to be trapezoidal.

So, maximum force is given by,

$$F_{\max} = \frac{W}{(kt + \text{Shear})} = \frac{200}{(0.2 \times 5 + 20) \times 10^{-3}}$$
$$= \frac{200}{21} \times 10^{-3} = 9.52 \times 10^3 \simeq 10\,\text{kN}$$

SOL 10.31 Option (D) is correct.

The cutting forces decrease with an increase in cutting speed, but it is substantially smaller than the increase in speed. With the increase in speed, friction decreases at the tool chip interface. The thickness of chip reduces by increasing the speed.

SOL 10.32 Option (A) is correct.

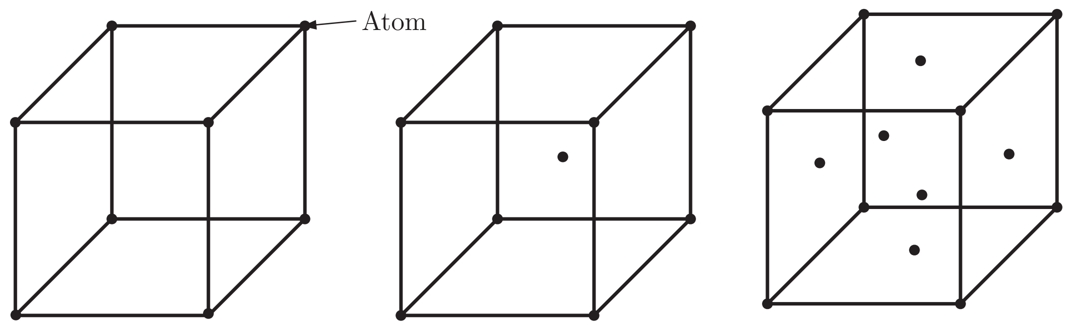
Two streams of liquid metal which are not hot enough to fuse properly result into a casting defect known as cold shut. This defect is same as in sand mould casting. The reasons are :-

(i) Cooling of die or loss of plasticity of the metal.

(ii) Shot speed less.

(iii) Air-vent or overflow is closed.

SOL 10.33 Option (B) is correct.



(i) Simple Cubic
Effective no. of Lattice
$\frac{1}{8}\times 8 = 1$

(ii) BCC
Effective no. of Lattice
$\frac{1}{8}\times 8 + 1 = 2$

(iii) FCC
Effective no. of Lattice
$\frac{1}{8}\times 8 + \frac{1}{2}\times 6 = 4$

SOL 10.34 Option (C) is correct.
Correct data structure for solid models is given by,
Vertices $\rightarrow$ edges $\rightarrow$ faces $\rightarrow$ solid parts

SOL 10.35 Option (D) is correct.
Given : $\alpha = 0^\circ$
We know that, shear strain

$$s = \cot\phi + \tan(\phi - \alpha) \qquad \alpha = 0^\circ$$

So,
$$s = \cot\phi + \tan\phi \qquad ...(i)$$

For minimum value of shear strain differentiate equation (i) w.r.t. ϕ

$$\frac{ds}{d\phi} = \frac{d}{d\phi}(\cot\phi + \tan\phi) = -\operatorname{cosec}^2\phi + \sec^2\phi \qquad ...(ii)$$

Again differentiate w.r.t. to ϕ,

$$\frac{d^2s}{d\phi^2} = -2\operatorname{cosec}\phi \times (-\operatorname{cosec}\phi\cot\phi) + 2\sec\phi \times (\sec\phi\tan\phi)$$
$$= +2\operatorname{cosec}^2\phi\cot\phi + 2\sec^2\phi\tan\phi \qquad .....(iii)$$

Using the principle of minima - maxima and put $\frac{ds}{d\phi} = 0$ in equation(ii)

$$-\operatorname{cosec}^2 + \sec^2\phi = 0$$

$$-\frac{1}{\sin^2\phi} + \frac{1}{\cos^2\phi} = 0$$

$$\frac{\cos^2\phi - \sin^2\phi}{\sin^2\phi \times \cos^2\phi} = 0$$

$$\cos^2\phi - \sin^2\phi = 0$$

$$\cos 2\phi = 0$$
$$2\phi = \cos^{-1}(0) = \frac{\pi}{2}$$
$$\phi = \frac{\pi}{4}$$

From equation (iii), at $\phi = \frac{\pi}{4}$

$$\left(\frac{d^2 s}{d\phi^2}\right)_{\phi=\frac{\pi}{4}} = 2\operatorname{cosec}^2\frac{\pi}{4} \times \cot\frac{\pi}{4} + 2\sec^2\frac{\pi}{4}\tan\frac{\pi}{4}$$
$$\left(\frac{d^2 s}{d\phi^2}\right)_{\phi=\frac{\pi}{4}} = 2 \times 2 \times 1 + 2 \times 2 \times 1 = 8$$
$$\left(\frac{d^2 s}{d\phi^2}\right)_{\phi=\frac{\pi}{4}} > 0$$

Therefore it is minimum at $\phi = \frac{\pi}{4}$, so from equation (i),

$$(s)_{\min} = \cot\frac{\pi}{4} + \tan\frac{\pi}{4} = 1 + 1 = 2$$

SOL 10.36 Option (A) is correct.

Given : $L = 0.2\text{ mm}$, $A = 20\text{ mm} \times 20\text{ mm} = 400\text{ mm}^2$, $V = 12$ Volt

$\rho = 2\,\Omega\text{cm} = 2 \times 10\,\Omega\text{ mm}$, $Z = 55.85$, $v = 2$, $F = 96540$ Coulombs

We know that Resistance is given by the relation

$$R = \frac{\rho L}{A} = \frac{2 \times 10 \times 0.2}{20 \times 20} = 0.01\,\Omega$$
$$I = \frac{V}{R} = \frac{12}{0.01} = 1200\text{ A}$$

Rate of mass removal $\dot{m} = \frac{I}{F} \times \frac{Z}{v} = \frac{1200}{96540} \times \frac{55.85}{2} = 0.3471\text{ g/sec}$

SOL 10.37 Option (C) is correct.

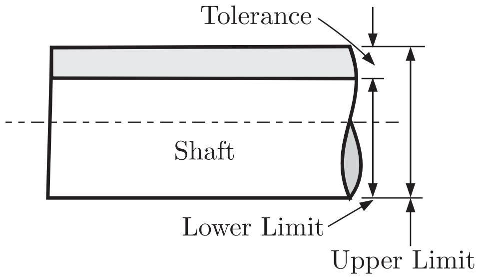
	NC code		Definition
P.	M05	3.	Spindle stop
Q.	G01	4.	Linear interpolation
R.	G04	2.	Dwell
S.	G09	1.	Absolute coordinate system

So, correct pairs are, P-3, Q-4, R-2, S-1

SOL 10.38 Option (A) is correct.

Since diameter 60 lies in the diameter step of $50 - 80\text{ mm}$, therefore the geometric mean diameter.

$$D = \sqrt{50 \times 80} = 63.246\text{ mm}$$



Fundamental tolerance unit.

$$i = 0.45D^{1/3} + 0.001D = 0.45(63.246)^{1/3} + 0.001 \times 63.246$$
$$= 1.856\,\mu\text{m} = 0.00186\text{ mm}$$

Standard tolerance for the hole of grades 8 (IT8)

$$= 25i = 25 \times 0.00186 = 0.0465\text{ mm}$$

Fundamental deviation for 'f' shaft

$$e_f = -5.5D^{0.41} = -5.5(63.246)^{0.41}$$
$$= -30.115\,\mu\text{m} = -0.030115\text{ mm}$$

Upper limit of shaft = Basic size + Fundamental deviation

$$= 60 - 0.030115 = 59.970\text{ mm}$$

Lower limit of shaft = Upper limit − Tolerance = 59.970 − 0.0465

$$= 59.924$$

SOL 10.39 Option (D) is correct.

	Column I		Column II
P.	Metallic Chills	4.	Progressive solidification
Q.	Metallic Chaplets	1.	Support for the core
R.	Riser	2.	Reservoir of the molten metal
S.	Exothermic Padding	3.	Control cooling of critical sections

So, correct pairs are P-4, Q-1, R-2, S-3

SOL 10.40 Option (A) is correct.

Given : $V_1 = 60\text{ m/min}$, $T_1 = 81\text{ min}$, $V_2 = 90\text{ m/min}$, $T_2 = 36\text{ min}$.

From the Taylor's tool life Equation

$$VT^n = \text{Constant (K)}$$

For case (I), $V_1 T_1^n = K$

$$60 \times (81)^n = K \quad ...(i)$$

For case (II), $V_2 T_2^n = K$

$$90 \times (36)^n = K \quad ...(ii)$$

By dividing equation (i) by equation (ii),

$$\frac{60 \times (81)^n}{90 \times (36)^n} = \frac{K}{K} = 1$$

$$\left(\frac{81}{36}\right)^n = \frac{90}{60}$$

$$\left(\frac{9}{4}\right)^n = \left(\frac{3}{2}\right)$$

Taking (log) both the sides,

$$n \log\left(\frac{9}{4}\right) = \log\left(\frac{3}{2}\right)$$

$$n \times 0.3522 = 0.1760$$

$$n = 0.5$$

Substitute $n = 0.5$ in equation (i), we get

$$K = 60 \times (81)^{0.5} = 540$$

So, $n = 0.5$ and $K = 540$

SOL 10.41 Option (C) is correct.

Take, $n = 0.5$ {from previous part}

From Taylor's tool life equation

$$VT^n = C$$

$$VT^{0.5} = C$$

$$V = \frac{1}{\sqrt{T}} \qquad \text{...(i)}$$

Given that cutting speed is halved

$$V_2 = \frac{1}{2} V_1 \qquad \Rightarrow \frac{V_2}{V_1} = \frac{1}{2}$$

Now, from equation (i),

$$\frac{V_2}{V_1} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{1}{2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{1}{4} = \frac{T_1}{T_2}$$

$$\frac{T_2}{T_1} = 4 \qquad \Rightarrow T_2 = 4T_1$$

Now, percentage increase in tool life is given by

$$= \frac{T_2 - T_1}{T_1} \times 100 = \frac{4T_1 - T_1}{T_1} \times 100$$

$$= \frac{3T_1}{T_1} \times 100 = 300\%$$

SOL 10.42 Option (C) is correct

Coon's surface is obtained by blending four boundary curves. The main advantage of Coon's surface is its ability to fit a smooth surface through digitized points in space such as those used in reverse engineering.

SOL 10.43 Option (C) is correct.
Internal gear cutting operation can be performed by shaping with pinion cutter. In the case of 'rotating pinion type cutter', such an indexing is not required, therefore, this type is more productive and so common.

SOL 10.44 Option (B) is correct.
Since metal shrinks on solidification and contracts further on cooling to room temperature, linear dimensions of patterns are increased in respect of those of the finished casting to be obtained. This is called the "Shrinkage allowance".
The riser can compensate for volume shrinkage only in the liquid or transition stage and not in the solid state.
So, Volume of metal that compensated from the riser $= 3\% + 4\% = 7\%$

SOL 10.45 Option (D) is correct.
Interconversion between ASA (American Standards Association) system and ORS (Orthogonal Rake System)

$$\tan\alpha_s = \sin\phi\tan\alpha - \cos\phi\tan i$$

where α_s = Side rake angle

α = orthogonal rake angle

ϕ = principle cutting edge angle $= 0 \le \phi \le 90^\circ$

i = inclination angle ($i = 0$ for ORS)

$\alpha_s = \alpha$ (Given)

$$\tan\alpha_s = \sin\phi\tan\alpha - \cos\phi\tan(0^\circ)$$

$$\tan\alpha_s = \sin\phi\tan\alpha$$

$$\frac{\tan\alpha_s}{\tan\alpha} = \sin\phi$$

$$1 = \sin\phi$$

$$\phi = \sin^{-1}(1) = 90^\circ$$

SOL 10.46 Option (B) is correct.
Given : $\rho = 6000\text{ kg/m}^3 = 6\text{ gm/cm}^3,\ F = 96500\text{ coulomb/mole}$
$MRR = 50\text{ mm}^3/\text{s} = 50\times10^{-3}\text{ cm}^3/\text{s},\ I = 2000\text{ A}$

For Iron : Atomic weight $= 56$
Valency $= 2$

For Metal P :Atomic weight $= 24$

Valency $= 4$

The metal Removal rate

$$MRR = \frac{eI}{F\rho}$$

$$50 \times 10^{-3} = \frac{e \times 2000}{96500 \times 6}$$

$$e = \frac{50 \times 10^{-3} \times 96500 \times 6}{2000} = 14.475$$

Let the percentage of the metal P in the alloy is x.

So,
$$\frac{1}{e} = \frac{100 - x}{100} \times \frac{V_{Fe}}{A_{t_{Fe}}} + \frac{x}{100} \times \frac{V_P}{A_{tP}}$$

$$\frac{1}{14.475} = \frac{100 - x}{100} \times \frac{2}{56} + \frac{x}{100} \times \frac{4}{24}$$

$$\frac{1}{14.475} = \left(1 - \frac{x}{100}\right)\frac{1}{28} + \frac{x}{100} \times \frac{1}{6}$$

$$\frac{1}{14.475} = x\left[\frac{1}{600} - \frac{1}{2800}\right] + \frac{1}{28}$$

$$\frac{1}{14.475} - \frac{1}{28} = x \times \frac{11}{8400}$$

$$\frac{541}{16212} = \frac{11x}{8400}$$

$$x = \frac{541 \times 8400}{16212 \times 11} \simeq 25$$

SOL 10.47 Option None of these.

Given : $t_i = 20$ mm, $t_f = 18$ mm, $b = 100$ mm,
$R = 250$ mm, $N = 10$ rpm, $\sigma_0 = 300$ MPa

We know, Roll strip contact length is given by,

$$L = \theta \times R = \sqrt{\frac{t_i - t_f}{R}} \times R$$

$$= \sqrt{R(t_i - t_f)}$$

So,
$$L = \sqrt{250 \times 10^{-3}(20 - 18)10^{-3}}$$

$$= 22.36 \times 10^{-3}$$

Rolling load,
$$F = Lb\sigma_0$$

$$= 22.36 \times 10^{-3} \times 100 \times 10^{-3} \times 300 \times 10^{6}$$

$$= 670.8 \text{ kN}$$

Power
$$P = F \times v = 670.8 \times \left(\frac{\pi DN}{60}\right)$$

$$= 670.8 \times \left(\frac{3.14 \times 0.5 \times 10}{60}\right) = 175.5 \text{ kW}$$

SOL 10.48 Option (B) is correct.

Given : $\eta_m = 0.5$, $\eta_h = 0.7$, $A = 5\text{ mm}^2$, $E_u = 10\text{ J/mm}^3$, $P = 2\text{ kW}$,
V (mm/s) = ?

Total energy required to melt,

$$E = E_u \times A \times V = 10 \times 5 \times v = 50\text{ VJ/sec}$$

Power supplied for welding,

$$P_s = P \times \eta_h \times \eta_m = 2 \times 10^3 \times 0.5 \times 0.7 = 700\text{ W}$$

From energy balance,

Energy required to melt = Power supplied for welding

$$50V = 700 \qquad \Rightarrow \quad V = 14\text{ mm/sec}$$

SOL 10.49 Option (A) is correct.

Seamless cylinders and tubes can be made by hot drawing or cupping.

The thickness of the cup is reduced and its length increased by drawing it through a series of dies having reduced clearance between the die and the punch. Due to reduction in its thickness, blanks shows a tendency to wrinkle up around the periphery because of buckling due to circumferential compression an due to this compression blank holder pressure increases.

SOL 10.50 Option (C) is correct.

The feed drive serves to transmit power from the spindle to the second operative unit of the lathe, that is, the carriage. It, thereby converts the rotary motion of the spindle into linear motion of the carriage.

So, Q and E are connected & U_s is placed between Q and E.

SOL 10.51 Option (C) is correct.

A dial indicator (gauge) or clock indicator is a very versatile and sensitive instrument. It is used for :

(i) determining errors in geometrical form, for example, ovality, out-of roundness, taper etc.

(ii) determining positional errors of surface

(iii) taking accurate measurements of deformation.

Here equal deflections are shown in both the sensor P and sensor Q. So drill spindle rotational axis is parallel to the drill spindle tape hole axis.

SOL 10.52 Option (D) is correct.

Given : $\tau_s = 250\text{ MPa}$, $V = 180\text{ m/min}$, $f = 0.20\text{ mm/rev}$
$d = 3\text{ mm}$, $r = 0.5$, $\alpha = 7^\circ$

We know from merchant's theory,

Shear plane angle $\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha} = \frac{0.5\cos 7^\circ}{1 - 0.7\sin 7^\circ} = \frac{0.496}{0.915} = 0.54$

$$\phi = \tan^{-1}(0.54) = 28.36 \simeq 28^\circ$$

Average stress on the shear plane area are

$$\tau_s = \frac{F_s}{A_s} \qquad \Rightarrow F_s = \tau_s \times A_s$$

where, A_s is the shear plane area $= \frac{bt}{\sin\phi}$

for orthogonal operation $b \cdot t = d \cdot f$

So, $F_s = \frac{\tau_s \times d \times f}{\sin\phi} = \frac{250 \times 3 \times 0.20}{\sin 28^\circ} = 319.50 \simeq 320\text{ N}$

SOL 10.53 Option (B) is correct.

Now we have to find cutting force (F_c) and frictional force (F_t).

From merchant's theory, $2\phi + \beta - \alpha = 90^\circ$

$$\beta = 90^\circ + \alpha - 2\phi = 90^\circ + 7 - 2 \times 28 = 41^\circ$$

We know that

$$\frac{F_c}{F_s} = \frac{\cos(\beta - \alpha)}{\cos(\phi + \beta - \alpha)} \qquad F_s = \text{Share force}$$

$$F_c = 320 \times \frac{\cos(41^\circ - 7^\circ)}{\cos(28^\circ + 41^\circ - 7^\circ)} = 320 \times 1.766 \simeq 565\text{ N}$$

And $F_s = F_c\cos\phi - F_t\sin\phi$

So, $F_t = \frac{F_c\cos\phi - F_s}{\sin\phi} = \frac{565 \times \cos 28^\circ - 320}{\sin 28^\circ} = \frac{178.865}{0.47}$

$$= 381.56\text{ N} \simeq 381\text{ N}$$

SOL 10.54 Option (B) is correct.

Given : $N = 200$ step/rev., $p = 4$ mm, $U = \frac{1}{4}$, $f = 10000$ Pulse/ min.

In a CNC machine basic length unit (BLU) represents the smallest distance.

Revolution of motor in one step $= \frac{1}{200}$ rev./step

Movement of lead screw $= \frac{1}{200} \times \frac{1}{4} = \frac{1}{800}$ rev. of load screw

Movement from lead screw is transferred to table.

i.e. Movement of table $= \frac{1}{800} \times \text{Pitch} = \frac{1}{800} \times 4 = \frac{1}{200}$

$$= 0.005 = 5\text{ microns.}$$

SOL 10.55 Option (C) is correct.

We know BLU = Revolution of motor $\times$ Gear ratio $\times$ pitch

$$= \frac{1}{200} \times \frac{1}{2} \times 4 = \frac{1}{100} = 10 \text{ micros}$$

We see that f is unchanged and value of Gear ratio is changed by 1/2.

SOL 10.56 Option (B) is correct.

The carbon alloy having less than 2% carbon are called "steels" and those containing over 2% carbon are called cast irons.

Now, steel may further be classified into two groups.

(i) Steels having less than 0.83% carbon are called "hypo-eutectoid steels"

(ii) Those having more than 0.83% carbon called "hyper-eutectoid steels"

SOL 10.57 Option (D) is correct.

The hot chamber die casting process is used for low melting temperature alloys.

Tin is a low melting temperature alloy.

SOL 10.58 Option (C) is correct.

Friction welding is defined as " A solid state welding process wherein coalescence is produced by heat obtained from mechanically induced sliding motion between rubbing surfaces.

SOL 10.59 Option (B) is correct.

Given : $D = 150 \text{ mm}, \; V = 90 \text{ m/min}, \; f = 0.24 \text{ mm/rev.}$

$d = 2 \text{ mm}, \; t_c = 0.48 \text{ mm}, \; \alpha = 0^\circ, \; \lambda = 90^\circ$

Uncut chip thickness, $t = f \sin \lambda = 0.24 \times \sin 90^\circ = 0.24 \text{ mm}$

Chip thickness ratio, $r = \frac{t}{t_c} = \frac{0.24}{0.48} = \frac{1}{2}$

From merchant's theory,

Shear angle, $\tan \phi = \frac{r \cos \alpha}{1 - r \sin \alpha} = \frac{0.5 \cos 0^\circ}{1 - 0.5 \times \sin 0^\circ} = 0.5$

$$\phi = \tan^{-1}(0.5) = 26.56^\circ$$

SOL 10.60 Option (C) is correct.

A spindle motor is a small, high precision, high reliability electric motor that is used to rotate the shaft or spindle used in machine tools for performing a wide rang of tasks like drilling, grinding, milling etc.

A stepper motor have not all these characteristic due to change of direction of rotation with time interval.

SOL 10.61 Option (D) is correct.

According to Caine's relation

Solidification time, $(T) = q\left(\frac{V}{A}\right)^2$

Where : $V =$ Volume, $A =$ Surface area, $Q =$ Flow rate

q = constant of proportionality depends upon composition of cast metal

Using the subscript c for the cube and subscript s for the sphere.

Given : $V_c = V_s$ So, $T \propto \frac{1}{A^2}$

So, $$\frac{T_c}{T_s} = \left(\frac{A_s}{A_c}\right)^2 = \left(\frac{4\pi r^2}{6l^2}\right)^2 = \left(\frac{4\pi}{6}\right)^2\left(\frac{r}{l}\right)^4$$

SOL 10.62 Question (A) is correct.

Metal removel rate depends upon current density and it increases with current. The MRR increase with thermal conductivity also

$$\text{Wear ratio} = \frac{\text{Volume of metal removed work}}{\text{Volume of metal removed tool}}$$

The volume of metal removed from the tool is very less compare to the volume of metal removed from the work.

So, Wear ration $\propto$ volume of metal removed work.

Hence, both the wear rate and MRR are expected to be high.

SOL 10.63 Option (D) is correct.

Given : $E = 2\text{ J/mm}^3$, $V = 120\text{ m/min}$, $f = 0.2\text{ mm/rev.} = t$, $d = 2\text{ mm} = b$

The specific energy. $E = \frac{F_c}{b \cdot t}$

In orthogonal cutting $b \times t = d \times f$

$$F_c = E \times b \times t = E \times d \times f$$

$$= 2 \times 10^9 \times 2 \times 10^{-3} \times 0.2 \times 10^{-3} = 800\text{ N}$$

SOL 10.64 Option (A) is correct.

Given : $\text{OCV} = 80\text{ V}$, $\text{SCC} = 800\text{ A}$

In Case (I) : $I = 500\text{ A}$ and $L = 5.0\text{ mm}$

And in, Case (II) : $I = 460\text{ A}$, and $L = 7.0\text{ mm}$

We know that, for welding arc,

$$E = a + bL \qquad \text{...(i)}$$

And For power source,

$$E = \text{OCV} - \left(\frac{\text{OCV}}{\text{SCC}}\right)I = 80 - \left(\frac{80}{800}\right)I \qquad \text{...(ii)}$$

Where : $I =$ Arc current, $E =$ Arc voltage

For stable arc,

$$\text{Welding arc} = \text{Power source}$$

$$80 - \left(\frac{80}{800}\right)I = a + bL \qquad \text{...(iii)}$$

Find the value of a & b, from the case (I) & (II)

For case (I), $I = 500\text{ A},\ L = 5\text{ mm}$

So, $80 - \left(\frac{80}{800}\right) \times 500 = a + 5b$ From equation (iii)

$$80 - 50 = a + 5b$$

$$a + 5b = 30 \qquad \text{...(iv)}$$

For case II, $I = 460\text{ A},\ L = 7\text{ mm}$

So, $80 - \frac{80}{800} \times 460 = a + 7b$ From equation(iii)

$$80 - 46 = a + 7b$$

$$a + 7b = 34 \qquad \text{...(v)}$$

Subtracting equation (iv) from equation (v),

$$(a + 7b) - (a + 5b) = 34 - 30$$

$$2b = 4 \qquad \Rightarrow b = 2$$

From equation (iv), put $b = 2$

$$a + 5 \times 2 = 30 \qquad \Rightarrow a = 20$$

Substituting the value of a & b in equation (i), we get

$$E = 20 + 2L$$

SOL 10.65 Option (C) is correct.

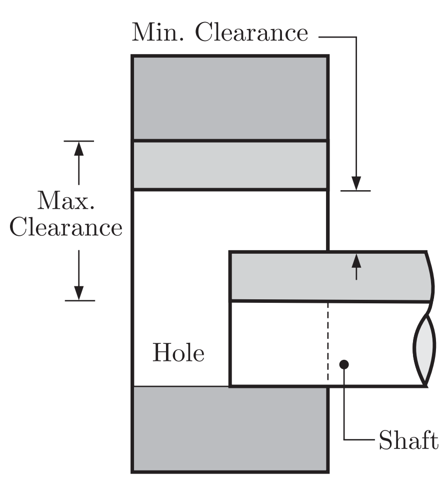
Given : Hole, $40^{+0.050}_{+0.000}$ mm

$$\text{Minimum hole size} = 40\text{ mm}$$

$$\text{Minimum clearance} = 0.01\text{ mm}$$

$$\text{Maximum size of hole} = 40 + 0.050 = 40.050\text{ mm}$$

$$\text{Tolerance of shaft} = 0.04\text{ mm}$$



Given that the mating shaft has a clearance fit with minimum clearance of 0.01 mm.

So, Maximum size of shaft $=$ Minimum hole size $-$ Minimum clearance

$$= 40 - 0.01 = 39.99 \text{ mm}$$

And Minimum size of shaft $=$ Maximum shaft size $-$ Tolerance of shaft

$$= 39.99 - 0.04 = 39.95$$

Maximum clearance,

$$c = \text{Maximum size of hole} - \text{Minimum size of shaft}$$

$$= 40.050 - 39.95 = 0.1 \text{ mm}$$

SOL 10.66 Option (C) is correct

Given : $\lambda = 90^\circ$, $F_c = 1000$ N, $F_t = 800$ N, $\phi = 25^\circ$, $\alpha = 0^\circ$

We know that, from the merchant's theory,

$$\frac{\text{Friction force}(F)}{\text{Normal force}(N)} = \mu = \frac{F_c \tan\alpha + F_t}{F_c - F_t \tan\alpha}$$

Substitute the values, we get

$$\frac{F}{N} = \frac{1000 \tan 0^\circ + 800}{1000 - 800 \tan 0^\circ} = \frac{800}{1000} = 0.80$$

SOL 10.67 Option (C) is correct.

Given : $w = 2$ mm, $I = 10$ kA $= 10^4$ A, $t = 10$ milli second $= 10^{-2}$ sec.

$T_a = 293$ K, $T_m = 1793$ K, $\rho = 7000$ kg/m^3, $L_f = 300$ kJ/kg

$c = 800$ J/kg K, $R = 500$ micro - ohm $= 500 \times 10^{-6}$ ohm

Radius of sphere, $r = 2 \text{ mm} = 2 \times 10^{-3}$ m

Heat supplied at the contacting area of the element to be welded is

$$Q_s = I^2 Rt = (10^4)^2 \times 500 \times 10^{-6} \times 10^{-2} = 500 \text{ J}$$

As fusion zone is spherical in shape.

Mass, $m = \rho \times v = 7000 \times \frac{4}{3} \times 3.14 \times (2 \times 10^{-3})^3$

$$= 2.344 \times 10^{-4} \text{ kg}$$

Total heat for melting (heat input)

$$Q_i = mL_f + mc(T_m - T_a)$$

Where $mL_f =$ Latent heat

Substitute the values, we get

$$Q_i = 2.344 \times 10^{-4}[300 \times 10^3 + 800(1793 - 293)]$$

$$= 2.344 \times 10^{-4}[300 \times 10^3 + 800 \times 1500] = 351.6 \text{ J}$$

$$\text{Efficiency } \eta = \frac{\text{Heat input}(Q_i)}{\text{Heat supplied}(Q_s)} \times 100$$

$$\eta = \frac{351.6}{500} \times 100 = 70.32\% \simeq 70.37\%$$

SOL 10.68 Option (C) is correct.

Given : $d_i = 200\text{ mm}$, $h_i = l_i = 60\text{ mm}$, $d_f = 400\text{ mm}$

Volume of disc remains unchanged during the whole compression process.

So, Initial volume = Final volume.

$$\frac{\pi}{4} d_i^2 \times l_i = \frac{\pi}{4} d_f^2 \times l_f$$

$$\frac{l_f}{l_i} = \frac{d_i^2}{d_f^2}$$

$$l_f = 60 \times \left(\frac{200}{400}\right)^2 = 60 \times \frac{1}{4} = 15\text{ mm}$$

Strain, $\varepsilon = \frac{\Delta l}{l} = \frac{l_i - l_f}{l_f} = \frac{60 - 15}{15} = 3$

True strain, $\varepsilon_0 = \ln(1 + \varepsilon) = \ln(1 + 3) = 1.386$

SOL 10.69 Option (D) is correct.

Let, Bite angle $= \theta$

$D = 400\text{ mm}$, $t_i = 16\text{ mm}$, $t_f = 10\text{ mm}$

Bite angle, $\tan\theta = \sqrt{\frac{t_i - t_f}{R}} = \sqrt{\frac{16 - 10}{200}} = \sqrt{0.03}$

$$\theta = \tan^{-1}(0.173) = 9.815^\circ \simeq 9.936^\circ$$

SOL 10.70 Option (D) is correct.

	Processes		Associated state of stress
P.	Blanking	3.	Shear
Q.	Stretch Forming	1.	Tension
R.	Coining	2.	Compression
S.	Deep Drawing	4.	Tension and Compression

So, correct pairs are, P-3, Q-1, R-2, S-4

SOL 10.71 Option (A) is correct.

Blanking force F_b is directly proportional to the thickness of the sheet 't' and diameter of the blanked part 'd'.

$$F_b \propto d \times t \qquad F_b = \tau \times d \times t \quad ...(i)$$

For case (I) : $F_{b1} = 5.0\text{ kN}$, $d_1 = d$, $t_1 = t$

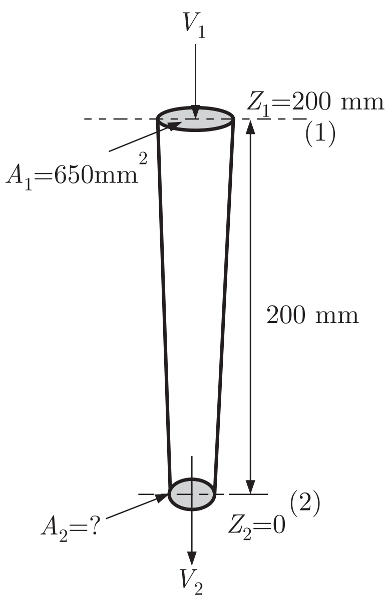
For case (II) : $d_2 = 1.5d$, $t_2 = 0.4t$, $F_{b2} = ?$

From equation (i) $\frac{F_{b2}}{F_{b1}} = \frac{d_2 t_2}{d_1 t_1}$

$$F_{b2} = 5 \times \frac{1.5d \times 0.4t}{d \times t} = 3\,\text{kN}$$

SOL 10.72 Option (C) is correct.

Let molten metal enters at section 1st and leaves the object at section 2nd



Given : $A_1 = 650\text{ mm}^2$, $Q = 6.5 \times 10^5\text{ mm}^3/\text{sec}$, $g = 10^4\text{ mm}/\text{sec}^2$

Now, for section 1st, flow rate

$$Q = A_1 V_1$$

$$V_1 = \frac{Q}{A_1} = \frac{6.5 \times 10^5}{650} = 1000\text{ mm}/\text{sec}$$

Applying Bernoulli's equation at section 1st and 2nd.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

But $p_1 = p_2 = \text{atmosphere pressure}$

So, $$\frac{V_1^2}{2g} + Z_1 = \frac{V_2^2}{2g} + Z_2$$

$$\frac{(1000)^2}{2 \times 10^4} + 200 = \frac{V_2^2}{2 \times 10^4} + 0$$

$$(50 + 200) \times 2 \times 10^4 = V_2^2$$

$$V_2^2 = 500 \times 10^4 = 5 \times 10^6$$

$$V_2 = 2.236 \times 10^3\text{ mm}/\text{sec} = 2236\text{ mm}/\text{sec}$$

We know that, flow rate remains constant during the process (from continuity equation). So, for section 2nd

$$Q = A_2 V_2$$

$$A_2 = \frac{Q}{V_2} = \frac{6.5 \times 10^5}{2236} = 290.7 \text{ mm}^2$$

SOL 10.73 Option (A) is correct.

	Parts		Manufacturing Process
P.	Computer chip	4.	Photochemical Machining
Q.	Metal forming dies and molds	3.	Electrodischarge Machining
R.	Turbine blade	1.	Electrochemical Machining
S.	Glass	2.	Ultrasonic Machining

So, correct pairs are, P-4, Q-3, R-1, S-2

SOL 10.74 Option (B) is correct.

Given : $T_1 = 24$ min, $T_2 = 12$ min, $V_1 = 90$ m/min, $V_2 = 120$ m/min

We have calculate velocity, when tool life is 20 minute.

First of all we the calculate the values of n, From the Taylor's tool life equation.

$$VT^n = C$$

For case 1st and 2nd, we can write

$$V_1 T_1^n = V_2 T_2^n$$

$$\left(\frac{T_1}{T_2}\right)^n = \frac{V_2}{V_1}$$

$$\left(\frac{24}{12}\right)^n = \frac{120}{90}$$

$$(2)^n = 1.33$$

Taking log both the sides,

$$n \log 2 = \log 1.33$$

$$n \times 0.301 = 0.124$$

$$n = 0.412$$

For V_3, we can write from tool life equation,

$$V_1 T_1^n = V_3 T_3^n$$

$$90 \times (24)^{0.412} = V_3 (20)^{0.412}$$

$$333.34 = V_3 \times 3.435$$

$$V_3 = 97 \text{ m/min}$$

SOL 10.75 Option (C) is correct.

Given : $D = 147$ mm, $l = 630$ mm, $f = 0.2$ mm/rev.

$d = 2$ mm, $V_3 = 97$ m/min

$$\text{Machining time } t = \frac{l}{fN}$$

$$V = \pi DN \text{ m/ min}$$

So, $$t = \frac{l \times \pi \times D}{fV}$$

$$t = \frac{0.63 \times 3.14 \times 0.147}{0.2 \times 10^{-3} \times 97} \qquad V = V_3$$

$$t = 15 \text{ min}$$

SOL 10.76 Option (D) is correct.

Investment casting uses an expandable pattern, which is made of wax or of a plastic by molding or rapid prototyping techniques. This pattern is made by injecting molten wax or plastic into a metal die in the shape of the pattern.

SOL 10.77 Option (D) is correct.

Spheroidizing may be defined as any heat treatment process that produces a rounded or globular form of carbide. High carbon steels are spheroidized to improve machinability, especially in continuous cutting operations.

SOL 10.78 Option (A) is correct.

NC contouring is a continuous path positioning system. Its function is to synchronize the axes of motion to generate a predetermined path, generally a line or a circular arc.

SOL 10.79 Option (A) is correct.

Ring gauges are used for gauging the shaft and male components i.e. measure the outside diameter. It does not able to measure the roundness of the given shaft.

SOL 10.80 Option (B) is correct.

Given : $\sigma_u = 400 \text{ MPa}$, $\frac{\Delta L}{L} = 35\% = 0.35 = \varepsilon_0$

Let, true stress is σ and true strain is ε.

True strain, $$\varepsilon = \ln(1 + \varepsilon_0) = \ln(1 + 0.35) = 0.30$$

True stress, $$\sigma = \sigma_u(1 + \varepsilon_0) = 400(1 + 0.35) = 540 \text{ MPa}$$

We know, at Ultimate tensile strength,

$$n = \varepsilon = 0.3$$

Relation between true stress and true strain is given by,

$$\sigma = K\varepsilon^n \qquad ...(i)$$

$$K = \frac{\sigma}{\varepsilon^n} = \frac{540}{(0.30)^{0.30}} = 774.92 \simeq 775$$

So, From equation (i) $\sigma = 775\varepsilon^{0.3}$

SOL 10.81 Option (B) is correct.

We know that,Time taken to fill the mould with top gate is given by,

$$t_A = \frac{A_m H_m}{A_g \sqrt{2gH_g}}$$

Where A_m = Area of mould

H_m = Height of mould

A_g = Area of gate

H_g = Height of gate

Given that, total liquid head is maintained constant and it is equal to the mould height.

So, $H_m = H_g$

$$t_A = \frac{A_m \sqrt{H_m}}{A_g \sqrt{2g}} \qquad ...(i)$$

Time taken to fill with the bottom gate is given by,

$$t_B = \frac{2A_m}{A_g \sqrt{2g}} \times (\sqrt{H_g} - \sqrt{H_g - H_m})$$

$$t_B = \frac{2A_m}{A_g \sqrt{2g}} \times \sqrt{H_m} \qquad H_m = H_g \; ...(ii)$$

By Dividing equation (ii) by equation (i),

$$\frac{t_B}{t_A} = 2$$

$$t_B = 2t_A$$

SOL 10.82 Option (C) is correct.

Given : $t_i = 4\,\text{mm}$, $D = 300\,\text{mm}$, $\mu = 0.1$, $t_f = ?$

We know that,

For single pass without slipping, minimum possible thickness is given by the relation.

$$(t_i - t_f) = \mu^2 R$$

$$t_f = t_i - \mu^2 R = 4 - (0.1)^2 \times 150 = 2.5\,\text{mm}$$

SOL 10.83 Option (B) is correct.

Given, $d_i = 10\,\text{mm}$, $d_f = 8\,\text{mm}$, $\sigma_0 = 400\,\text{MPa}$

The expression for the drawing force under frictionless condition is given by

$$F = \sigma_{mean} A_f \ln\left(\frac{A_i}{A_f}\right) = 400 \times 10^6 \times \frac{\pi}{4} \times (0.008)^2 \ln\left[\frac{\frac{\pi}{4}(0.001)^2}{\frac{\pi}{4}(0.008)^2}\right]$$

$$= 20096 \times \ln(1.5625) = 8.968 \text{ kN} \simeq 8.97 \text{ kN}$$

SOL 10.84 Option (D) is correct.

	Column I		Column II
P.	Wrinkling	4.	Insufficient blank holding force
Q.	Orange peel	3.	Large grain size
R.	Stretcher strains	1.	Yield point elongation
S.	Earing	2.	Anisotropy

So correct pairs are, P-4, Q-3, R-1, S-2

SOL 10.85 Option (B) is correct.

Given, $V = 25$ Volt, $I = 300$ A, $\eta = 0.85$, $V = 8$ mm/sec

We know that the power input by the heat source is given by,

$$\text{Voltage} = 25 \text{ Volt}$$

$$P = \text{Voltage} \times I$$

Heat input into the work piece $= P \times$ efficiency of heat/transfer

$$H_i = \text{Voltage} \times I \times \eta = 25 \times 300 \times 0.85 = 6375 \text{ J/sec}$$

$$\text{Heat energy input (J/mm)} = \frac{H_i}{V}$$

$$H_i(\text{J/mm}) = \frac{6375}{8} = 796.9 \simeq 797 \text{ J/mm}$$

(D) Zero rake angle, high shear angle and high cutting speed

SOL 10.86 Option (A) is correct.

In common grinding operation, the average rake angle of the grains is highly negative, such as -60° or even lower and smaller the shear angle. From this, grinding chips under go much larger deformation than they do in other cutting process. The cutting speeds are very high, typically 30 m/s

SOL 10.87 Option (D) is correct.

	Process	Metal Removal Rate(MRR) (in mm^3/sec)
1.	ECM	2700
2.	USM	14
3.	EBM	0.15
4.	LBM	0.10
5.	EDM	14.10

So the processes which has maximum MRR in increasing order is,

LBM, EBM, USM, EDM, ECM

SOL 10.88 Option (D) is correct.

	Column I		Column II
P.	Charpy test	4.	Toughness
Q.	Knoop test	2.	Microhardness
R.	Spiral test	1.	Fluidity
S.	Cupping test	3.	Formability

So, correct pairs are, P-4, Q-2, R-1, S-3

SOL 10.89 Option (D) is correct.

Given : $t = 0.5\text{ mm}$, $V = 20\text{ m/min}$, $\alpha = 15^\circ$, $w = 5\text{ mm}$, $t_c = 0.7\text{ mm}$, $F_t = 200\text{ N}$, $F_c = 1200\text{ N}$

We know, from the merchant's theory

Chip thickness ratio, $r = \dfrac{t}{t_c} = \dfrac{0.5}{0.7} = 0.714$

For shear angle, $\tan\phi = \dfrac{r\cos\alpha}{1 - r\sin\alpha}$

Substitute the values, we get

$$\tan\phi = \frac{0.714\cos 15^\circ}{1 - 0.714\sin 15^\circ} = \frac{0.689}{0.815} = 0.845$$

$$\phi = \tan^{-1}(0.845) = 40.2^\circ$$

Shear strain, $s = \cot\phi + \tan(\phi - \alpha)$

$$s = \cot(40.2^\circ) + \tan(40.2^\circ - 15^\circ)$$

$$= \cot 40.2^\circ + \tan 25.2 = 1.183 + 0.470 = 1.65$$

SOL 10.90 Option (B) is correct.

From merchants, theory

$$\mu = \frac{F}{N} = \frac{F_c\sin\alpha + F_t\cos\alpha}{F_c\cos\alpha - F_t\sin\alpha} = \frac{F_c\tan\alpha + F_t}{F_c - F_t\tan\alpha}$$

$$= \frac{1200\tan 15^\circ + 200}{1200 - 200 \times \tan 15^\circ} = \frac{521.539}{1146.41} = 0.455 \simeq 0.46$$

SOL 10.91 Option (A) is correct.

We know, from merchant's theory, frictional force of the tool acting on the tool-chip interface is

$$F = F_c\sin\alpha + F_t\cos\alpha$$

$$= 1200\sin 15^\circ + 200\cos 15^\circ = 503.77\text{ N}$$

Chip velocity, $V_c = \dfrac{\sin\phi}{\cos(\phi - \alpha)} \times V$

$$= \frac{\sin(40.2^\circ)}{\cos(40.2^\circ - 15^\circ)} \times 20 = 14.27 \text{ m/min}$$

Total energy required per unit time during metal cutting is given by,

$$E = F_c \times V = 1200 \times \frac{20}{60} = 400 \text{ Nm/sec}$$

Energy consumption due to friction force F,

$$E_f = F \times V_c = 503.77 \times \frac{14.27}{60} \text{ Nm/sec}$$

$$= 119.81 \text{ Nm/sec}$$

Percentage of total energy dissipated due to friction at tool-chip interface is

$$E_d = \frac{E_f}{E} \times 100 = \frac{119.81}{400} \times 100 \simeq 30\%$$

SOL 10.92 Option (D) is correct.

	List-I (Equipment)		List-II (Process)
P.	Hot Chamber Machine	3.	Die casting
Q.	Muller	5.	Sand mixing
R.	Dielectric Baker	2.	Core making
S.	Sand Blaster	1.	Cleaning

So, correct pairs are, P-3, Q-5, R-2, S-1

SOL 10.93 Option (A) is correct.

When the temperature of a solid metal increases, its intramolecular bonds are brake and strength of solid metal decreases. Due to decrease its strength, the elongation of the metal increases, when we apply the load i.e. ductility increases.

SOL 10.94 Option (D) is correct.

We know that,

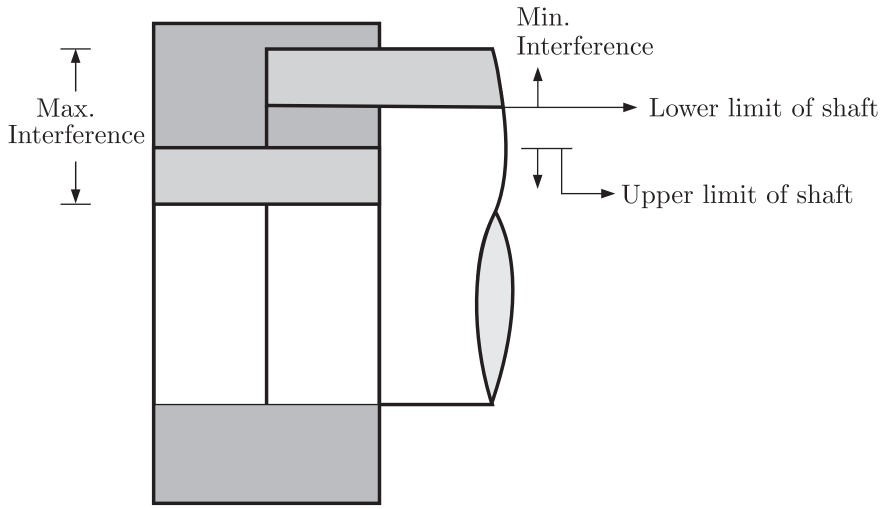
The strength of the brazed joint depend on (a) joint design and (b) the adhesion at the interfaces between the workpiece and the filler metal.

The strength of the brazed joint increases up to certain gap between the two joining surfaces beyond which it decreases.

SOL 10.95 Option (B) is correct.

In ECM, the principal of electrolysis is used to remove metal from the workpiece. The ECM method has also been developed for machining new hard and tough materials (for rocket and aircraft industry) and also hard refractory materials.

SOL 10.96 Option (A) is correct.



The interference is the amount by which the actual size of a shaft is larger than the actual finished size of the mating hole in an assembly.
For interference fit, lower limit of shaft should be greater than the upper limit of the hole (from figure).

SOL 10.97 Option (C) is correct.
According to 3-2-1 principle, only the minimum locating points should be used to secure location of the work piece in any one plane.

(A) The workpiece is resting on three pins A, B, C which are inserted in the base of fixed body.

The workpiece cannot rotate about the axis XX and YY and also it cannot move downward. In this case, the five degrees of freedom have been arrested.

(B) Two more pins D and E are inserted in the fixed body, in a plane perpendicular to the plane containing, the pins A, B and C. Now the workpiece cannot rotate about the Z-axis and also it cannot move towards the left. Hence the addition of pins D and E restrict three more degrees of freedom.

(C) Another pin F in the second vertical face of the fixed body, arrests degree of freedom 9.

SOL 10.98 Option (A) is correct.
Arc welding, Laser cutting of sheet and milling operations are the continuous path operations.

SOL 10.99 Option (A) is correct.

We know,

Machining cost = Machining time $\times$ Direct labour cost.

If cutting speed increases then machining time decreases and machining cost also decreases and due to increase in cutting speed tool changing cost increases.

So, Curve 1 $\rightarrow$ Machining cost

Curve 2 $\rightarrow$ Non-productive cost

Curve 3 $\rightarrow$ Tool changing cost

SOL 10.100 Option (B) is correct.

Given : $l = 20\,\text{cm} = 0.2\,\text{m},\ A = 1\,\text{cm}^2 = 10^{-4}\,\text{m}^2$

$V = 1000\,\text{cm}^3 = 1000 \times 10^{-6}\,\text{m}^3 = 10^{-3}\,\text{m}^3$

Velocity at the base of sprue is,

$$V = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.2} = 1.98\,\text{m/sec}$$

From the continuity equation flow rate to fill the mould cavity is,

Filling rate $\dot{Q} = \text{Area} \times \text{Velocity} = AV$

$$\frac{v}{t} = AV \qquad v = \text{Volume}$$

$$t = \frac{v}{AV} = \frac{10^{-3}}{10^{-4} \times 1.98} = \frac{10}{1.98} = 5.05\,\text{sec.}$$

SOL 10.101 Option (C) is correct.

Given :

$\rho = 8000\,\text{kg/m}^3,\ t = 0.1\,\text{sec.},\ d = 5\,\text{mm},\ w = 1.5\,\text{mm},\ L_f = 1400\,\text{kJ/kg},$
$R = 200\,\mu\Omega$

First of all calculate the mass,

$$\rho = \frac{m}{V}$$

$$m = \rho \times V = \rho \times \frac{\pi}{4} d^2 \times t$$

$$= 8000 \times \frac{\pi}{4} \times (5 \times 10^{-3})^2 \times 1.5 \times 10^{-3}$$

$$= 235.5 \times 10^{-6}\,\text{kg} = 2.35 \times 10^{-4}\,\text{kg}$$

Total heat for fusion,

$$Q = mL_f \qquad L = \text{Latent heat}$$

$$= 2.35 \times 10^{-4} \times 1400 \times 10^3 = 329\,\text{J} \qquad \text{...(i)}$$

We also know that, the amount of heat generated at the contacting area of the element to be welded is,

$$Q = I^2 Rt$$

$$329 = I^2 \times 200 \times 10^{-6} \times 0.1 \qquad \text{From equation (i)}$$

$$I^2 = \frac{329}{200 \times 10^{-7}} = 16.45 \times 10^6$$

$$I = \sqrt{16.45 \times 10^6} = 4056 \text{ A} \simeq 4060 \text{ A}$$

SOL 10.102 Option (C) is correct.

Given : $\alpha = 1\,\text{radian} \times \frac{180}{\pi} = \left(\frac{180}{\pi}\right)^\circ$, $r = 100\,\text{mm}$, $k = 0.5$, $t = 2\,\text{mm}$

Here, $r > 2t$

So, $k = 0.5t$

Bend allowance $B = \frac{\alpha}{360} \times 2\pi(r + k)$

$$= \frac{180}{\pi} \times \frac{2\pi}{360}(100 + 0.5 \times 2) = 101\,\text{mm}$$

SOL 10.103 Option (B) is correct.

Given : Side of the plate$= 600\,\text{mm}$, $V = 8\,\text{m/min}$, $f = 0.3\,\text{mm/stroke}$

$$\frac{\text{Return time}}{\text{Cutting time}} = \frac{1}{2}$$

The tool over travel at each end of the plate is $20\,\text{mm}$. So length travelled by the tool in forward stroke,

$$L = 600 + 20 + 20 = 640\text{ mm}$$

$$\text{Number of stroke required} = \frac{\text{Thickness of flat plate}}{\text{Feed rate/stroke}}$$

$$= \frac{30}{0.3} = 100\,\text{strokes}$$

Distance travelled in 100 strokes is,

$$d = 640 \times 100$$

$$= 64000\text{ mm} = 64\text{ m}$$

So, Time required for forward stroke

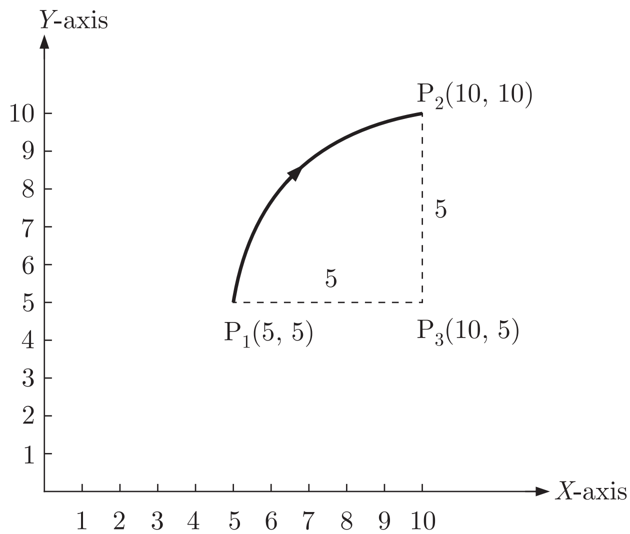
$$t = \frac{d}{V} = \frac{64}{8} = 8\text{ min}$$

$$\text{Return time} = \frac{1}{2} \times 8 = 4\text{ min}$$

Machining time, $T_M = \text{Cutting time} + \text{Return time}$

$$= 8 + 4 = 12\text{ min}$$

SOL 10.104 Option (A) is correct.



So, N010 →represent start the operation

GO2 →represent circular (clock wise) interpolation

X10Y10 →represent final coordinates

X5Y5 →represent starting coordinate

R5 →represent radius of the arc

So, NC tool path command is, N010 GO2 X10 Y10 X5 Y5 R5

SOL 10.105 Option (B) is correct.

Tool designation or tool signature under ASA, system is given in the order. Back rake, Side rake, End relief, Side relief, End cutting edge angle, Side cutting edge angle and nose radius that is

$$\alpha_b\text{-}\alpha_s\text{-}\theta_e\text{-}\theta_s\text{-}C_e\text{-}C_s\text{-}R$$

Given : For tool P, tool signature, 5°-5°-6°-6°-8°-30°-0

For tool Q : 5°-5°-7°-7°-8°-15°-0

We know that,

$$h = \frac{\text{feed}}{\tan(\text{SCEA}) + \cot(\text{ECEA})} = \frac{f}{\tan(C_s) + \cot(C_e)}$$

For tool P, $$h_P = \frac{f_P}{\tan 30^\circ + \cot 8^\circ}$$

For tool Q $$h_Q = \frac{f_Q}{\tan 15^\circ + \cot 8^\circ}$$

for same machining condition $f_P = f_Q$

Hence, $$\frac{h_P}{h_Q} = \frac{\tan 15^\circ + \cot 8^\circ}{\tan 30^\circ + \cot 8^\circ}$$

SOL 10.106 Option (C) is correct.

We know that maximum possible clearance occurs between minimum shaft size and maximum hole size.

Maximum size of shaft $= 25 + 0.040 = 25.040$ mm

Minimum size of shaft $= 25 - 0.100 = 24.99$ mm

Maximum size of hole $= 25 + 0.020 = 25.020$ mm

Minimum size of hole $= 25 - 0.000 = 25.00$ mm

$$25.020 - 24.99 = 0.03 \text{ mm} = 30 \text{ microns}$$

SOL 10.107 Option (A) is correct.

Given:-$NO20$ $GO2$ $X45.0$ $Y25.0$ $R5.0$

Here term $X45.0$ $Y25.0$ $R5.0$ will produce circular motion because radius is consider in this term and $GO2$ will produce clockwise motion of the tool.

SOL 10.108 Option (A) is correct

In EDM, the thermal energy is employed to melt and vaporize tiny particles of work material by concentrating the heat energy on a small area of the work-piece.

SOL 10.109 Option (D) is correct

Given : $I = 5000$ A, $R = 200\,\mu\Omega = 200 \times 10^{-6}\,\Omega$, $\Delta t = 0.2$ second

Heat generated, $H_g = I^2(R \triangle t)$

$$H_g = (5000)^2 \times 200 \times 10^{-6} \times 0.2$$

$$= 25 \times 10^6 \times 40 \times 10^{-6} = 1000 \text{ Joule}$$

SOL 10.110 Option (B) is correct

Two streams of liquid metal which are not hot enough to fuse properly result into a casting defect, known as Misrun/cold shut.

It occurs due to insufficient fluidity of the molten metal.

SOL 10.111 Option (C) is correct.

Gray cast iron is the most widely used of all cast irons. In fact, it is common to speak of gray cast iron just as cast iron.

It contains 3 to 4% C and 2.5 % Si.

SOL 10.112 Option (B) is correct.

For hole size $= 20.000^{+0.050}_{+0.010}$ mm

Maximum hole size $= 20.000 + 0.050 = 20.050$ mm

Minimum hole size $= 20.000 + 0.010 = 20.010$

So, Hole tolerance = Maximum hole size − Minimum hole size

$$= 20.050 - 20.010 = 0.040 \text{ mm}$$

Gauge tolerance can be 10% of the hole tolerance (Given).

So, Gauge tolerance $= 10\%$ of 0.040

$$= \frac{10}{100} \times 0.040 = 0.0040 \text{ mm}$$

$$\text{Size of Go Gauge} = \text{Minimum hole size} + \text{Gauge tolerance}$$

$$= 20.010 + 0.0040 = 20.014 \text{ mm}$$

$$\text{Size of NO-GO Gauge} = \text{Maximum hole size} - \text{Gauge tolerance}$$

$$= 20.050 - 0.004 = 20.046 \text{ mm}$$

SOL 10.113 Option (A) is correct.

Given : $d = 10\text{ mm},\ t = 3\text{ mm},\ \tau_s = 400\text{ N/mm}^2,\ t_1 = 2\text{ mm},\ p = 40\% = 0.4$

We know that, when shear is applied on the punch, the blanking force is given by,

$$F_B = \pi dt\left(\frac{t \times p}{t_1}\right) \times \tau_s \quad \text{Where } t \times p = \text{Punch travel}$$

Substitute the values, we get

$$F_B = 3.14 \times 10 \times 3\left(\frac{3 \times 0.4}{2}\right) \times 400$$

$$= 94.2 \times 0.6 \times 400 = 22.6 \text{ kN}$$

SOL 10.114 Option (B) is correct

Given : $D = 10\text{ mm},\ t = 20\text{ mm},\ N = 300\text{ rpm},\ f = 0.2\text{ mm/rev.}$

Point angle of drill, $2\alpha_p = 120^\circ \quad \Rightarrow \alpha_p = 60^\circ$

Drill over-travel $= 2$ mm

We know that, break through distance,

$$A = \frac{D}{2\tan\alpha_p} = \frac{10}{2\tan 60^\circ} = 2.89 \text{ mm}$$

Total length travelled by the tool,

$$L = t + A + 2$$

$$= 20 + 2.89 + 2 = 24.89 \text{ mm}$$

So, time for drilling, $$t = \frac{L}{f \cdot N} = \frac{24.89}{0.2 \times 300} = 0.415 \text{ min}$$

$$= 0.415 \times 60 \text{ sec} = 24.9 \simeq 25 \text{ sec}$$

SOL 10.115 Option (D) is correct.

Given : Dimension of block $= 200 \times 100 \times 10$ mm

Shrinkage allowance, $X = 1\%$

We know that, since metal shrinks on solidification and contracts further on cooling to room temperature, linear dimensions of patterns are increased in respect of those of the finished casting to be obtained.

So, $v_c = 200 \times 100 \times 10 = 2 \times 10^5 \text{ mm}^2$

Shrinkage allowance along length,

$$S_L = LX = 200 \times 0.01 = 2 \text{ mm}$$

Shrinkage allowance along breadth,

$$S_B = 100 \times 0.01 = 1 \text{ mm}$$

or Shrinkage allowance along height,

$$S_H = 10 \times 0.01 = 0.1 \text{ mm}$$

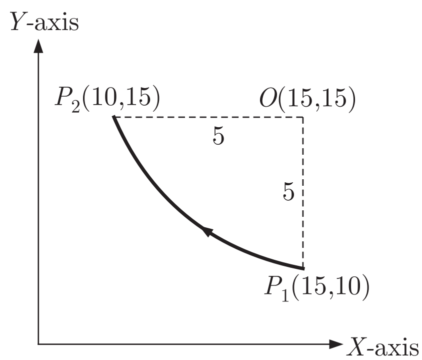
Volume of pattern will be

$$v_p = [(L + S_L)(B + S_B)(S + S_H)] \text{ mm}^3$$

$$= 202 \times 101 \times 10.01 \text{ mm}^3 = 2.06 \times 10^5 \text{ mm}^3$$

So, $$\frac{\text{Volume of Pattern } v_p}{\text{Volume of Casting } v_c} = \frac{2.06 \times 10^5}{2 \times 10^5} = 1.03$$

SOL 10.116 Option (C) is correct



From the figure, the centre of circular arc with radius 5 is

$$[15, (10 + 5)] = [15, 15] \qquad \text{From point } P_1$$

$$[(10 + 5), 15] = [15, 15] \qquad \text{From point } P_2$$

SOL 10.117 Option (B) is correct.

Given : $V = 40 \text{ m/min}$, $d = 0.3 \text{ mm}$, $\alpha = 5^\circ$, $t = 1.5 \text{ mm}$, $F_c = 900 \text{ N}$, $F_t = 450 \text{ N}$

We know from the merchant's analysis

$$\mu = \frac{F}{N} = \frac{F_c \sin\alpha + F_t \cos\alpha}{F_c \cos\alpha - F_t \sin\alpha}$$

Where F = Frictional resistance of the tool acting on the chip.

N = Force at the tool chip interface acting normal to the cutting face of the tool.

$$\mu = \frac{900\tan 5^\circ + 450}{900 - 450\tan 5^\circ}$$

$$= \frac{528.74}{860.63} = 0.614$$

Now, Frictional angle, $\beta = \tan^{-1}\mu = \tan^{-1}(0.614) = 31.5^\circ$

SOL 10.118 Option (B) is correct.

Given : $t_i = 25\,\text{mm},\ t_f = 20\,\text{mm},\ D = 600\,\text{mm},\ N = 100\,\text{rpm}$

Let, Angle substended by the deformation zone at the roll centre is θ in radian and it is given by the relation.

$$\theta(\text{radian}) = \sqrt{\frac{t_i - t_f}{R}}$$

$$= \sqrt{\frac{25-20}{300}} = \sqrt{0.0166} = 0.129\,\text{radian}$$

Roll strip contact length is

$$L = \theta \times R \qquad\qquad \text{Angle} = \frac{\text{Arc}}{R}$$

$$L = 0.129 \times 300 = 38.73\,\text{mm} \simeq 39\,\text{mm}$$

SOL 10.119 Option (C) is correct.

Given : $VT^n = C$

Let V and T are the initial cutting speed & tool life respectively.

Case (I) : The relation between cutting speed and tool life is,

$$VT^n = C \qquad ...(i)$$

Case (II) : In this case doubling the cutting speed and tool life reduces to $1/8^{\text{th}}$ of original values.

So, $$(2V) \times \left(\frac{T}{8}\right)^n = C \qquad ...(ii)$$

On dividing equation (i) by equation (ii),

$$\frac{VT^n}{2V\left(\frac{T}{8}\right)^n} = 1$$

$$T^n = 2\left(\frac{T}{8}\right)^n$$

$$\frac{1}{2} = \left(\frac{1}{8}\right)^n$$

$$\left(\frac{1}{2}\right)^1 = \left(\frac{1}{2}\right)^{3n}$$

Compare powers both the sides,

$$1 = 3n \qquad \Rightarrow n = \frac{1}{3}$$

SOL 10.120 Option (B) is correct.

	Feature to be inspected		Instrument
P.	Pitch and Angle errors of screw thread	5.	Sine bar
Q.	Flatness error of a surface	2.	Optical Interferometer
R.	Alignment error of a machine slideway	1.	Auto collimator
S.	Profile of a cam	6.	Tool maker's Microscope

So, correct pairs are, P-5, Q-2, R-1, S-6

SOL 10.121 Option (B) is correct.

	Product		Process
P.	Molded luggage	4.	Transfer molding
Q.	Packaging containers for Liquid	5.	Blow molding
R.	Long structural shapes	2.	Hot rolling
S.	Collapsible tubes	3.	Impact extrusion

So, correct pairs are, P-4 Q-5 R-2 S-3

SOL 10.122 Option (D) is correct.

	Operation		Process
P.	Deburring (internal surface)	2.	Abrasive Flow Machining
Q.	Die sinking	3.	Electric Discharge Machining
R.	Fine hole drilling in thin sheets	5.	Laser beam Machining
S.	Tool sharpening	6.	Electrochemical Grinding

So, Correct pairs are, P-2, Q-3, R-5, S-6

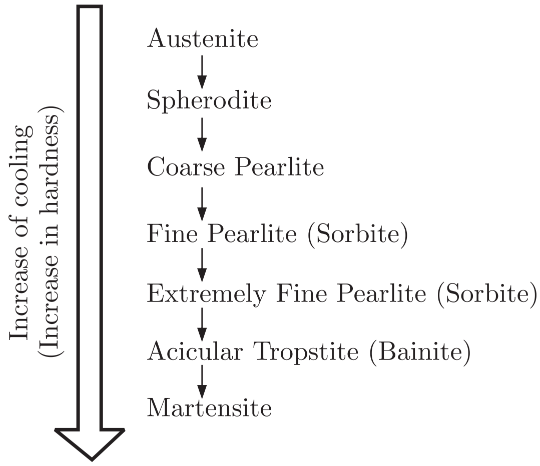
SOL 10.123 Option (C) is correct.

	Process		Characteristics
P.	Tempering	4.	Both hardness and brittleness are reduced
Q.	Austempering	1.	Austenite is converted into bainite
R.	Martempering	2.	Austenite is converted into martensite

So, correct pairs are, P-4, Q-1, R-2

SOL 10.124 Option (D) is correct.

Steel can be cooled from the high temperature region at a rate so high that the austenite does not have sufficient time to decompose into sorbite or troostite. In this case the austenite is transformed into martensite. Martensite is ferromagnetic, very hard & brittle.



So hardness is increasing in the order,

Spherodite $\rightarrow$ Coarse Pearlite $\rightarrow$ Fine Pearlite $\rightarrow$ Martensite

SOL 10.125 Option (C) is correct.

Permeability or porosity of the moulding sand is the measure of its ability to permit air to flow through it.

So, hardness of green sand mould increases by restricted the air permitted in the sand i.e. decrease its permeability.

SOL 10.126 Option (B) is correct.

In OAW, Acetylene (C_2H_2) produces higher temperature (in the range of 3200° C)than other gases, (which produce a flame temperature in the range of 2500° C) because it contains more available carbon and releases heat when its components (C & H) dissociate to combine with O_2 and burn.

SOL 10.127 Option (C) is correct.

Cold forming or cold working can be defined as the plastic deforming of metals and alloys under conditions of temperature and strain rate.

Theoretically, the working temperature for cold working is below the recrystallization temperature of the metal/alloy (which is about one-half the absolute melting temperature.)

SOL 10.128 Option (D) is correct.

Quality screw threads are produced by only thread casting.
Quality screw threads are made by die-casting and permanent mould casting are very accurate and of high finish, if properly made.

SOL 10.129 Option (D) is correct.
In EDM, the thermal energy is employed to melt and vaporize tiny particles of work-material by concentrating the heat energy on a small area of the work-piece.
A powerful spark, such as at the terminals of an automobile battery, will cause pitting or erosion of the metal at both anode & cathode. No force occurs between tool & work.

SOL 10.130 Option (B) is correct.
Since 25 mm lies in the diameter step 18 & 30 mm, therefore the geometric mean diameter,

$$D = \sqrt{18 \times 30} = 23.24 \text{ mm}$$

We know that standard tolerance unit,

$$i\,(\text{microns}) = 0.45\sqrt[3]{D} + 0.001D$$

$$i = 0.45\sqrt[3]{23.24} + 0.001 \times 23.24 = 1.31 \text{ microns}$$

Standard tolerance for hole 'h' of grade $7\,(IT\,7)$,

$$IT\,7 = 16i = 16 \times 1.31 = 20.96 \text{ microns}$$

Hence, lower limit for shaft = Upper limit of shaft – Tolerance

$$= 25 - 20.96 \times 10^{-3} \text{ mm} = 24.979 \text{ mm}$$

SOL 10.131 Option (B) is correct.
Hardness is greatly depend on the carbon content present in the steel.
Cyaniding is case-hardening with powered potassium cyanide or potassium ferrocyanide mixed with potassium bichromate, substituted for carbon. Cyaniding produces a thin but very hard case in a very short time.

SOL 10.132 Option (B) is correct.
Given : $q = 0.97 \times 10^6 \text{ s/m}^2$, $D = 200 \text{ mm} = 0.2 \text{ m}$

From the caine's relation solidification time, $T = q\left(\frac{V}{A}\right)^2$

Volume $V = \frac{4}{3}\pi R^3$

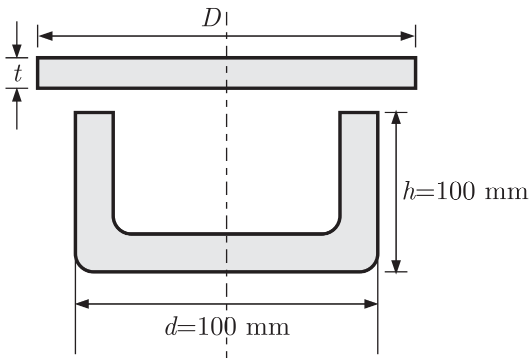
Surface Area $A = 4\pi R^2$

So, $$T = 0.97 \times 10^6 \left(\frac{\frac{4}{3}\pi R^3}{4\pi R^2}\right)^2 = 0.97 \times 10^6 \left(\frac{R}{3}\right)^2$$

$$= \frac{0.97}{9} \times 10^6 \left(\frac{0.2}{2}\right)^2 = 1078 \text{ sec}$$

SOL 10.133 Option (C) is correct.

Given : $d = 100$ mm, $h = 100$ mm, $R = 0.4$ mm



Here we see that $d > 20r$

If $d \geq 20r$, blank diameter in cup drawing is given by,

$$D = \sqrt{d^2 + 4dh}$$

Where, D = diameter of flat blank

d = diameter of finished shell

h = height of finished shell

Substitute the values, we get

$$D = \sqrt{(100)^2 + 4 \times 100 \times 100} = \sqrt{50000}$$

$$= 223.61 \text{ mm} \simeq 224 \text{ mm}$$

SOL 10.134 Option (B) is correct.

Given : $d_i = 100$ mm, $d_f = 50$ mm, $T = 700^\circ$ C, $k = 250$ MPa

Extrusion force is given by,

$$F_e = kA_i \ln\left(\frac{A_i}{A_f}\right) = k\frac{\pi}{4}d_i^2 \ln\left(\frac{\frac{\pi}{4}d_i^2}{\frac{\pi}{4}d_f^2}\right) = k\frac{\pi}{4}d_i^2 \ln\left(\frac{d_i}{d_f}\right)^2$$

Substitute the values, we get

$$F_e = 250 \times \frac{\pi}{4}(0.1)^2 \ln\left(\frac{0.1}{0.05}\right)^2$$

$$= 1.96 \ln 4 = 2.717 \text{ MN} \simeq 2.72 \text{ MN}$$

SOL 10.135 Option (A) is correct.

Given : $D = 20$ mm, $t = 2$ mm, Punch or diameter clearance = 3%

Required punch diameter will be,

$$d = D - 2 \times (3\% \text{ of thickness})$$

$$= 20 - 2 \times \frac{3}{100} \times 2 = 19.88 \text{ mm}$$

SOL 10.136 Option (A) is correct.

Given : For case (I) :

$N = 50$ rpm, $f = 0.25$ mm/rev., $d = 1$ mm

Number of cutting tools $= 10$

Number of components produce $= 500$

So, Velocity $V_1 = N \times f = 50 \times 0.25 = 12.5$ mm/min.

For case (II) :

$N = 80$ rpm, $f = 0.25$ mm/rev., $d = 1$ mm

Number of cutting tools, $= 10$

Number of components produce $= 122$

So, Velocity $V_2 = N \times f = 80 \times 0.25 = 20$ mm/min

From the tool life equation between cutting speed & tool life, $VT^n = C$,

$$V_1 T_1^n = V_2 T_2^n \qquad \text{where } C = \text{constant} \qquad \text{...(i)}$$

Tool life = Number of components produce × Tool constant

For case (I), $T_1 = 500k$ $k =$ tool constant

For case (II), $T_2 = 122k$

From equation (i),

$$12.5 \times (500k)^n = 20 \times (122k)^n$$

$$\left(\frac{500k}{122k}\right)^n = \frac{20}{12.5} = 1.6$$

Taking log both the sides,

$$n \ln\left(\frac{500}{122}\right) = \ln(1.6)$$

$$n(1.41) = 0.47$$

$$n = 0.333$$

Let the no. of components produced be n_1 by one cutting tool at 60 r.p.m.

So, tool life, $T_3 = n_1 k$

Velocity, $V_3 = 60 \times 0.25 = 15$ mm/min feed remains same

Now, tool life T_1 if only 1 component is used,

$$T_1' = \frac{500k}{10}$$

So, $V_1(T_1')^n = V_3(T_3)^n$

Substitute the values, we get

$$V_1\left(\frac{500k}{10}\right)^n = 15(n_1 k)^n$$

$$\left(\frac{50k}{n_1 k}\right)^n = \frac{15}{12.5}$$

$$\frac{50}{n_1} = (1.2)^{1/0.333} = 1.73$$

$$n_1 = \frac{50}{1.73} = 28.90 \simeq 29$$

SOL 10.137 Option (B) is correct.

Given : $p = 2\,\text{mm}$, $d = 14.701\,\text{mm}$

We know that, in case of ISO metric type threads,

$$2\theta = 60^\circ \qquad \Rightarrow \quad \theta = 30^\circ$$

And in case of threads, always rollers are used.

For best size of rollers, $d = \frac{p}{2}\sec\theta = \frac{2}{2}\sec 30^\circ = 1.155\,\text{mm}$

Hence, rollers of $1.155\,\text{mm}$ diameter (1.155ϕ) is used.

SOL 10.138 Option (D) is correct.

The total number of straight fringes that can be observed on both slip gauges is 13.

SOL 10.139 Option (A) is correct.

Given : $P = 35.00 \pm 0.08\,\text{mm}$, $Q = 12.00 \pm 0.02\,\text{mm}$

$$R = 13.00^{+0.04}_{-0.02}\,\text{mm} = 13.01 \pm 0.03\,\text{mm}$$

From the given figure, we can say

$$P = Q + W + R$$

$$W = P - (Q + R)$$

$$= (35.00 \pm 0.08) - [(12.00 \pm 0.02) + (13.01 \pm 0.03)]$$

$$= (35 - 12 - 13.01)^{+0.08-0.02-0.03}_{-0.08+0.02+0.03}$$

$$= 9.99^{+0.03}_{-0.03} = 9.99 \pm 0.03\,\text{mm}$$

SOL 10.140 Option (D) is correct.

	Working material		Type of Joining
P.	Aluminium	5.	Gas Tungsten Arc Welding
Q.	Die steel	4.	Atomic Hydrogen Welding
R.	Copper Wire	2.	Soldering
S.	Titanium sheet	6.	Laser Beam Welding

So, correct pairs are, P - 5, Q - 4, R - 2, S - 6

SOL 10.141 Option (A) is correct

Given : $N = 200\,\text{rpm},\ f = 0.25\,\text{mm/revolution},\ d = 0.4\,\text{mm},\ \alpha = 10^\circ,\ \phi = 27.75^\circ$

Uncut chip thickness, $t = f(\text{feed, mm/rev.}) = 0.25\,\text{mm/rev.}$

Chip thickness ratio is given by,

$$r = \frac{t}{t_c} = \frac{\sin\phi}{\cos(\phi - \alpha)}$$

Where, t_c = thickness of the produced chip.

So,

$$t_c = \frac{t \times \cos(\phi - \alpha)}{\sin\phi}$$

$$= \frac{0.25 \times \cos(27.75 - 10)}{\sin(27.75)} = 0.511\,\text{mm}$$

Alternate :

We also find the value of t_c by the general relation,

$$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha} \qquad \text{where } r = \frac{t}{t_c}$$

SOL 10.142 Option (D) is correct.

We know that angle of friction,

$$\beta = \tan^{-1}\mu$$

or,

$$\mu = \tan\beta \qquad \text{...(i)}$$

For merchant and earnest circle, the relation between rake angle (α), shear angle (ϕ) and friction angle (β) is given by,

$$2\phi + \beta - \alpha = 90^\circ$$

$$\beta = 90^\circ + \alpha - 2\phi$$

$$= 90^\circ + 10 - 2 \times 27.75 = 44.5^\circ$$

Now, from equation (i),

$$\mu = \tan(44.5^\circ) = 0.98$$

SOL 10.143 Option (D) is correct.

A lead-screw with half nuts in a lathe, free to rotate in both directions had Acme threads. When it is used in conjunction with a split nut, as on the lead screw of a lathe, the tapered sides of the threads facilitate ready to engagement and disengagement of the halves of the nut when required.

SOL 10.144 Option (C) is correct.

From the pouring basin, the molten metal is transported down into the mould cavity by means of the sprue or downgate. It is a vertical channel that connects the pouring basin with runners and gates.

SOL 10.145 Option (D) is correct.
Hot rolling of metal means working of metals when heated sufficiently (above the recrystallizing temperature) to make them plastic and easily worked.

SOL 10.146 Option (B) is correct.
GTAW is also called as Tungsten Inert Gas welding (TIG). The electrode is non consumable since its melting point is about $3400^\circ C$.

SOL 10.147 Option (B) is correct.
In trepanning, the cutting tool produces a hole by removing a disk-shaped piece (core), usually from flat plates. A hole is produced without reducing all the material removed to chips, as is the case in drilling. Such drills are used in deep-hole drilling machines for making large hollow shafts, long machine tool spindles etc.

SOL 10.148 Option (B) is correct.
Because each abrasive grain usually removes only a very small amount of material at a time, high rates of material removal can be achieved only if a large number of these grains act together. This is done by using bonded abrasives, typically in the form of a grinding wheel. The abrasive grains are held together by a bonding material which acts as supporting posts or brace between the grains and also increases the hardness of the grinding wheel.

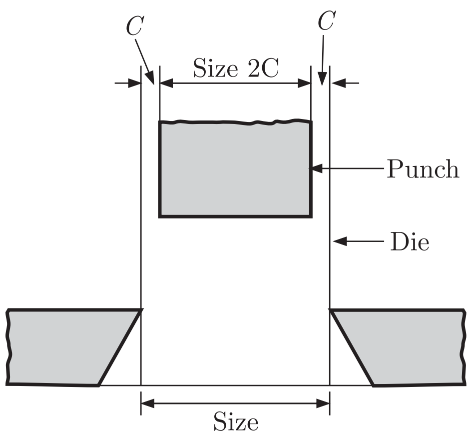
SOL 10.149 Option (D) is correct.
Centrifugal casting is the method of producing castings by pouring the molten metal into a rapidly rotating mould. Because of density differences, lighter elements such as dross, impurities and pieces of the refractory lining tend to collect at the centre of the casting. This results in better mould filling and a casting with a denser grain structure, which is virtually free of porosity.

SOL 10.150 Option (B) is correct.
Work hardening is when a metal is strained beyond the yield. An increasing stress is required to produce additional plastic deformation and the metal apparently becomes stronger and more difficult to deform.
Work hardening reduces ductility, which increases the chances of brittle failure.

SOL 10.151 Option (B) is correct.
A carburising flame is obtained when an excess of acetylene is supplied than

which is theoretically required. This excess amount of acetylene increases the temperature of the flame. So, the temperature of a carburising flame in gas welding is higher than that of a neutral or an oxidising flame.

SOL 10.152 Option (C) is correct.



The punch size is obtained by subtracting the clearance from the die-opening size. Clearance is the gap between the punch and the die. (From the figure)

SOL 10.153 Option (B) is correct.

When machining ductile materials, conditions of high local temperature and extreme pressure in the cutting zone and also high friction in the tool chip interface, may cause the work material to adhere or weld to the cutting edge of the tool forming the built-up edge. Low-cutting speed contributes to the formation of the built-up edge. Increasing the cutting speed, increasing the rake angle and using a cutting fluid contribute to the reduction or elimination of built-up edge.

SOL 10.154 Option (B) is correct.

Given : $t = 25\text{ mm}, \quad N = 300\text{ rpm}, \quad f = 0.25\text{ mm/rev}$

We know, time taken to drill a hole,

$$T = \frac{t}{fN} = \frac{25}{0.25 \times \frac{300}{60}} = \frac{25}{0.25 \times 5} = 20\text{ sec}$$

SOL 10.155 Option (C) is correct.

Since metal shrinks on solidification and contracts further on cooling to room temperature, linear dimensions of patterns are increased in respect of those of the finished casting to be obtained. This is called the "shrinkage allowance".

So, the temperature of solid phase drops from freezing to room temperature.

SOL 10.156 Option (B) is correct.

The blanking force is given by the relation,

$$F_b = \tau \times d \times t$$

Where, $\tau =$ shear strength of material.

SOL 10.157 Option (D) is correct.

In ECM, the principal of electrolysis is used to remove metal from the workpiece. The material removal is due to ion displacement. The principal of electrolysis is based on Faraday's law of electrolysis.

SOL 10.158 Option (C) is correct.

Electric arc welding is "a welding process wherein coalescence is produced by heating with an arc, with or without the use of filler metals.

No filler metal is used in butt weld. So, when the plate thickness changes, welding is achieved by changing the electrode size.

SOL 10.159 Option (A) is correct.

Allowance is an intentional difference between the maximum material limits of mating parts. For shaft, the maximum material limit will be its high limit and for hole, it will be its low limit. So, allowance refers to maximum clearance between shaft and hole.

SOL 10.160 Option (A) is correct.

Given : $H_g = 175\text{ mm},\ A_g = 200\text{ mm}^2,\ v_m = 10^6\text{ mm}^3,$

$g = 10\text{ m/sec}^2 = 10^4\text{ mm/sec}^2$

Time required to fill the mould is given by,

$$t = \frac{v_m}{A_g\sqrt{2gH_g}} = \frac{10^6}{200 \times \sqrt{2 \times 10^4 \times 175}} = 2.67\text{ sec}$$

SOL 10.161 Option (B) is correct.

The maximum reduction taken per pass in wire drawing, is limited by the strength of the deformed product. The exit end of the drawn rod will fracture at the die exit, when

$$\frac{\sigma_d}{\sigma_o} = 1, \text{ if there is no strain hardening.}$$

For zero back stress, the condition will be,

$$\frac{1+B}{B}\left[1-(1-RA)^B\right] = 1 \qquad \text{...(i)}$$

In wire drawing, co-efficient of friction of the order 0.1 are usually obtained.

Now, $B = \mu\cot\alpha$

$\mu = 0.1$ and $\alpha = 6^\circ$

$$B = \mu \cot 6^\circ = 0.9515$$

From equation (i),

$$1 - (1 - RA)^B = \frac{B}{1+B} = \frac{0.9515}{1+0.9515} = 0.49$$

$$(1 - RA)^B = 0.51$$

$$1 - RA = (0.51)^{\frac{1}{0.9515}} = 0.49$$

$$RA = 1 - 0.49 = 0.51$$

The approximate option is (B).

SOL 10.162 Option (C) is correct.

Given : $\alpha = 10^\circ,\ r = 0.4$

Shear angle $\tan\phi = \dfrac{r\cos\alpha}{1 - r\sin\alpha} = \dfrac{0.4\cos 10^\circ}{1 - 0.4\sin 10^\circ} = 0.4233$

$$\tan\phi = 0.4233$$

$$\phi = \tan^{-1}(0.4233) = 22.94^\circ$$

SOL 10.163 Option (A) is correct.

Given : $I = 15000\text{ A},\ \ t = 0.25\text{ sec},\ \ R = 0.0001\,\Omega$

The heat generated to form the weld is,

$$Q = I^2Rt = (15000)^2 \times 0.0001 \times 0.25 = 5625\text{ W-sec}$$

SOL 10.164 Option (C) is correct.

According to 3-2-1 principle, only the minimum locating points should be used to secure location of the work piece in any one plane.

(A) The workpiece is resting on three pins A, B, C which are inserted in the base of fixed body.

The workpiece cannot rotate about the axis XX and YY and also it cannot move downward. In this case, the five degrees of freedom have been arrested.

(B) Two more pins D and E are inserted in the fixed body, in a plane perpendicular to the plane containing, the pins A, B and C. Now the workpiece cannot rotate about the Z-axis and also it cannot move towards the left. Hence the addition of pins D and E restrict three more degrees of freedom.

(C) Another pin F in the second vertical face of the fixed body, arrests degree of freedom 9.

SOL 10.165 Option (B) is correct.

Given : Initial point (5, 4), Final point (7, 2), Centre (5, 4)

So, the G, N codes for this motion are $N010\ GO2\ X7.0\ Y2.0$ 15.0 $J2.0$

where, $GO2 \rightarrow$ Clockwise circular interpolation

$X7.0\ Y2.0 \rightarrow$ Final point

$I5.0\ J2.0 \rightarrow$ Centre point

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CHAPTER 11

INDUSTRIAL ENGINEERING

YEAR 2012 ONE MARK

MCQ 11.1 Which one of the following is NOT a decision taken during the aggregate production planning stage ?

(A) Scheduling of machines

(B) Amount of labour to be committed

(C) Rate at which production should happen

(D) Inventory to be carried forward

YEAR 2012 TWO MARKS

• Common Data For Q.2 and Q.3

For a particular project, eight activities are to be carried out. Their relationships with other activities and expected durations are mentioned in the table below.

Activity	Predecessors	Durations (days)
a	-	3
b	a	4
c	a	5
d	a	4
e	b	2
f	d	9
g	c, e	6
h	f, g	2

MCQ 11.2 The critical path for the project is

(A) $a - b - e - g - h$

(B) $a - c - g - h$

(C) $a - d - f - h$

(D) $a - b - c - f - h$

MCQ 11.3 If the duration of activity f alone is changed from 9 to 10 days, then the

(A) critical path remains the same and the total duration to complete the project changes to 19 days.

(B) critical path and the total duration to complete the project remains the same.

(C) critical path changes but the total duration to complete the project remains the same.

(D) critical path changes and the total duration to complete the project changes to 17 days.

YEAR 2011 ONE MARK

MCQ 11.4 Cars arrive at a service station according to Poisson's distribution with a mean rate of 5 per hour. The service time per car is exponential with a mean of 10 minutes. At steady state, the average waiting time in the queue is

(A) 10 minutes (B) 20 minutes

(C) 25 minutes (D) 50 minutes

MCQ 11.5 The word 'kanban' is most appropriately associated with

(A) economic order quantity

(B) just-in-time production

(C) capacity planning

(D) product design

YEAR 2011 TWO MARKS

• Common Data For Q.6 and Q.7

One unit of product P_1 requires 3 kg of resources R_1 and 1 kg of resources R_2. One unit of product P_2 requires 2 kg of resources R_1 and 2 kg of resources R_2. The profits per unit by selling product P_1 and P_2 are Rs. 2000 and Rs. 3000 respectively. The manufacturer has 90 kg of resources R_1 and 100 kg of resources R_2.

MCQ 11.6 The unit worth of resources R_2, i.e., dual price of resources R_2 in Rs. per kg is

(A) 0 (B) 1350

(C) 1500 (D) 2000

MCQ 11.7 The manufacturer can make a maximum profit of Rs.

(A) 60000 (B) 135000

(C) 150000 (D) 200000

YEAR 2010 ONE MARK

MCQ 11.8 The demand and forecast for February are 12000 and 10275, respectively. Using single exponential smoothening method (smoothening coefficient $= 0.25$), forecast for the month of March is

(A) 431 (B) 9587

(C) 10706 (D) 11000

MCQ 11.9 Little's law is a relationship between

(A) stock level and lead time in an inventory system

(B) waiting time and length of the queue in a queuing system

(C) number of machines and job due dates in a scheduling problem

(D) uncertainty in the activity time and project completion time

MCQ 11.10 Vehicle manufacturing assembly line is an example of

(A) product layout (B) process layout

(C) manual layout (D) fixed layout

MCQ 11.11 Simplex method of solving linear programming problem uses

(A) all the points in the feasible region

(B) only the corner points of the feasible region

(C) intermediate points within the infeasible region

(D) only the interior points in the feasible region

YEAR 2010 TWO MARKS

MCQ 11.12 Annual demand for window frames is 10000. Each frame cost Rs. 200 and ordering cost is Rs. 300 per order. Inventory holding cost is Rs. 40 per frame per year. The supplier is willing of offer 2% discount if the order quantity is 1000 or more, and 4% if order quantity is 2000 or more. If the total cost is to be minimized, the retailer should

(A) order 200 frames every time

(B) accept 2% discount

(C) accept 4% discount

(D) order Economic Order Quantity

MCQ 11.13 The project activities, precedence relationships and durations are described in the table. The critical path of the project is

Activity	Precedence	Duration (in days)
P	-	3
Q	-	4
R	P	5
S	Q	5
T	R, S	7
U	R, S	5
V	T	2
W	U	10

(A) P-R-T-V (B) Q-S-T-V

(C) P-R-U-W (D) Q-S-U-W

• Common Data For Q.14 and Q.15

Four jobs are to be processed on a machine as per data listed in the table.

Job	Processing time (in days)	Due date
1	4	6
2	7	9
3	2	19
4	8	17

MCQ 11.14 If the Earliest Due Date (EDD) rule is used to sequence the jobs, the number of jobs delayed is

(A) 1 (B) 2

(C) 3 (D) 4

MCQ 11.15 Using the Shortest Processing Time (SPT) rule, total tardiness is

(A) 0 (B) 2

(C) 6 (D) 8

YEAR 2009 ONE MARK

MCQ 11.16 The expected time (t_e) of a $PERT$ activity in terms of optimistic time t_0, pessimistic time (t_p) and most likely time (t_l) is given by

(A) $t_e = \frac{t_o + 4t_l + t_p}{6}$

(B) $t_e = \frac{t_o + 4t_p + t_l}{6}$

(C) $t_e = \frac{t_o + 4t_l + t_p}{3}$

(D) $t_e = \frac{t_o + 4t_p + t_l}{3}$

MCQ 11.17 Which of the following forecasting methods takes a fraction of forecast error into account for the next period forecast ?

(A) simple average method

(B) moving average method

(C) weighted moving average method

(D) exponential smoothening method

YEAR 2009 TWO MARKS

MCQ 11.18 A company uses 2555 units of an item annually. Delivery lead time is 8 days. The reorder point (in number of units) to achieve optimum inventory is

(A) 7

(B) 8

(C) 56

(D) 60

MCQ 11.19 Consider the following Linear Programming Problem (LPP):

Maximize $Z = 3x_1 + 2x_2$

Subject to

$$x_1 \leq 4$$
$$x_2 \leq 6$$
$$3x_1 + 2x_2 \leq 18$$
$$x_1 \geq 0, x_2 \geq 0$$

(A) The LPP has a unique optimal solution

(B) The LPP is infeasible.

(C) The LPP is unbounded.

(D) The LPP has multiple optimal solutions.

MCQ 11.20 Six jobs arrived in a sequence as given below:

Jobs	Processing Time (days)
I	4
II	9
III	5
IV	10
V	6
VI	8

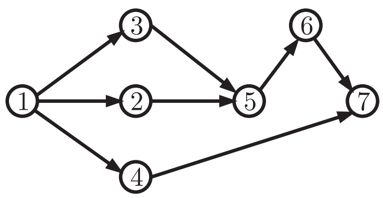
Average flow time (in days) for the above jobs using Shortest Processing time rule is

(A) 20.83 (B) 23.16

(C) 125.00 (D) 139.00

• Common Data For Q.21 and Q.22

Consider the following PERT network:



The optimistic time, most likely time and pessimistic time of all the activities are given in the table below:

Activity	Optimistic time (days)	Most likely time (days)	Pessimistic time (days)
1 - 2	1	2	3
1 - 3	5	6	7
1 - 4	3	5	7
2 - 5	5	7	9
3 - 5	2	4	6
5 - 6	4	5	6
4 - 7	4	6	8
6 - 7	2	3	4

MCQ 11.21 The critical path duration of the network (in days) is

(A) 11 (B) 14

(C) 17 (D) 18

MCQ 11.22 The standard deviation of the critical path is

(A) 0.33 (B) 0.55

(C) 0.77 (D) 1.66

YEAR 2008 ONE MARK

MCQ 11.23 In an $M/M/1$ queuing system, the number of arrivals in an interval of length T is a Poisson random variable (i.e. the probability of there being arrivals in an interval of length T is $\frac{e^{-\lambda T}(\lambda T)^n}{n!}$). The probability density function $f(t)$ of the inter-arrival time is

(A) $\lambda^2(e^{-\lambda^2 t})$

(B) $\frac{e^{-\lambda^2 t}}{\lambda^2}$

(C) $\lambda e^{-\lambda t}$

(D) $\frac{e^{-\lambda t}}{\lambda}$

MCQ 11.24 A set of 5 jobs is to be processed on a single machine. The processing time (in days) is given in the table below. The holding cost for each job is Rs. K per day.

Job	Processing time
P	5
Q	2
R	3
S	2
T	1

A schedule that minimizes the total inventory cost is

(A) T-S-Q-R-P

(B) P-R-S-Q-T

(C) T-R-S-Q-P

(D) P-Q-R-S-T

YEAR 2008 TWO MARKS

MCQ 11.25 For the standard transportation linear programme with m source and n destinations and total supply equaling total demand, an optimal solution (lowest cost) with the smallest number of non-zero x_{ij} values (amounts from source i to destination j) is desired. The best upper bound for this number is

(A) mn

(B) $2(m+n)$

(C) $m+n$

(D) $m+n-1$

MCQ 11.26 A moving average system is used for forecasting weekly demand $F_1(t)$ and $F_2(t)$ are sequences of forecasts with parameters m_1 and m_2, respectively, where m_1 and $m_2 (m_1 > m_2)$ denote the numbers of weeks over which the moving averages are taken. The actual demand shows a step increase from d_1 to d_2 at a certain time. Subsequently,

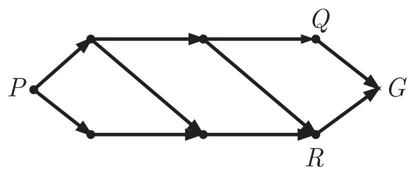
(A) neither $F_1(t)$ nor $F_2(t)$ will catch up with the value d_2

(B) both sequences $F_1(t)$ and $F_2(t)$ will reach d_2 in the same period

(C) $F_1(t)$ will attain the value d_2 before $F_2(t)$

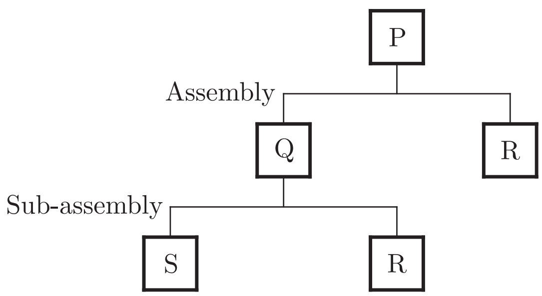
(D) $F_2(t)$ will attain the value d_2 before $F_1(t)$

MCQ 11.27 For the network below, the objective is to find the length of the shortest path from node P to node G.
Let d_{ij} be the length of directed arc from node i to node j.
Let S_j be the length of the shortest path from P to node j. Which of the following equations can be used to find S_G ?



(A) $S_G = \text{Min}\{S_Q, S_R\}$
(B) $S_G = \text{Min}\{S_Q - d_{QG}, S_R - d_{RG}\}$
(C) $S_G = \text{Min}\{S_Q + d_{QG}, S_R + d_{RG}\}$
(D) $S_G = \text{Min}\{d_{QG}, d_{RG}\}$

MCQ 11.28 The product structure of an assembly P is shown in the figure.



Estimated demand for end product P is as follows

Week	1	2	3	4	5	6
Demand	1000	1000	1000	1000	1200	1200

ignore lead times for assembly and sub-assembly. Production capacity (per week) for component R is the bottleneck operation. Starting with zero inventory, the smallest capacity that will ensure a feasible production plan up to week 6 is

(A) 1000
(B) 1200
(C) 2200
(D) 2400

• Common Data For Q.29 and Q.30

Consider the Linear Programme (LP)

Max $4x + 6y$

Subject to
$$3x + 2y \le 6$$
$$2x + 3y \le 6$$
$$x, y \ge 0$$

MCQ 11.29 After introducing slack variables s and t, the initial basic feasible solution is represented by the table below (basic variables are $s = 6$ and $t = 6$, and the objective function value is 0)

	-4	-6	0	0	0
s	3	2	1	0	6
t	2	3	0	1	6
	x	y	s	t	RHS

After some simplex iterations, the following table is obtained

	0	0	0	2	12
s	$5/3$	0	1	$-1/3$	2
y	$2/3$	1	0	$1/3$	2
	x	y	s	t	RHS

From this, one can conclude that

(A) the LP has a unique optimal solution

(B) the LP has an optimal solution that is not unique

(C) the LP is infeasible

(D) the LP is unbounded

MCQ 11.30 The dual for the LP in Q. 29 is

(A) Min $6u + 6v$
subject to
$3u + 2v \geq 4$
$2u + 3v \geq 6$
$u, v \geq 0$

(B) Max $6u + 6v$
subject to
$3u + 2v \leq 4$
$2u + 3v \leq 6$
$u, v \geq 0$

(C) Max $4u + 6v$
subject to
$3u + 2v \geq 6$
$2u + 3v \geq 6$
$u, v \geq 0$

(D) Min $4u + 6v$
subject to
$3u + 2v \leq 6$
$2u + 3v \leq 6$
$u, v \geq 0$

YEAR 2007 TWO MARKS

MCQ 11.31 Capacities of production of an item over 3 consecutive months in regular time are 100, 100 and 80 and in overtime are 20, 20 and 40. The demands over those 3 months are 90, 130 and 110. The cost of production in regular time and overtime are respectively Rs. 20 per item and Rs. 24 per item.

Inventory carrying cost is Rs. 2 per item per month. The levels of starting and final inventory are nil. Backorder is not permitted. For minimum cost of plan, the level of planned production in overtime in the third month is

(A) 40 (B) 30

(C) 20 (D) 0

MCQ 11.32 The maximum level of inventory of an item is 100 and it is achieved with infinite replenishment rate. The inventory becomes zero over one and half month due to consumption at a uniform rate. This cycle continues throughout the year. Ordering cost is Rs. 100 per order and inventory carrying cost is Rs. 10 per item per month. Annual cost (in Rs.) of the plan, neglecting material cost, is

(A) 800 (B) 2800

(C) 4800 (D) 6800

MCQ 11.33 In a machine shop, pins of 15 mm diameter are produced at a rate of 1000 per month and the same is consumed at a rate of 500 per month. The production and consumption continue simultaneously till the maximum inventory is reached. Then inventory is allowed to reduced to zero due to consumption . The lot size of production is 1000. If backlog is not allowed, the maximum inventory level is

(A) 400 (B) 500

(C) 600 (D) 700

MCQ 11.34 The net requirements of an item over 5 consecutive weeks are 50-0-15-20-20. The inventory carrying cost and ordering cost are Rs. 1 per item per week and Rs. 100 per order respectively. Starting inventory is zero. Use " Least Unit Cost Technique" for developing the plan. The cost of the plan (in Rs.) is

(A) 200 (B) 250

(C) 225 (D) 260

YEAR 2006 ONE MARK

MCQ 11.35 The number of customers arriving at a railway reservation counter is Poisson distributed with an arrival rate of eight customers per hour. The reservation clerk at this counter takes six minutes per customer on an average with an exponentially distributed service time. The average number of the customers in the queue will be

(A) 3 (B) 3.2

(C) 4 (D) 4.2

MCQ 11.36 In an MRP system, component demand is

(A) forecasted

(B) established by the master production schedule

(C) calculated by the MRP system from the master production schedule

(D) ignored

YEAR 2006 TWO MARKS

MCQ 11.37 An manufacturing shop processes sheet metal jobs, wherein each job must pass through two machines ($M1$ and $M2$, in that order). The processing time (in hours) for these jobs is

Machine	Jobs					
	P	Q	R	S	T	U
$M1$	15	32	8	27	11	16
$M2$	6	19	13	20	14	7

The optimal make-span (in-hours) of the shop is

(A) 120

(B) 115

(C) 109

(D) 79

MCQ 11.38 Consider the following data for an item.
Annual demand : 2500 units per year, Ordering cost : Rs.100 per order, Inventory holding rate : 25% of unit price
Price quoted by a supplier

Order quantity (units)	Unit price (Rs.)
< 500	10
≥ 500	9

The optimum order quantity (in units) is

(A) 447

(B) 471

(C) 500

(D) ≥ 600

MCQ 11.39 A firm is required to procure three items (P, Q, and R). The prices quoted for these items (in Rs.) by suppliers $S1$, $S2$ and $S3$ are given in table. The management policy requires that each item has to be supplied by only one supplier and one supplier supply only one item. The minimum total cost (in Rs.) of procurement to the firm is

Item	Suppliers		
	$S1$	$S2$	$S3$
P	110	120	130
Q	115	140	140
R	125	145	165

(A) 350 (B) 360

(C) 385 (D) 395

MCQ 11.40 A stockist wishes to optimize the number of perishable items he needs to stock in any month in his store. The demand distribution for this perishable item is

Demand (in units)	2	3	4	5
Probability	0.10	0.35	0.35	0.20

The stockist pays Rs. 70 for each item and he sells each at Rs. 90. If the stock is left unsold in any month, he can sell the item at Rs. 50 each. There is no penalty for unfulfilled demand. To maximize the expected profit, the optimal stock level is

(A) 5 units (B) 4 units

(C) 3 units (D) 2 units

MCQ 11.41 The table gives details of an assembly line.

Work station	I	II	III	IV	V	VI
Total task time at the workstation (in minutes)	7	9	7	10	9	6

What is the line efficiency of the assembly line ?

(A) 70% (B) 75%

(C) 80% (D) 85%

MCQ 11.42 The expected completion time of the project is

(A) 238 days (B) 224 days

(C) 171 days (D) 155 days

MCQ 11.43 The standard deviation of the critical path of the project is

(A) $\sqrt{151}$ days (B) $\sqrt{155}$ days

(C) $\sqrt{200}$ days (D) $\sqrt{238}$ days

YEAR 2005 ONE MARK

MCQ 11.44 An assembly activity is represented on an Operation Process Chart by the symbol

(A) ☐ (B) A

(C) D (D) O

MCQ 11.45 The sales of a product during the last four years were 860, 880, 870 and 890 units. The forecast for the fourth year was 876 units. If the forecast for the fifth year, using simple exponential smoothing, is equal to the forecast using a three period moving average, the value of the exponential smoothing constant α is

(A) $\frac{1}{7}$ (B) $\frac{1}{5}$

(C) $\frac{2}{7}$ (D) $\frac{2}{5}$

MCQ 11.46 Consider a single server queuing model with Poisson arrivals ($\lambda = 4/\text{hour}$) and exponential service ($\mu = 4/\text{hour}$). The number in the system is restricted to a maximum of 10. The probability that a person who comes in leaves without joining the queue is

(A) $\frac{1}{11}$ (B) $\frac{1}{10}$

(C) $\frac{1}{9}$ (D) $\frac{1}{2}$

YEAR 2005 TWO MARKS

MCQ 11.47 A component can be produced by any of the four processes I, II, III and IV. Process I has a fixed cost of Rs. 20 and variable cost of Rs. 3 per piece. Process II has a fixed cost Rs. 50 and variable cost of Rs. 1 per piece. Process III has a fixed cost of Rs. 40 and variable cost of Rs. 2 per piece. Process IV has a fixed cost of Rs. 10 and variable cost of Rs. 4 per piece. If the company wishes to produce 100 pieces of the component, form economic point of view it should choose

(A) Process I (B) Process II

(C) Process III (D) Process IV

MCQ 11.48 A welding operation is time-studied during which an operator was pace-rated as 120%. The operator took, on an average, 8 minutes for producing the weld-joint. If a total of 10% allowances are allowed for this operation. The expected standard production rate of the weld-joint (in units per 8 hour day) is

(A) 45 (B) 50

(C) 55 (D) 60

MCQ 11.49 The distribution of lead time demand for an item is as follows:

Lead time demand	Probability
80	0.20
100	0.25
120	0.30
140	0.25

The reorder level is 1.25 times the expected value of the lead time demand. The service level is

(A) 25% (B) 50%

(C) 75% (D) 100%

MCQ 11.50 A project has six activities (A to F) with respective activity duration 7, 5, 6, 6, 8, 4 days. The network has three paths A-B, C-D and E-F. All the activities can be crashed with the same crash cost per day. The number of activities that need to be crashed to reduce the project duration by 1 day is

(A) 1 (B) 2

(C) 3 (D) 6

MCQ 11.51 A company has two factories $S1$, $S2$, and two warehouses $D1$, $D2$. The supplies from $S1$ and $S2$ are 50 and 40 units respectively. Warehouse $D1$ requires a minimum of 20 units and a maximum of 40 units. Warehouse $D2$ requires a minimum of 20 units and, over and above, it can take as much as can be supplied. A balanced transportation problem is to be formulated for the above situation. The number of supply points, the number of demand points, and the total supply (or total demand) in the balanced transportation problem respectively are

(A) 2, 4, 90 (B) 2, 4, 110

(C) 3, 4, 90 (D) 3, 4, 110

• Common Data For Q.52 and Q.53

Consider a linear programming problem with two variables and two constraints. The objective function is : Maximize $X_1 + X_2$. The corner points of the feasible region are (0, 0), (0, 2), (2, 0) and (4/3, 4/3)

MCQ 11.52 If an additional constraint $X_1 + X_2 \leq 5$ is added, the optimal solution is

(A) $\left(\frac{5}{3}, \frac{5}{3}\right)$ (B) $\left(\frac{4}{3}, \frac{4}{3}\right)$

(C) $\left(\frac{5}{2}, \frac{5}{2}\right)$ (D) (5, 0)

MCQ 11.53 Let Y_1 and Y_2 be the decision variables of the dual and v_1 and v_2 be the slack variables of the dual of the given linear programming problem. The optimum dual variables are

(A) Y_1 and Y_2
(B) Y_1 and v_1
(C) Y_1 and v_2
(D) v_1 and v_2

YEAR 2004 ONE MARK

MCQ 11.54 In PERT analysis a critical activity has

(A) maximum Float
(B) zero Float
(C) maximum Cost
(D) minimum Cost

MCQ 11.55 For a product, the forecast and the actual sales for December 2002 were 25 and 20 respectively. If the exponential smoothing constant (α) is taken as 0.2, then forecast sales for January 2003 would be

(A) 21
(B) 23
(C) 24
(D) 27

MCQ 11.56 There are two products P and Q with the following characteristics

Product	Demand (Units)	Order cost (Rs/order)	Holding Cost (Rs./ unit/ year)
P	100	50	4
Q	400	50	1

The economic order quantity (EOQ) of products P and Q will be in the ratio

(A) 1 : 1
(B) 1 : 2
(C) 1 : 4
(D) 1 : 8

YEAR 2004 TWO MARKS

MCQ 11.57 A standard machine tool and an automatic machine tool are being compared for the production of a component. Following data refers to the two machines.

	Standard Machine Tool	Automatic Machine Tool
Setup time	30 min	2 hours
Machining time per piece	22 min	5 min
Machine rate	Rs. 200 per hour	Rs. 800 per hour

The break even production batch size above which the automatic machine tool will be economical to use, will be

(A) 4 (B) 5

(C) 24 (D) 225

MCQ 11.58 A soldering operation was work-sampled over two days (16 hours) during which an employee soldered 108 joints. Actual working time was 90% of the total time and the performance rating was estimated to be 120 per cent. If the contract provides allowance of 20 percent of the time available, the standard time for the operation would be

(A) 8 min (B) 8.9 min

(C) 10 min (D) 12 min

MCQ 11.59 An electronic equipment manufacturer has decided to add a component sub-assembly operation that can produce 80 units during a regular 8-hours shift. This operation consist of three activities as below

Activity	Standard time (min)
M. Mechanical assembly	12
E. Electric wiring	16
T. Test	3

For line balancing the number of work stations required for the activities M, E and T would respectively be

(A) 2, 3, 1 (B) 3, 2, 1

(C) 2, 4, 2 (D) 2, 1, 3

MCQ 11.60 A maintenance service facility has Poisson arrival rates, negative exponential service time and operates on a 'first come first served' queue discipline. Breakdowns occur on an average of 3 per day with a range of zero to eight. The maintenance crew can service an average of 6 machines per day with a range of zero to seven. The mean waiting time for an item to be serviced would be

(A) $\frac{1}{6}$ day (B) $\frac{1}{3}$ day

(C) 1 day (D) 3 day

MCQ 11.61 A company has an annual demand of 1000 units, ordering cost of Rs. 100 / order and carrying cost of Rs. 100/ unit/year. If the stock-out cost are estimated to be nearly Rs. 400 each time the company runs out-of-stock, then safety stock justified by the carrying cost will be

(A) 4 (B) 20

(C) 40 (D) 100

MCQ 11.62 A company produces two types of toys : P and Q. Production time of Q is twice that of P and the company has a maximum of 2000 time units per day. The supply of raw material is just sufficient to produce 1500 toys (of any type) per day. Toy type Q requires an electric switch which is available @ 600 pieces per day only. The company makes a profit of Rs. 3 and Rs. 5 on type P and Q respectively. For maximization of profits, the daily production quantities of P and Q toys should respectively be

(A) 1000, 500 (B) 500, 1000

(C) 800, 600 (D) 1000, 1000

YEAR 2003 ONE MARK

MCQ 11.63 The symbol used for Transport in work study is

(A) $\Rightarrow$ (B) T

(C) ▭ (D) ∇

YEAR 2003 TWO MARKS

MCQ 11.64 Two machines of the same production rate are available for use. On machine 1, the fixed cost is Rs. 100 and the variable cost is Rs. 2 per piece produced. The corresponding numbers for the machine 2 are Rs. 200 and Re. 1 respectively. For certain strategic reasons both the machines are to be used concurrently. The sales price of the first 800 units is Rs. 3.50 per unit and subsequently it is only Rs. 3.00. The breakeven production rate for each machine is

(A) 75 (B) 100

(C) 150 (D) 600

MCQ 11.65 A residential school stipulates the study hours as 8.00 pm to 10.30 pm. Warden makes random checks on a certain student 11 occasions a day during the study hours over a period of 10 days and observes that he is studying on 71 occasions. Using 95% confidence interval, the estimated minimum hours of his study during that 10 day period is

(A) 8.5 hours (B) 13.9 hours

(C) 16.1 hours (D) 18.4 hours

MCQ 11.66 The sale of cycles in a shop in four consecutive months are given as 70, 68, 82, 95. Exponentially smoothing average method with a smoothing factor of 0.4 is used in forecasting. The expected number of sales in the next month is

(A) 59 (B) 72

(C) 86 (D) 136

MCQ 11.67 Market demand for springs is 8,00,000 per annum. A company purchases these springs in lots and sells them. The cost of making a purchase order is Rs.1200. The cost of storage of springs is Rs.120 per stored piece per annum. The economic order quantity is

(A) 400

(B) 2,828

(C) 4,000

(D) 8,000

MCQ 11.68 A manufacturer produces two types of products, 1 and 2, at production levels of x_1 and x_2 respectively. The profit is given is $2x_1 + 5x_2$. The production constraints are

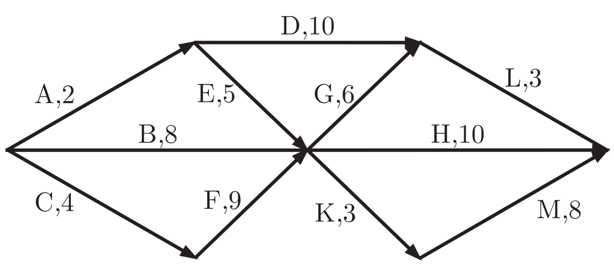
$$x_1 + 3x_2 \le 40$$
$$3x_1 + x_2 \le 24$$
$$x_1 + x_2 \le 10$$
$$x_1 > 0,\ x_2 > 0$$

The maximum profit which can meet the constraints is

(A) 29 (B) 38

(C) 44 (D) 75

MCQ 11.69 A project consists of activities A to M shown in the net in the following figure with the duration of the activities marked in days



The project can be completed

(A) between 18, 19 days (B) between 20, 22 days

(C) between 24, 26 days (D) between 60, 70 days

MCQ 11.70 The principles of motion economy are mostly used while conducting

(A) a method study on an operation

(B) a time study on an operation

(C) a financial appraisal of an operation

(D) a feasibility study of the proposed manufacturing plant

YEAR 2002 ONE MARK

MCQ 11.71 The standard time of an operation while conducting a time study is

(A) mean observed time + allowances

(B) normal time + allowances

(C) mean observed time $\times$ rating factor + allowances

(D) normal time $\times$ rating factor + allowances

MCQ 11.72 In carrying out a work sampling study in a machine shop, it was found that a particular lathe was down for 20% of the time. What would be the 95% confidence interval of this estimate, if 100 observations were made ?

(A) $(0.16, 0.24)$ (B) $(0.12, 0.28)$

(C) $(0.08, 0.32)$ (D) None of these

MCQ 11.73 An item can be purchased for Rs. 100. The ordering cost is Rs. 200 and the inventory carrying cost is 10% of the item cost per annum. If the annual demand is 4000 unit, the economic order quantity (in unit) is

(A) 50 (B) 100

(C) 200 (D) 400

YEAR 2002 TWO MARKS

MCQ 11.74 Arrivals at a telephone booth are considered to be Poisson, with an average time of 10 minutes between successive arrivals. The length of a phone call is distributed exponentially with mean 3 minutes. The probability that an arrival does not have to wait before service is

(A) 0.3 (B) 0.5

(C) 0.7 (D) 0.9

MCQ 11.75 The supplies at three sources are 50, 40 and 60 unit respectively whilst the demands at the four destinations are 20, 30, 10 and 50 unit. In solving this transportation problem

(A) a dummy source of capacity 40 unit is needed

(B) a dummy destination of capacity 40 unit is needed

(C) no solution exists as the problem is infeasible

(D) no solution exists as the problem is degenerate

MCQ 11.76 A project consists of three parallel paths with mean durations and variances of $(10, 4)$, $(12, 4)$ and $(12, 9)$ respectively. According to the standard PERT assumptions, the distribution of the project duration is

(A) beta with mean 10 and standard deviation 2

(B) beta with mean 12 and standard deviation 2

(C) normal with mean 10 and standard deviation 3

(D) normal with mean 12 and standard deviation 3

YEAR 2001 ONE MARK

MCQ 11.77 Production flow analysis (PFA) is a method of identifying part families that uses data from

(A) engineering drawings
(B) production schedule
(C) bill of materials
(D) route sheets

MCQ 11.78 When using a simple moving average to forecast demand, one would

(A) give equal weight to all demand data

(B) assign more weight to the recent demand data

(C) include new demand data in the average without discarding the earlier data

(D) include new demand data in the average after discarding some of the earlier demand data

YEAR 2001 TWO MARKS

MCQ 11.79 Fifty observations of a production operation revealed a mean cycle time of 10 min. The worker was evaluated to be performing at 90% efficiency. Assuming the allowances to be 10% of the normal time, the standard time (in second) for the job is

(A) 0.198
(B) 7.3
(C) 9.0
(D) 9.9

****

SOLUTION

SOL 11.1 Option (A) is correct.

Costs relevant to aggregate production planning is as given below.

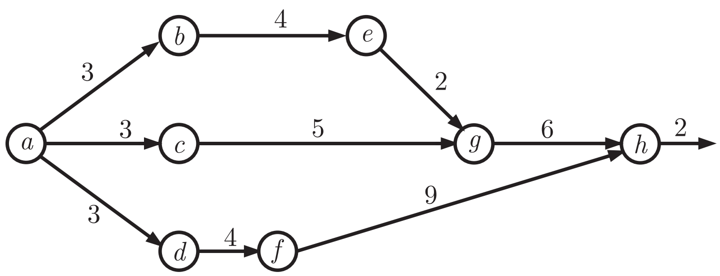
(i) Basic production cost : Material costs, direct labour costs, and overhead cost.

(ii) Costs associated with changes in production rate : Costs involving in hiring, training and laying off personnel, as well as, overtime compensation.

(iii) Inventory related costs.

Hence, from above option (A) is not related to these costs. Therefore option (A) is not a decision taken during the APP.

SOL 11.2 Option (C) is correct.



For path	Duration
$a - b - e - g - h$	$= 3 + 4 + 2 + 6 + 2 = 17$ days
$a - c - g - h$	$= 3 + 5 + 6 + 2 = 16$ days
$a - d - f - h$	$= 3 + 4 + 9 + 2 = 18$ days

The critical path is one that takes longest path.

Hence, path $a - d - f - h = 18$ days is critical path

SOL 11.3 Option (A) is correct.

From previous question

For critical path

a-d-f-h $= 18$ days, the duration of activity f alone is changed from 9 to 10 days, then

$$a - d - f - h = 3 + 4 + 10 + 2 = 19 \text{ days}$$

Hence critical path remains same and the total duration to complete the project changes to 19 days.

SOL 11.4 Option (D) is correct.

Given : $\lambda = 5$ per hour, $\mu = \frac{1}{10} \times 60$ per hour $= 6$ per hour

Average waiting time of an arrival

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{5}{6(6-5)}$$

$$= \frac{5}{6}\text{ hours} = 50\text{ min}$$

SOL 11.5 Option (B) is correct.

Kanban Literally, a "Visual record"; a method of controlling materials flow through a Just-in-time manufacturing system by using cards to authorize a work station to transfer or produce materials.

SOL 11.6 Option (A) is correct.

Since, in Z_j Row of final (second) obtimum table the value of slack variable S_2 showns the unit worth or dual price of Resource R_2 and the value of S_2 in given below table is zero. Hence the dual Price of Resource R_2 is zero.

$$\text{Max } Z = 2000P_1 + 3000P_2$$

S.T. $3P_1 + 2P_2 \le 90$ $\rightarrow R_1$ – Resource

$P_1 + 2P_2 \le 100$ $\rightarrow R_2$ – Resource

$P_1, P_2 \ge 0$

Solution : $Z = 2000P_1 + 3000P_2 + 0.S_1 + 0.S_2$

S.T. $3P_1 + 2P_2 + S_1 = 90$

$P_1 + 2P_2 + S_2 = 100$

$P_1 \ge 0,\ P_2 \ge 0,\ S_1 \ge 0,\ S_2 \ge 0$

First table :-

		C_j	2000	3000	0	0
C_B	S_B	P_B	P_1	P_2	S_1	S_2
0	S_1	90	3	$\boxed{2}$ $\rightarrow$	1	0
0	S_2	100	1	2	0	1
	Z_j		0	0	0	0
	$Z_j - C_j$		-2000	-3000 $\uparrow$	0	0

Second Table :-

		C_j	2000	3000	0	0
C_B	S_B	P_B	P_1	P_2	S_1	S_2
3000	P_2	$\boxed{45}$	3/2	1	1/2	0

0	S_2	10	-2	0	-1	1
		Z_j	4500	3000	1500	$\boxed{0}$ → unit worth of R_2
		$Z_j - C_j$	2500	0	1500	0

SOL 11.7 Option (B) is correct.

Since all $Z_j - C_j \geq 0$, an optimal basic feasible solution has been attained. Thus, the optimum solution to the given LPP is

$$\text{Max} Z = 2000 \times 0 + 3000 \times 45 = \text{Rs.}135000 \text{ with } P_1 = 0 \text{ and } P_2 = 45$$

SOL 11.8 Option (C) is correct.

Given, forecast for February $F_{t-1} = 10275$

Demand for February $D_{t-1} = 12000$

Smoothing coefficient $\alpha = 0.25$

Which is The forecast for the next period is given by,

$$\begin{aligned} F_t &= \alpha(D_{t-1}) + (1-\alpha) \times F_{t-1} \\ &= 0.25 \times (12000) + (1-0.25) \times (10275) \\ &= 10706.25 \simeq 10706 \end{aligned}$$

Hence, forecast for the month of march is 10706.

SOL 11.9 Option (B) is correct.

Little's law is a relationship between average waiting time and average length of the queue in a queuing system.

The little law establish a relation between Queue length (L_q), Queue waiting time (W_q) and the Mean arrival rate λ.

So, $$L_q = \lambda W_q$$

SOL 11.10 Option (A) is correct.

Vehicle manufacturing assembly line is an example of product layout.

A product-oriented layout is appropriate for producing one standardized product, usually in large volume. Each unit of output requires the same sequence of operations from beginning to end.

SOL 11.11 Option (D) is correct.

Simplex method provides an algorithm which consists in moving from one point of the region of feasible solutions to another in such a manner that the value of the objective function at the succeeding point is less (or more, as the case may be) than at the preceding point. This procedure of jumping from one point to another is then repeated. Since the number of points is finite, the method leads to an optimal point in a finite number of steps. Therefore simplex method only uses the interior points in the feasible region.

SOL 11.12 Option (C) is correct.

Given : $D = 10000$

Ordering cost C_o = Rs. 300 per order

Holding cost C_h = Rs. 40 per frame per year

Unit cost, C_u = Rs. 200

$$EOQ = \sqrt{\frac{2C_oD}{C_h}} = \sqrt{\frac{2\times 300\times 10000}{40}} \simeq 387 \text{ units}$$

Total cost = Purchase cost + holding cost + ordering cost

For $EOQ = 387$ units

$$\text{Total cost} = D\times C_u + \frac{Q}{2}\times C_h + \frac{D}{Q}\times C_o$$

Where $Q = EOQ = 387$ units

$$\text{Total cost} = 10000\times 200 + \frac{387}{2}\times 40 + \frac{10000}{387}\times 300$$

$$= 2000000 + 7740 + 7752 = \text{Rs. } 2015492$$

Now supplier offers 2% discount if the order quantity is 1000 or more.

For $Q = 1000$ units

$$\text{Total cost} = 10000\times(200\times 0.98) + \frac{1000}{2}\times 40 + \frac{10000}{1000}\times 300$$

$$= 1960000 + 20000 + 3000 = \text{Rs. } 1983000$$

Supplier also offers 4% discount if order quantity is 2000 or more.

For $Q = 2000$ units

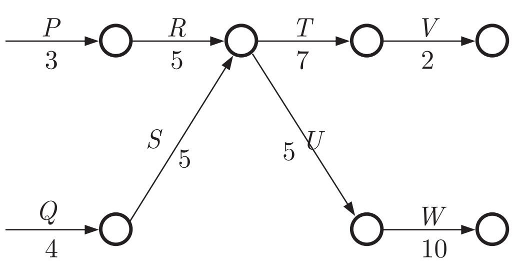
$$\text{Total cost} = 10000\times(200\times 0.96) + \frac{2000}{2}\times 40 + \frac{10000}{2000}\times 300$$

$$= 1920000 + 40000 + 1500 = \text{Rs. } 1961500$$

It is clearly see that the total cost is to be minimized, the retailer should accept 4% discount.

SOL 11.13 Option (D) is correct.

We have to draw a arrow diagram from the given data.



Here Four possible ways to complete the work.

	Path	Total duration (days)
(i)	$P - R - T - V$	$T = 3 + 5 + 7 + 2 = 17$
(ii)	$Q - S - T - V$	$T = 4 + 5 + 7 + 2 = 18$
(iii)	$Q - S - U - W$	$T = 4 + 5 + 5 + 10 = 24$
(iv)	$P - R - U - W$	$T = 3 + 5 + 5 + 10 = 23$

The critical path is the chain of activities with the longest time durations.

So, Critical path $= Q - S - U - W$

SOL 11.14 Option (C) is correct.

In the Earliest due date (EDD) rule, the jobs will be in sequence according to their earliest due dates.

Table shown below :

Job	Processing time (in days)	Due date	Operation start	Operation end
1	4	6	0	$0 + 4 = 4$
2	7	9	4	$4 + 7 = 11$
4	8	17	11	$11 + 8 = 19$
3	2	19	19	$19 + 2 = 21$

We see easily from the table that, job 2, 4, & 3 are delayed.

Number of jobs delayed is 3.

SOL 11.15 Option (D) is correct.

By using the shortest processing time (SPT) rule & make the table

Job	Processing time (in days)	Flow time		Due date	Tradiness
		Start	End		
3	2	0	2	19	0
1	4	2	$2 + 4 = 6$	6	0
2	7	6	$6 + 7 = 13$	9	4
4	8	13	$13 + 8 = 21$	17	4

So, from the table

Total Tradiness $= 4 + 4 = 8$

SOL 11.16 Option (A) is correct.

Under the conditions of uncertainty, the estimated time for each activity for PERT network is represented by a probability distribution. This probability distribution of activity time is based upon three different time estimates made for each activity. These are as follows.

t_o = the optimistic time, is the shortest possible time to complete the activity if

all goes well.

t_p = the pessimistic time, is the longest time that an activity could take if every

thing goes wrong

t_l = the most likely time, is the estimate of normal time an activity would take.

The expected time (t_e) of the activity duration can be approximated as the arithmetic mean of $(t_o + t_p)/2$ and $2t_l$. Thus

$$(t_e) = \frac{1}{3}\left[2t_l + \frac{(t_o + t_p)}{2}\right] = \frac{t_o + 4t_l + t_p}{6}$$

SOL 11.17 Option (D) is correct.

Exponential smoothing method of forecasting takes a fraction of forecast error into account for the next period forecast.

The exponential smoothed average u_t, which is the forecast for the next period $(t+1)$ is given by.

$$\begin{aligned} u_t &= \alpha y_t + \alpha(1-\alpha) y_{t-1} + ...\alpha(1-\alpha)^n y_{t-n} + ...\infty \\ &= \alpha y_t + (1-\alpha)[\alpha y_{t-1} + \alpha(1-\alpha) y_{t-2} + ... + \alpha(1-\alpha)^n y_{t-(n-1)} + ...] \\ &= u_{t-1} + \alpha(y_t - u_{t-1}) \\ &= u_{t-1} + \alpha e_t \end{aligned}$$

where $e_t = (y_t - u_{t-1})$ is called error and is the difference between the least observation, y_t and its forecast a period earlier, u_{t-1}.

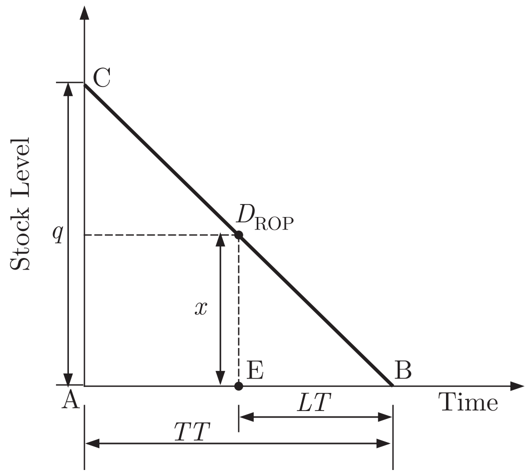
The value of α lies between 0 to 1.

SOL 11.18 Option (C) is correct.

In figure,

$$\begin{aligned} \text{ROP} &= \text{Reorder point} \\ LT &= \text{Lead Time} = 8 \text{ days} \\ TT &= \text{Total Time} = 365 \text{ days} \\ q &= \text{stock level} = 2555 \text{ units} \end{aligned}$$

Let the reorder quantity be x



Now from the similar triangles
ΔABC & ΔBDE

$$\frac{q}{TT} = \frac{x}{LT}$$

$$\Rightarrow \qquad \frac{2555}{365} = \frac{x}{8}$$

$$x = \frac{2555}{365} \times 8 = 56 \text{ Units}$$

Alternate Method
Given,

$$\text{Demand in a year } D = 2555 \text{ Units}$$

$$\text{Lead time } T = 8 \text{ days}$$

Now, Number of orders to be placed in a year

$$N = \frac{\text{Number. of days in a year}}{\text{Lead Time}} = \frac{365}{8} \text{ orders}$$

Now, quantity ordered each time or reorder point.

$$Q = \frac{\text{Demand in a years}}{\text{Number of orders}} = \frac{2555}{\frac{365}{8}} = 56 \text{ Units}$$

SOL 11.19 Option (D) is correct.
Given objective function

$$Z_{max} = 3x_1 + 2x_2$$

and constraints are

$$x_1 \leq 4 \qquad \text{...(i)}$$

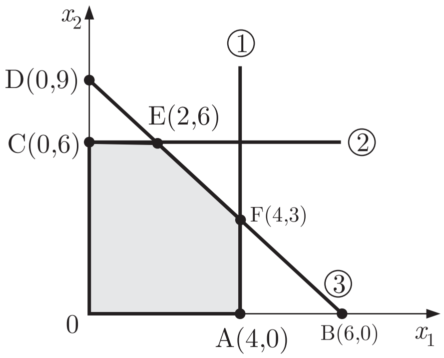
$$x_2 \leq 6 \qquad \text{...(ii)}$$

$$3x_1 + 2x_2 \leq 18 \qquad \text{...(iii)}$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

Plot the graph from the given constraints and find the common area.



Now, we find the point of intersection E & F.

For E, $3x_1 + 2x_2 = 18$

(E is the intersection point of equation. (ii) & (iii))

$$x_2 = 6$$

So, $3x_1 + 12 = 18$

$$x_1 = 2$$

For F, $3x_1 + 2x_2 = 18$

$$x_1 = 4$$

So, $3 \times 4 + 2x_2 = 18$

$$x_2 = 3$$

Hence,

$$E(2,6) \text{ or } F(4,3)$$

Now at point $E(2,6)$

$$Z = 3 \times 2 + 2 \times 6$$
$$= 18$$

At point $F(4,3)$

$$Z = 3 \times 4 + 2 \times 3$$
$$= 18$$

The objective function and the constraint (represent by equation (iii)) are equal.

Hence, the objective function will have the multiple solutions as at point E & F, the value of objective function ($Z = 3x_1 + 2x_2$) is same.

SOL 11.20 Option (A) is correct.

In shortest processing time rule, we have to arrange the jobs in the increasing order of their processing time and find total flow time.

So, job sequencing are I - III - V - VI - II - IV

Jobs	Processing Time (days)	Flow time (days)
I	4	4
III	5	$4+5=9$
V	6	$9+6=15$
VI	8	$15+8=23$
II	9	$23+9=32$
IV	10	$32+10=42$

Now Total flow time $T = 4+9+15+23+32+42$

$$= 125$$

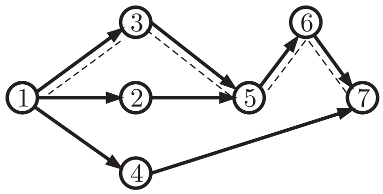
$$\text{Average flow time} = \frac{\text{Total flow time}}{\text{Number of jobs}}$$

$$T_{average} = \frac{125}{6} = 20.83 \text{ days}$$

SOL 11.21 Option (D) is correct.

Make the table and calculate the excepted time and variance for each activity

Activity	Optimistic time (days) t_o	Most likely time (days) t_m	Pessimistic time (days) t_p	Expected Time (days) $t_e = \frac{t_o + 4t_m + t_p}{6}$	Variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1 - 2	1	2	3	$\frac{1+8+3}{6} = 2$	$\left(\frac{3-1}{6}\right)^2 = \frac{1}{9}$
1 - 3	5	6	7	$\frac{5+24+7}{6} = 6$	$\left(\frac{7-5}{6}\right)^2 = \frac{1}{9}$
1 - 4	3	5	7	$\frac{3+20+7}{6} = 5$	$\left(\frac{7-3}{6}\right)^2 = \frac{4}{9}$
2 - 5	5	7	9	$\frac{5+28+9}{6} = 7$	$\left(\frac{9-5}{6}\right)^2 = \frac{4}{9}$
3 - 5	2	4	6	$\frac{2+16+6}{6} = 4$	$\left(\frac{6-2}{6}\right)^2 = \frac{4}{9}$
5 - 6	4	5	6	$\frac{4+20+6}{6} = 5$	$\left(\frac{6-4}{6}\right)^2 = \frac{1}{9}$
4 - 7	4	6	8	$\frac{4+24+8}{6} = 6$	$\left(\frac{8-4}{6}\right)^2 = \frac{4}{9}$
6 - 7	2	3	4	$\frac{2+12+4}{6} = 3$	$\left(\frac{4-2}{6}\right)^2 = \frac{1}{9}$



Now, the paths of the network & their durations are given below in tables.

	Paths	Expected Time duration (in days)
i	Path 1-3-5-6-7	$T = 6 + 4 + 5 + 3 = 18$
ii	Path 1-2-5-6-7	$T = 2 + 7 + 5 + 3 = 17$
iii	Path 1-4-7	$T = 5 + 6 = 11$

Since path 1-3-5-6-7 has the longest duration, it is the critical path of the network and shown by dotted line.
Hence,
The expected duration of the critical path is 18 days.

SOL 11.22 Option (C) is correct.
The critical path is 1-3-5-6-7
Variance along this critical path is,

$$\sigma^2 = \sigma^2_{1-3} + \sigma^2_{3-5} + \sigma^2_{5-6} + \sigma^2_{6-7}$$
$$= \frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{1}{9} = \frac{7}{9}$$

We know,

$$\text{Standard deviation} = \sqrt{\text{Variance}\,(\sigma^2)}$$
$$= \sqrt{\frac{7}{9}} = 0.88$$

The most appropriate answer is 0.77.

SOL 11.23 Option (C) is correct.
The most common distribution found in queuing problems is poisson distribution. This is used in single-channel queuing problems for random arrivals where the service time is exponentially distributed.
Probability of n arrivals in time t

$$P = \frac{(\lambda T)^n \cdot e^{-\lambda T}}{n!} \qquad \text{where } n = 0, 1, 2.......$$

So, Probability density function of inter arrival time (time interval between two consecutive arrivals)

$$f(t) = \lambda \cdot e^{-\lambda t}$$

SOL 11.24 Option (A) is correct.

Total inventory cost will be minimum, when the holding cost is minimum. Now, from the Johnson's algorithm, holding cost will be minimum, when we process the least time consuming job first. From this next job can be started as soon as possible.

Now, arrange the jobs in the manner of least processing time.

T-S-Q-R-P or T-Q-S-R-P (because job Q and S have same processing time).

SOL 11.25 Option (D) is correct.

In a transportation problem with m origins and n destinations, if a basic feasible solution has less than $m+n-1$ allocations (occupied cells), the problem is said to be a degenerate transportation problem.

So, the basic condition for the solution to be optimal without degeneracy is.

$$\text{Number of allocations} = m+n-1$$

SOL 11.26 Option (D) is correct.

Here $F_1(t)$ & $F_2(t)$ = Forecastings

m_1 & m_2 = Number of weeks

A higher value of m results in better smoothing. Since here $m_1 > m_2$ the weightage of the latest demand would be more in $F_2(t)$.

Hence, $F_2(t)$ will attain the value of d_2 before $F_1(t)$.

SOL 11.27 Option (C) is correct.

There are two paths to reach from node P to node G.

(i) Path P-Q-G (ii) Path P-R-G

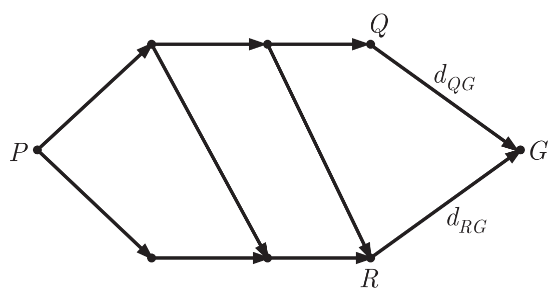
For Path P-Q-G,

Length of the path $S_G = S_Q + d_{QG}$

For path P-R-G,

Length of the path $S_G = S_R + d_{RG}$

So, shortest path $S_G = \text{Min}\{S_Q + d_{QG}, S_R + d_{RG}\}$



SOL 11.28 Option (C) is correct.

From the product structure we see that 2 piece of R is required in production of 1 piece P.

So, demand of R is double of P

Week	Demand (P)	Demand (R)	Inventory level I = Production – Demand
1	1000	2000	$R - 2000$
2	1000	2000	$2R - 4000$
3	1000	2000	$3R - 6000$
4	1000	2000	$4R - 8000$
5	1200	2400	$5R - 10400$
6	1200	2400	$6R - 12800$

We know that for a production system with bottleneck the inventory level should be more than zero.

So,

$$6R - 12800 \geq 0$$

For minimum inventory

$$6R - 12800 = 0$$
$$6R = 12800$$
$$R = 2133$$
$$\simeq 2200$$

Hence, the smallest capacity that will ensure a feasible production plan up to week 6 is 2200.

SOL 11.29 Option (B) is correct.

The LP has an optimal solution that is not unique, because zero has appeared in the non-basic variable (x and y) column, in optimal solution.

SOL 11.30 Option (A) is correct.

The general form of LP is

$$\text{Max } Z = CX$$

Subject to $AX \leq B$

And dual of above LP is represented by

$$\text{Min } Z = B^T Y$$

Subject to $A^T Y \geq C^T$

So, the dual is $\text{Min } 6u + 6v$

Subject to $3u + 2v \geq 4$

$$2u + 3v \geq 6$$
$$u, v \geq 0$$

SOL 11.31 Option (B) is correct.

We have to make a table from the given data.

Month	Production (Pieces)		Demand	Excess or short form (pieces)	
	In regular time	In over time		Regular	Total
1	100	20	90	10	$10 + 20 = 30$
2	100	20	130	-30	$-30 + 20 = -10$
3	80	40	110	-30	$-30 + 40 = 10$

From the table,

For 1st month there is no need to overtime, because demand is $90\,\text{units}$ and regular time production is $100\,\text{units}$, therefore $10\,\text{units}$ are excess in amount. For 2nd month the demand is $130\,\text{unit}$ and production capacity with overtime is $100 + 20 = 120\,\text{units}$, therefore $10\,\text{units}$ $(130 - 120 = 10)$ are short in amount, which is fulfilled by $10\,\text{units}$ excess of 1st month. So at the end of 2nd month there is no inventory.

Now for the 3rd month demand is $110\,\text{units}$ and regular time production is $80\,\text{units}$. So remaining $110 - 80 = 30\,\text{units}$ are produced in overtime to fulfill the demand for minimum cost of plan.

SOL 11.32 Option (D) is correct.

Total annual cost = Annual holding cost + Annual ordering cost

Maximum level of inventory $N = 100$

So, Average inventory $= \frac{N}{2} = 50$

Inventory carrying cost C_h = Rs. 10 per item per month

$= \text{Rs.}\,10 \times 12$ per item per year

$= \text{Rs.}\,120$ per item per year

So, Annual holding cost $= \frac{N}{2} \times C_h$

$C_{hA} = 50 \times 120$

$= \text{Rs.}6000$ item per year

And, Ordering cost $C_o = 100$ per order

Number of orders in a year $= \frac{12}{1.5}$ order

$$= 8 \text{ order}$$

So, Annual ordering cost C_{oA} = ordering cost per order × no. of orders

$$= 100 \times 8$$

$$= \text{Rs.}800 \text{ per order}$$

Hence,

$$\text{Total Annual cost} = 6000 + 800$$

$$= \text{Rs.}6800$$

SOL 11.33 Option (B) is correct.

Given :

Number of items produced per moth

$$K = 1000 \text{ per month}$$

Number of items required per month

$$R = 500 \text{ per month}$$

$$\text{Lot size } q_0 = 1000$$

When backlog is not allowed, the maximum inventory level is given by,

$$I_m = \frac{K - R}{K} \times q_o = \frac{1000 - 500}{1000} \times 1000 = 500$$

SOL 11.34 Option (B) is correct.

Given :

$$C_h = \text{Rs. 1 per item per week}$$

$$C_o = \text{Rs. 100 per order}$$

Requirements = 50 - 0 - 15 - 20 - 20

Total cost is the cost of carrying inventory and cost of placing order.

Case (I) Only one order of 105 units is placed at starting.

Weeks	Quantity			Cost		
	Inventory	Used	Carried forward	Order	Carrying	Total
1.	105 (ordered)	50	55	100	55	155
2.	55	0	55	0	55	55
3.	55	15	40	0	40	40
4.	40	20	20	0	20	20
5.	20	20	0	0	0	0

Total cost of plan = 155 + 55 + 40 + 20 = 270 Rs.

Case (II) Now order is placed two times, 50 units at starting and 55 units after 2[nd] week.

Weeks	Quantity			Cost		
	Inventory	Used	Carried forward	Ordering Rs.	Carrying Rs.	Total Rs.
1.	50 (ordered)	50	0	100	0	100
2.	0	0	0	0	0	0
3.	55 (ordered)	15	40	100	40	140
4.	40	20	20	0	20	20
5.	20	20	0	0	0	0

Total cost of plan $= 100 + 140 + 20 = 260$ Rs.

Case (III) The order is placed two times, 65 units at starting and 40 units after 3rd week.

Weeks	Quantity			Cost		
	Inventory	Used	Carried forward	Ordering Rs.	Carrying Rs.	Total Rs.
1.	65 (ordered)	50	15	100	15	115
2.	15	0	15	0	15	15
3.	15	15	0	0	0	0
4.	40 (ordered)	20	20	100	20	120
5.	20	20	0	0	0	0

Total cost of plan $= 115 + 15 + 120 = 250$ Rs.

Case (IV) Now again order is placed two times, 85 units at starting and 20 units after 4thweek.

Weeks	Quantity			Cost		
	Inventory	Used	Carried forward	Order	Carrying	Total
1.	85 (ordered)	50	35	100	35	135
2.	35	0	35	0	35	35
3.	35	15	20	0	20	20
4.	20	20	0	0	0	0
5.	20 (ordered)	20	0	100	0	100

Total cost of plan $= 135 + 35 + 20 + 100 = 290$ Rs.
So, The cost of plan is least in case (III) & it is 250 Rs.

SOL 11.35 Option (B) is correct.

Given :

$$\lambda = 8 \text{ per hour}$$
$$\mu = 6 \text{ min per customer}$$
$$= \frac{60}{6} \text{ customer/hours} = 10 \text{ customer/hour}$$

We know, for exponentially distributed service time.
Average number of customers in the queue.

$$L_q = \frac{\lambda}{\mu} \times \frac{\lambda}{(\mu - \lambda)} = \frac{8}{10} \times \frac{8}{(10-8)} = 3.2$$

SOL 11.36 Option (C) is correct.

MRP (Material Requirement Planning) :
MRP function is a computational technique with the help of which the master schedule for end products is converted into a detailed schedule for raw materials and components used in the end product.
Input to MRP
(i) Master production schedule.
(ii) The bill of material
(iii) Inventory records relating to raw materials.

SOL 11.37 Option (B) is correct.

First finding the sequence of jobs, which are entering in the machine. The solution procedure is described below :
By examining the rows, the smallest machining time of 6 hours on machine $M2$. Then scheduled Job P last for machine $M2$

					P

After entering this value, the next smallest time of 7 hours for job U on machine $M2$. Thus we schedule job U second last for machine $M2$ as shown below

				U	P

After entering this value, the next smallest time of 8 hours for job R on machine $M1$. Thus we schedule job R first as shown below.

R				U	P

After entering this value the next smallest time of 11 hours for job T on machine $M1$. Thus we schedule job T after the job R.

R	T			U	P

After this the next smallest time of 19 hours for job Q on machine $M2$. Thus schedule job Q left to the U and remaining job in the blank block.
Now the optimal sequence as :

R	T	S	Q	U	P

Then calculating the elapsed time corresponding to the optimal sequence, using the individual processing time given in the problem.
The detailed are shown in table.

Jobs	$M1$		$M2$	
	In	Out	In	Out
R	0	8	8	$8+13=21$
T	8	$8+11=19$	21	$21+14=35$
S	19	$19+27=46$	46	$46+20=66$
Q	46	$46+32=78$	78	$78+19=97$
U	78	$78+16=94$	97	$97+7=104$
P	94	$94+15=109$	109	$109+6=115$

We can see from the table that all the operations (on machine 1st and machine 2nd) complete in 115 hours. So the optimal make-span of the shop is 115 hours.

SOL 11.38 Option (C) is correct.

Given : $D = 2500$ units per year

$C_o =$ Rs. 100 per order

$C_h = 25\%$ of unit price

Case (I) : When order quantity is less than 500 units.

Then, Unit price $= 10$ Rs.

and $C_h = 25\%$ of $10 = 2.5$ Rs.

$$EOQ = \sqrt{\frac{2C_0D}{C_h}} = \sqrt{\frac{2\times 100\times 2500}{2.5}}$$

$$Q = 447.21 \simeq 447 \text{ units}$$

$$\text{Total cost} = D \times \text{unit cost} + \frac{Q}{2} \times c_h + \frac{D}{Q} \times c_o$$

$$= 2500 \times 10 + \frac{447}{2} \times 2.5 + \frac{2500}{447} \times 100$$

$$= 25000 + 558.75 + 559.75 = 26118 \text{ Rs.}$$

Case (II) : when order Quantity is 500 units. Then unit prize $= 9$ Rs.

$$\text{and } c_h = 25\% \text{ of } 9 = 2.25 \text{ Rs.}$$

$$Q = 500 \text{ units}$$

$$\text{Total cost} = 2500 \times 9 + \frac{500}{2} \times 2.25 + \frac{2500}{500} \times 100$$

$$= 22500 + 562.5 + 500 = 23562.5 \text{ Rs.}$$

So, we may conclude from both cases that the optimum order quantity must be equal to 500 units.

SOL 11.39 Option (C) is correct.

Given, In figure

	$S1$	$S2$	$S3$
P	110	120	130
Q	115	140	140
R	125	145	165

Step (I) : Reduce the matrix :

In the effectiveness matrix, subtract the minimum element of each row from all the element of that row. The resulting matrix will have at least one zero element in each row.

	$S1$	$S2$	$S3$
P	0	10	20
Q	0	25	25
R	0	20	40

Step (II) : Mark the column that do not have zero element. Now substract the minimum element of each such column for all the elements of that column.

	$S1$	$S2$	$S3$
P	0	0	0
Q	0	15	5
R	0	10	20

Step (III) : Check whether an optimal assignment can be made in the reduced matrix or not.

For this, Examine rows successively until a row with exactly one unmarked zero is obtained. Making square (☐) around it, cross (×) all other zeros in the same column as they will not be considered for making any more assignment in that column. Proceed in this way until all rows have been examined.

	$S1$	$S2$	$S3$
P	[0]	~~0~~	~~0~~
Q	~~0~~	15	5
R	~~0~~	10	20

In this there is not one assignment in each row and in each column.

Step (IV) : Find the minimum number of lines crossing all zeros. This consists of following substep

(A) Right marked ( ) the rows that do not have assignment.

(B) Right marked ( ) the column that have zeros in marked column (not already marked).

(C) Draw straight lines through all unmarked rows and marked columns.

	$S1$	$S2$	$S3$	
P	[0]	~~0~~	~~0~~	
Q	~~0~~	15	5	✓
R	~~0~~	10	20	✓
	✓			

Step (V) : Now take smallest element & add, where two lines intersect.

No change, where single line & subtract this where no lines in the block.

	$S1$	$S2$	$S3$	
P	5	[0]	~~0~~	
Q	~~0~~	10	[0]	✓
R	[0]	5	15	✓
	✓			

So, minimum cost is $= 120 + 140 + 125 = 385$

SOL 11.40 Option (A) is correct.

Profit per unit sold $= 90 - 70 = 20$ Rs.

Loss per unit unsold item $= 70 - 50 = 20$ Rs.

Now consider all the options :

Cases	Units in stock	Unit sold (Demand)	Profit	Probability	Total profit
Option (D)	2	2	$2\times20=40$	0.1	4
Option (C)	3	2	$2\times20-1\times20=20$	0.1	2
	3	3	$3\times20=60$	0.35	21
					23
Option (B)	4	2	$2\times20-2\times20=0$	0	0
	4	3	$3\times20-1\times20=40$	0.35	14
	4	4	$4\times20=80$	0.35	28
					42
Option (A)	5	2	$2\times20-3\times20=-20$	0.10	-2
	5	3	$3\times20-2\times20=20$	0.35	7
	5	4	$4\times20-1\times20=60$	0.35	21
	5	5	$5\times20=100$	0.20	20
					46

Thus, For stock level of 5 units, profit is maximum.

SOL 11.41 Option (C) is correct.

$$\text{Total time used} = 7+9+7+10+9+6 = 48\text{ min}$$

$$\text{Number of work stations} = 6$$

Maximum time per work station (cycle time) $= 10$ min

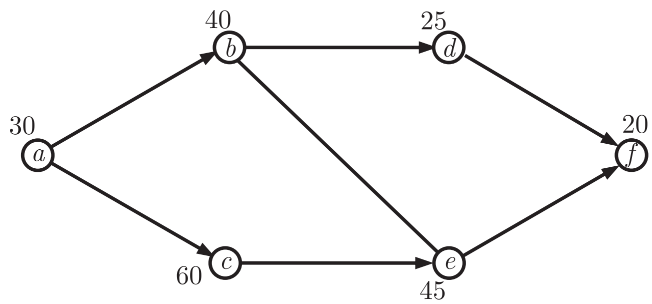
We know,

$$\text{Line efficiency } \eta_L = \frac{\text{Total time used}}{\text{Number of work stations} \times \text{cycle time}}$$

$$\eta_L = \frac{48}{6\times10} = 0.8 = 80\%$$

SOL 11.42 Option (D) is correct.

We have to make a network diagram from the given data.



For simple projects, the critical path can be determined quite quickly by enumerating all paths and evaluating the time required to complete each. There are three paths between a and f. The total time along each path is

(i) For path a-b-d-f

$$T_{abdf} = 30 + 40 + 25 + 20 = 115 \text{ days}$$

(ii) For path a-c-e-f

$$T_{acef} = 30 + 60 + 45 + 20 = 155 \text{ days}$$

(iii) For path a-b-e-f

$$T_{abef} = 30 + 40 + 45 + 20 = 135 \text{ days}$$

Now, path a-c-e-f be the critical path time or maximum excepted completion time $T = 155$ days

SOL 11.43 Option (A) is correct.

The critical path of the network is a-c-e-f.

Now, for variance.

Task	Variance (days^2)
a	25
c	81
e	36
f	9

Total variance for the critical path

$$V_{critical} = 25 + 81 + 36 + 9$$

$$= 151 \text{ days}^2$$

We know the standard deviation of critical path is

$$\sigma = \sqrt{V_{critical}} = \sqrt{151} \text{ days}$$

SOL 11.44 Option (D) is correct.

In operation process chart an assembly activity is represented by the symbol O

SOL 11.45 Option (C) is correct.

Gives :

Sales of product during four years were 860, 880, 870 and 890 units.

Forecast for the fourth year $u_4 = 876$

Forecast for the fifth year, using simple exponential smoothing, is equal to the forecast using a three period moving average.

So, $$u_5 = \frac{1}{3}(880 + 870 + 890)$$

$$u_5 = 880 \text{ unit}$$

By the exponential smoothing method.

$$u_5 = u_4 + \alpha(x_4 - u_4)$$
$$880 = 876 + \alpha(890 - 876)$$
$$4 = \alpha(14)$$
$$\alpha = \frac{4}{14} = \frac{2}{7}$$

SOL 11.46 Option (A) is correct.

Given : $\lambda = 4/\text{hour}, \quad \mu = 4/\text{hour}$

The sum of probability $\sum_{n=0}^{n=10} P_n = 1$ $\qquad n = 10$

$$P_0 + P_1 + P_2 ..... + P_{10} = 1$$

In the term of traffic intensity $\rho = \frac{\lambda}{\mu} \quad \Rightarrow \rho = \frac{4}{4} = 1$

So,

$$P_0 + \rho P_0 + \rho^2 P_0 + \rho^3 P_0 + ...\rho^{10} P_0 = 1 \qquad P_1 = \rho P_0, P_2 = \rho^2 P_0 \text{ and so on}$$
$$P_0(1 + 1 + 1 + ...) = 1$$
$$P_0 \times 11 = 1$$
$$P_0 = \frac{1}{11}$$

Hence, the probability that a person who comes in leaves without joining the queue is,

$$P_{11} = \rho^{11} \cdot P_0$$
$$P_1 = 1^{11} \times \frac{1}{11} = \frac{1}{11}$$

SOL 11.47 Option (B) is correct.

For economic point of view, we should calculate the total cost for all the four processes.

$$\text{Total cost} = \text{Fixed cost} + \text{Variable cost} \times \text{Number of piece}$$

For process (I) :

$$\text{Fixed cost} = 20 \text{ Rs.}$$
$$\text{Variable cost} = 3 \text{ Rs. per piece}$$
$$\text{Number of pieces} = 100$$
$$\text{Total cost} = 20 + 3 \times 100 = 320 \text{ Rs.}$$

For process (II) :

$$\text{Total cost} = 50 + 1 \times 100 = 150 \text{ Rs.}$$

For process (III) :

$$\text{Total cost} = 40 + 2 \times 100 = 240 \text{ Rs.}$$

For process (IV) :

$$\text{Total cost} = 10 + 4 \times 100 = 410 \text{ Rs.}$$

Now, we can see that total cost is minimum for process (II). So process (II) should choose for economic point of view.

SOL 11.48 Option (A) is correct.

Given : $\text{Rating factor} = 120\%$

$$\text{Actual time } T_{actual} = 8 \text{ min}$$
$$\text{Normal time } T_{normal} = \text{actual time} \times \text{Rating factor}$$
$$T_{normal} = 8 \times \frac{120}{100} = 9.6 \text{ min}$$

10% allowance is allowed for this operation.

So, standard time,

$$T_{standard} = \frac{T_{normal}}{1 - \frac{10}{100}} = \frac{9.6}{0.9} = 10.67 \text{ min}$$

Hence, standard production rate of the weld joint

$$= \frac{8 \times 60}{10.67} = 45 \text{ units}$$

SOL 11.49 Option (D) is correct.

The expected value of the lead time demand

$$= 80 \times 0.20 + 100 \times 0.25 + 120 \times 0.30 + 140 \times 0.25$$
$$= 112$$

Reorder level is 1.25 time the lead time demand.

So, reorder value $= 1.25 \times 112 = 140$

Here both the maximum demand or the reorder value are equal.

Hence, service level $= 100\%$

SOL 11.50 Option (C) is correct.

The 3 activity need to be crashed to reduce the project duration by 1 day.

SOL 11.51 Option (C) is correct.

First we have to make a transportation model from the given details.

Warehouse \ Factories	S_1	S_2	Supply
D_1			20 to 40 units
D_2			≥ 20
Demand	50	40	

We know,

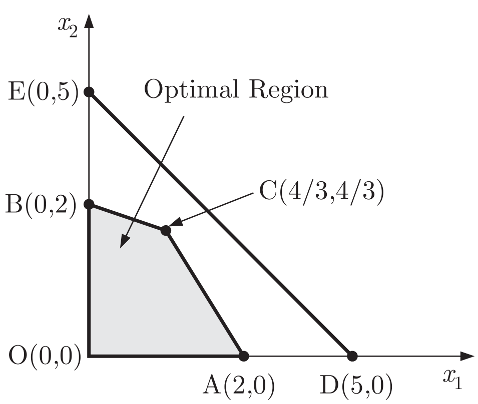
Basic condition for transportation model is balanced, if it contains no more than $m+n-1$ non-negative allocations, where m is the number of rows and n is the number of columns of the transportation problem.

$$\text{So, Number of supply point (allocations)} = m+n-1$$
$$= 2+2-1 = 3$$
$$\text{Number of demand points} = 4\,(\text{No. of blank blocks})$$
$$\text{Total supply or demand} = 50+40 = 90$$

SOL 11.52 Option (B) is correct.

Given : Objective function $Z = X_1 + X_2$

From the given corners we have to make a graph for X_1 and X_2



From the graph, the constraint $X_1 + X_2 \leq 5$ has no effect on optimal region.

Now, checking for optimal solution

	Point	$Z = X_1 + X_2$
(i)	$O(0,0)$	$Z = 0$
(ii)	$A(2,0)$	$Z = 2+0 = 2$
(iii)	$B(0,2)$	$Z = 0+2 = 2$
(iv)	$C(4/3,4/3)$	$Z = 4/3+4/3 = 8/3$

The optimal solution occurs at point $C(4/3, 4/3)$

SOL 11.53 Option (D) is correct.

We know,

The inequality constraints are changed to equality constraints by adding or subtracting a non-negative variable from the left-hand sides of such constraints. These variable is called slack variables or simply slacks.

They are added if the constraints are $(\leq)$ and subtracted if the constraints are $(\geq)$. These variables can remain positive throughout the process of solution and their values in the optimal solution given useful information about the problem.

Hence, Optimum dual variables are v_1 and v_2.

SOL 11.54 Option (B) is correct.

PERT (Programme Evaluation and Review Technique) uses even oriented network in which successive events are joined by arrows.

Float is the difference between the maximum time available to perform the activity and the activity duration. In PERT analysis a critical activity has zero float.

SOL 11.55 Option (C) is correct.

Given :

$$\text{Forecast sales for December } u_t = 25$$
$$\text{Actual sales for December } X_t = 20$$
$$\text{Exponential smoothing constant } \alpha = 0.2$$

We know that, Forecast sales for January is given by

$$u_{t+1} = u_t + \alpha[X_t - u_t]$$
$$= 25 + 0.2(20 - 25)$$
$$= 25 + 0.2 \times (-5) = 25 - 1 = 24$$

Hence, Forecast sales for January 2003 would be 24.

SOL 11.56 Option (C) is correct.

For product P : $D = 100 \text{ units}, C_o = 50 \text{ Rs./order}, C_h = 4 \text{ Rs./unit/year}$

Economic order quantity (EOQ) for product P,

$$(\text{EOQ})_P = \sqrt{\frac{2C_oD}{C_h}}$$

$$(\text{EOQ})_P = \sqrt{\frac{2 \times 50 \times 100}{4}} = \sqrt{2500} = 50 \qquad ...(i)$$

For product Q :

$$D = 400 \text{ Units } \; C_o = 50 \text{ Rs. order}, \; C_h = 1 \text{ Rs. Unit/year}$$

EOQ For Product Q,

$$(\text{EOQ})_Q = \sqrt{\frac{2C_oD}{C_h}}$$

$$= \sqrt{\frac{2\times 50\times 400}{1}} = \sqrt{40000} = 200 \qquad \text{...(ii)}$$

From equation (i) & (ii),

$$\frac{(\text{EOQ})_P}{(\text{EOQ})_Q} = \frac{50}{200} = \frac{1}{4}$$

$$(\text{EOQ})_P : (\text{EOQ})_Q = 1:4$$

SOL 11.57 Option (D) is correct.

Let, The standard machine tool produce x_1 number of components.

For standard machine tool,

$$\text{Total cost} = \text{Fixed cost} + \text{Variable cost}\times\text{Number. of components}$$

$$(\text{TC})_{SMT} = \left[\frac{30}{60} + \frac{22}{60}\times x_1\right]\times 200$$

$$= \frac{30}{60}\times 200 + \frac{22}{60}\times x_1\times 200 = 100 + \frac{220}{3}x_1 \qquad \text{...(i)}$$

If automatic machine tool produce x_2 Number of components, then the total cost for automatic machine tool is

$$(\text{TC})_{AMT} = \left(2 + \frac{5}{60}x_2\right)800$$

$$= 1600 + \frac{200}{3}x_2 \qquad \text{....(ii)}$$

Let, at the breakeven production batch size is x and at breakeven point.

$$(\text{TC})_{SMT} = (\text{TC})_{AMT}$$

$$100 + \frac{220x}{3} = 1600 + \frac{200x}{3}$$

$$\frac{220x}{3} - \frac{200x}{3} = 1600 - 100$$

$$\frac{20x}{3} = 1500$$

$$x = \frac{1500\times 3}{20} = 225$$

So, breakeven production batch size is 225.

SOL 11.58 Option (D) is correct.

Given :

$$\text{Total time } T = 16\text{ hours} = 16\times 60 = 960\text{ min}$$

Actual working time was 90% of total time

So, $\quad$ Actual time, $T_{actual} = 90\%$ of 960

$$= \frac{90}{100} \times 960, \; T_{actual} = 864 \text{ min}$$

Performance rating was 120 percent.

So, Normal time, $T_{normal} = 120\% \text{ of } 864$

$$= \frac{120}{100} \times 864 = 1036.8 \text{ min}$$

Allowance is 20% of the total available time.

So total standard time $T_{standard} = \dfrac{T_{normal}}{\left(1 - \frac{20}{100}\right)} = \dfrac{1036.8}{1 - 0.2} = \dfrac{1036.8}{0.8}$

$$= 1296 \text{ min}$$

Number of joints soldered, $N = 108$

Hence,

$$\text{Standard time for operation} = \frac{1296}{108} = 12 \text{ min}$$

SOL 11.59 Option (A) is correct.

Given :

Number of units produced in a day $= 80$ units

Working hours in a day $= 8$ hours

Now, Time taken to produce one unit is,

$$T = \frac{8}{80} \times 60 = 6 \text{ min}$$

Activity	Standard time (min)	No. of work stations $(S.T/T)$
M. Mechanical assembly	12	$12/6 = 2$
E. Electric wiring	16	$16/6 = 2.666 = 3$
T. Test	3	$3/6 = 0.5 = 1$

Number of work stations are the whole numbers, not the fractions.

So, number of work stations required for the activities M, E and T would be 2, 3 and 1, respectively.

SOL 11.60 Option (A) is correct.

Given :

Mean arrival rate $\lambda = 3$ per day

Mean service rate $\mu = 6$ per day

We know that, for first come first serve queue.

Mean waiting time of an arrival,

$$t = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{3}{6(6-3)} = \frac{1}{6}\text{ day}$$

SOL 11.61 Option (C) is correct.

Given : $D = 1000$ units, $C_o = 100/\text{order}$, $C_h = 100$ unit/year

$C_s = 400$ Rs.

We know that, optimum level of stock out will be,

$$\text{S.O} = \sqrt{\frac{2DC_o}{C_h}} \times \sqrt{\frac{C_s}{C_h + C_s}}$$

$$\text{S.O} = \sqrt{\frac{2 \times 1000 \times 100}{100}} \times \sqrt{\frac{400}{100 + 400}}$$

$$= 44.72 \times 0.895 = 40$$

SOL 11.62 Option (A) is correct.

Solve this problem, by the linear programming model.

We have to make the constraints from the given conditions.

For production conditions

$$P + 2Q \le 2000 \quad \text{...(i)}$$

For raw material

$$P + Q \le 1500 \quad \text{...(ii)}$$

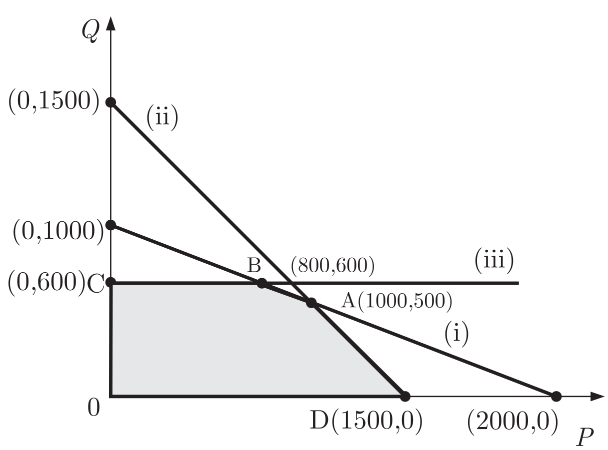
For electric switch

$$Q \le 600 \quad \text{...(iii)}$$

For maximization of profit, objective function

$$Z = 3P + 5Q \quad \text{...(iv)}$$

From the equations (i), (ii) & (iii), draw a graph for toy P and Q



Line (i) and line (ii) intersects at point A, we have to calculate the intersection point.

$$P + 2Q = 2000$$

$$P + Q = 1500$$

After solving there equations, we get $A(1000, 500)$

For point B,

$$P + 2Q = 2000$$
$$Q = 600$$
$$P = 2000 - 1200 = 800$$

So, $B(800, 600)$

Here shaded area shows the area bounded by the three line equations (common area)

This shaded area have five vertices.

	Vertices	Profit $Z = 3P + 5Q$
(i)	0(0, 0)	$Z = 0$
(ii)	$A(1000, 500)$	$Z = 3000 + 2500 = 5500$
(iii)	$B(800, 600)$	$Z = 2400 + 3000 = 5400$
(iv)	$C(0, 600)$	$Z = 3000$
(v)	$D(1500, 0)$	$Z = 4500$

So, for maximization of profit

$$P = 1000 \qquad \text{from point(ii)}$$
$$Q = 500$$

SOL 11.63 Option (A) is correct.

The symbol used for transport in work study is given by, $\Rightarrow$

SOL 11.64 Option (A) is correct.

Given : For machine $M1$:

$$\text{Fixed cost} = 100 \text{ Rs.}$$
$$\text{Variable cost} = 2 \text{ Rs. per piece}$$

For machine $M2$:

$$\text{Fixed cost} = 200 \text{ Rs.}$$
$$\text{Variable cost} = 1 \text{ Rs. per piece}$$

Let, n number of units are produced per machine, when both the machines are to be used concurrently.

We know that,

$$\text{Total cost} = \text{Fixed cost} + \text{Variable cost} \times \text{Number of units}$$

For $M1$, Total cost of production $= 100 + 2 \times n$

For $M2$, Total cost of production $= 200 + n$

Hence,

Total cost of production on machine $M1 \& M2$ is

$$= 100 + 2n + 200 + n = 300 + 3n$$

We know, Breakeven point is the point, where total cost of production is equal to the total sales price.

Assuming that Number of units produced are less than 800 units and selling price is Rs. 3.50 per unit.

So at breakeven point,

$$300 + 3n = 3.50(n + n)$$

$$300 + 3n = 3.50 \times 2n$$

$$300 = 4n$$

$$n = \frac{300}{4} = 75 \text{ units}$$

SOL 11.65 Option (C) is correct.

Warden checks the student 11 occasions a day during the study hours over a period of 10 days.

So, Total number of observations in 10 days.

$$= 11 \times 10 = 110 \text{ observations}$$

Study hours as 8.00 pm to 10.30 pm.

So, total study hours in 10 days

$$= 2.5 \times 10 = 25 \text{ hours.}$$

Number of occasions when student studying $= 71$

So, Probability of studying

$$P = \frac{\text{No. of observations when student studying}}{\text{Total observations}} = \frac{71}{110} = 0.645$$

Hence,

Minimum hours of his study during 10 day period is

$$T = P \times \text{Total study hours in 10 days} = 0.645 \times 25 = 16.1 \text{ hours}$$

SOL 11.66 Option (B) is correct.

We know, from the exponential and smoothing average method, the exponential smoothed average $u_{(t+1)}$ which is the forecast for the next period $(t+1)$ is given by

$$u_{(t+1)} = \alpha u_t + \alpha(1-\alpha)u_{t-1} + ...\alpha(1-\alpha)^n u_{t-n} + ...\infty$$

Now, for sales of the fifth month put $t = 4$ in the above equation,

So, $$u_5 = \alpha u_4 + \alpha(1-\alpha)u_3 + \alpha(1-\alpha)^2 u_2 + \alpha(1-\alpha)^3 u_1$$

where u_1, u_2, u_3 and u_4 are $70, 68, 82,$ and 95 respectively and $\alpha = 0.4$

Hence $$u_5 = 0.4 \times 95 + 0.4(1-0.4)82 + 0.4(1-0.4)^2 \times 68 + 0.4(1-0.4)^3 \times 70$$

$$u_5 = 38 + 19.68 + 9.792 + 6.048 = 73.52$$

SOL 11.67 Option (C) is correct.

Given :

$$D = 800000 \text{ per annum}$$
$$C_o = 1200 \text{ Rs.}$$
$$C_h = 120 \text{ per piece per annum}$$

We know that,

$$\text{Economic order quantity } (EOQ) = N = \sqrt{\frac{2C_oD}{C_h}}$$

$$N = \sqrt{\frac{2 \times 1200 \times 800000}{120}} = \sqrt{16 \times 10^6}$$
$$= 4 \times 10^3 = 4000$$

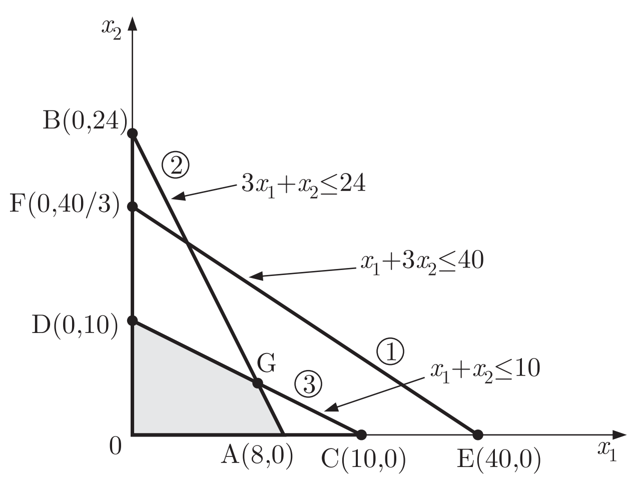
SOL 11.68 Option (A) is correct.

Given : Objective function, $Z = 2x_1 + 5x_2$

and
$$x_1 + 3x_2 \leq 40$$
$$3x_1 + x_2 \leq 24$$
$$x_1 + x_2 \leq 10$$
$$x_1 > 0$$
$$x_2 > 0$$

First we have to make a graph from the given constraints. For draw the graph, substitute alternatively $x_1 \& x_2$ equal to zero in each constraints to find the point on the $x_1 \& x_2$ axis.

Now shaded area shows the common area. Note that the constraint $x_1 + 3x_2 \leq 40$ does not affect the solution space and it is the redundant constraint. Finding the coordinates of point G by the equations.



$$3x_1 + x_2 = 24$$

$$x_1 + x_2 = 10$$

Subtract these equations,

$$(3x_1 - x_1) + 0 = 24 - 10$$

$$2x_1 = 14 \Rightarrow x_1 = 7$$

$$x_2 = 10 - x_1 = 10 - 7 = 3$$

So, point $G(7,3)$

So, maximum profit which can meet the constraints at $G(7,3)$ is

$$Z_{max} = 2 \times 7 + 5 \times 3 = 14 + 15 = 29$$

SOL 11.69 Option (C) is correct.

The various path and their duration are :-

Path	Duration (days)
A-D-L	$2 + 10 + 3 = 15$
A-E-G-L	$2 + 5 + 6 + 3 = 16$
A-E-H	$2 + 5 + 10 = 17$
B-H	$8 + 10 = 18$
C-F-K-M	$4 + 9 + 3 + 8 = 24$
C-F-H	$4 + 9 + 10 = 23$
A-E-K-M	$2 + 5 + 3 + 8 = 18$
B-G-L	$8 + 6 + 3 = 17$
B-K-M	$8 + 3 + 8 = 19$
C-F-G-L	$4 + 9 + 6 + 3 = 22$

Here maximum time along the path C-F-K-M. So, it is a critical path and project can be completed in 24 days.

SOL 11.70 Option (A) is correct.

The principal of motion economy are used while conduction a method study on an operation.

Method study consist of the sequence of operation, which are performing on a machine. From the sequencing, the idle time of the machine reduced to a certain amount and the operation becomes faster and smooth. Also the productivity of the plant increases by the principle of motion economy.

SOL 11.71 Option (B) is correct.

$$\text{Standard Time} = \text{Normal time} + \text{Allowance}$$

SOL 11.72 Option (B) is correct.

Percentage Error $E = 20\%$ or 0.20

Standard deviation $S = \sqrt{\dfrac{E \times (1-E)}{n}}$

where $n =$ No. of observation

$$S = \sqrt{\frac{0.20(1-0.20)}{100}} = 0.04$$

For 95% confidence level, $\sigma = \pm 2$

So, upper control limit UCL $= E + \sigma \times S$

$= 0.20 + 2 \times 0.04 = 0.28$

Lower control Limit LCL $= E - \sigma \times S$

$= 0.20 - 2 \times 0.04 = 0.12$

Hence 95% confidence interval of this estimate is (0.12, 0.28)

SOL 11.73 Option (D) is correct.

Given : $C_o = 200$ Rs

$D = 4000$ units per annum

$C_h = 10\%$ of $100 = 10$ Rs per annum

The Economic order quantity is,

$$EOQ = \sqrt{\frac{2C_oD}{C_h}} = \sqrt{\frac{2 \times 200 \times 4000}{10}} = 400 \text{ unit}$$

SOL 11.74 Option (C) is correct.

Given :

Average time between arrivals $= 10$ min

Mean arrival rate (Number of arrivals per unit time) $\lambda = 6$ per hour

Average time between call $= 3$ min

Mean service rate $\mu = \dfrac{60}{3} = 20$ per hour

So, the probability that an arrival does not have to wait before service is,

$$P_O = 1 - \frac{\lambda}{\mu} = 1 - \frac{6}{20} = 1 - 0.3 = 0.7$$

SOL 11.75 Option (B) is correct.

Total supply $= 50 + 40 + 60 = 150$ units

Total demand $= 20 + 30 + 10 + 50 = 110$ units

In this question, the total availability (supply) may not be equal to the total demand, i.e.,

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$$

Such problems are called unbalanced transportation problems.

Here total availability is more than the demand. So we add a dummy

destination to take up the excess capacity and the costs of shipping to this destination are set equal to zero.
So, a dummy destination of capacity 40 unit is needed.

SOL 11.76 Option (B) is correct.
In PERT analysis, a Beta distribution is assumed because it is unimodal, has non-negative end points, and is approximately symmetric.
Here three parallel paths are given. But the critical path is one with the longest time durations.
Two paths have same time duration of 12.

So, mean $= 12$

The PERT analysis has a beta (β) distribution and Standard deviation $= \sqrt{\text{variance}} = \sqrt{4} = 2$.

SOL 11.77 Option (D) is correct.
Production flow analysis (PFA) is a comprehensive method for material analysis, Part family formation, design of manufacturing cells and facility layout design. These informations are taken from the route sheet.

SOL 11.78 Option (D) is correct.
The simple moving average method can be used if the underlying demand pattern is stationary. This method include new demand data in the average after discarding some of the earlier demand data.

Let m_t = moving average at time t
y_t = demand in time t and
n = moving average period

$$m_{t+1} = \frac{y_{t+1} - y_{t-n+1}}{n}$$

SOL 11.79 Option (D) is correct.
Given : Mean cycle time $= 10$ min
The workers performing at 90% efficiency.

So, Normal time $= 10 \times \frac{90}{100} = 9$ min

Allowance $= 10\%$

$$\text{Standard time} = \text{Normal time} + \text{Allowance}$$
$$= 9 + 9 \times \frac{10}{100} = 9 + 0.9 = 9.9 \text{ min}$$

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CHAPTER 12

GENERAL APTITUDE

YEAR 2012 ONE MARK

MCQ 12.1 Choose the most appropriate alternative from the options given below to complete the following sentence :

Suresh's dog is the one ....was hurt in the stampede.

(A) that (B) which

(C) who (D) whom

MCQ 12.2 The cost function for a product in a firm is given by $5q^2$, where q is the amount of production. The firm can sell the product at a market price of Rs. 50 per unit. The number of units to be produced by the firm such that the profit maximized is

(A) 5 (B) 10

(C) 15 (D) 25

MCQ 12.3 Choose the most appropriate alternative from the options given below to complete the following sentence.

Despite several ....the mission succeeded in its attempt to resolve the conflict.

(A) attempts (B) setbacks

(C) meetings (D) delegations

MCQ 12.4 Which one of the following options is the closest in meaning to the word given below ?

Mitigate

(A) Diminish (B) Divulge

(C) Dedicate (D) Denote

MCQ 12.5 Choose the grammatically INCORRECT sentence :

(A) They gave us the money back less the service charges of Three Hundred Rupees.

(B) This country's expenditure is not less than that of Bangladesh.

(C) The committee initially asked for a funding of Fifty Lakh rupees, but later settled for a lesser sum.

(D) This country's expenditure on educational reforms is very less.

YEAR 2012 TWO MARKS

MCQ 12.6 Given the sequence of terms, $AD\ CG\ FK\ JP$, the next term is

(A) OV (B) OW

(C) PV (D) PW

MCQ 12.7 Wanted Temporary, Part-time persons for the post of Field Interviewer to conduct personal interviews to collect and collate economic data. Requirements : High School-pass, must be available for Day, Evening and Saturday work. Transportation paid, expenses reimbursed.
Which one of the following is the best inference from the above advertisement ?

(A) Gender-discriminatory

(B) Xenophobic

(C) Not designed to make the post attractive

(D)Not gender-discriminatory

MCQ 12.8 A political party order an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equations $y = 2x - 0.1x^2$ where y is the height of the arch in meters. The maximum possible height of the arch is

(A) 8 meters (B) 10 meters

(C) 12 meters (D) 14 meters

MCQ 12.9 An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable.
The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is

(A) 0.288 (B) 0.334

(C) 0.667 (D) 0.720

MCQ 12.10 Which of the following assertions are CORRECT ?

P : Adding 7 to each entry in a list adds 7 to the mean of the list

Q : Adding 7 to each entry in a list adds 7 to the standard deviation of the list

R : Doubling each entry in a list doubles the mean of the list

S : Doubling each entry in a list leaves the standard deviation of the list unchanged

(A) P, Q (B) Q, R

(C) P, R (D) R, S

YEAR 2011 ONE MARK

MCQ 12.11 Choose the word from the options given below that is most nearly opposite in meaning to the given word :

Amalgamate

(A) merge (B) split

(C) collect (D) separate

MCQ 12.12 Which of the following options is the closest in the meaning to the word below :

Inexplicable

(A) Incomprehensible (B) Indelible

(C) Inextricable (D) Infallible

MCQ 12.13 If $\log(P) = (1/2)\log(Q) = (1/3)\log(R)$, then which of the following options is TRUE ?

(A) $P^2 = Q^3R^2$ (B) $Q^2 = PR$

(C) $Q^2 = R^3P$ (D) $R = P^2Q^2$

MCQ 12.14 Choose the most appropriate word from the options given below to complete the following sentence.

In contemplated...............Singapore for my vacation but decided against it.

(A) to visit (B) having to visit

(C) visiting (D) for a visit

MCQ 12.15 Choose the most appropriate word from the options given below to complete the following sentence.

If you are trying to make a strong impression on your audience, you cannot do so by being understated, tentative or

(A) hyperbolic (B) restrained

(C) argumentative (D) indifferent

YEAR 2011 TWO MARKS

MCQ 12.16 A container originally contains 10 litres of pure spirit. From this container 1 litre of spirit is replaced with 1 litre of water. Subsequently, 1 litre of the mixture is again replaced with 1 litre of water and this processes is repeated one more time. How much spirit is now left in the container ?

(A) 7.58 litres (B) 7.84 litres

(C) 7 litres (D) 7.29 litres

MCQ 12.17 Few school curricula include a unit on how to deal with bereavement and grief, and yet all students at some point in their lives suffer from losses through death and parting.

Based on the above passage which topic would not be included in a unit on bereavement ?

(A) how to write a letter of condolence

(B) what emotional stages are passed through in the healing process

(C) what the leading causes of death are

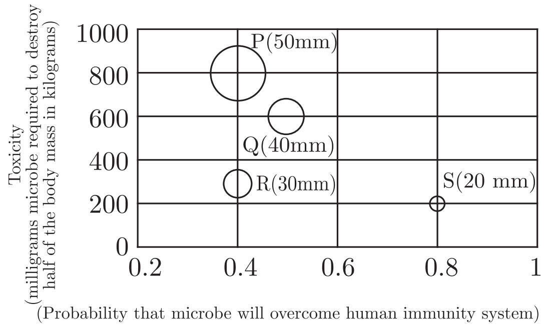
(D) how to give support to a grieving friend

MCQ 12.18 The variable cost (V) of manufacturing a product varies according to the equation $V = 4q$, where q is the quantity produced. The fixed cost (F) of production of same product reduces with q according to the equation $F = 100/q$. How many units should be produced to minimize the total cost $(V + F)$?

(A) 5 (B) 4

(C) 7 (D) 6

MCQ 12.19 P, Q, R and S are four types of dangerous microbes recently found in a human habitat. The area of each circle with its diameter printed in brackets represents the growth of a single microbe surviving human immunity system within 24 hours of entering the body. The danger to human beings varies proportionately with the toxicity, potency and growth attributed to a microbe shown in the figure below :



A pharmaceutical company is contemplating the development of a vaccine against the most dangerous microbe. Which microbe should the company target in its first attempt ?

(A) P (B) Q

(C) R (D) S

MCQ 12.20 A transporter receives the same number of orders each day. Currently, he has some pending orders (backlog) to be shipped. If he uses 7 trucks, then at the end of the 4th day he can clear all the orders. Alternatively, if he uses only 3 trucks, then all the orders are cleared at the end of the 10th day. What is the minimum number of trucks required so that there will be no pending order at the end of the 5th day ?

(A) 4 (B) 5

(C) 6 (D) 7

2010 ONE MARK

MCQ 12.21 Which of the following options is the closest in meaning to the word below ?
Circuitous

(A) Cyclic (B) Indirect

(C) Confusing (D) Crooked

MCQ 12.22 The question below consist of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.
Unemployed : Worker

(A) Fallow : Land (B) Unaware : Sleeper

(C) Wit : Jester (D) Renovated : House

MCQ 12.23 Choose the most appropriate word from the options given below to complete the following sentence :
If we manage to .... our natural resources, we would leave a better planet for our children.

(A) unhold (B) restrain

(C) cherish (D) conserve

MCQ 12.24 Choose the most appropriate word from the options given below to complete the following sentence :
His rather casual remarks on politics.................his lack of seriousness about the subject.

(A) masked (B) belied

(C) betrayed (D) suppressed

MCQ 12.25 25 persons are in a room 15 of them play hockey, 17 of them play football and 10 of them play hockey and football. Then the number of persons playing neither hockey nor football is

(A) 2 (B) 17

(C) 13 (D) 3

2010 TWO MARKS

MCQ 12.26 Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare ; and regretfully, their exist people in military establishments who think that chemical agents are useful fools for their cause.

Which of the following statements best sums up the meaning of the above passage ?

(A) Modern warfare has resulted in civil strife.

(B) Chemical agents are useful in modern warfare.

(C) Use of chemical agents in ware fare would be undesirable.

(D) People in military establishments like to use chemical agents in war.

MCQ 12.27 If $137 + 276 = 435$ how much is $731 + 672$?

(A) 534 (B) 1403

(C) 1623 (D) 1531

MCQ 12.28 5 skilled workers can build a wall in 20 days; 8 semi-skilled workers can build a wall in 25 days; 10 unskilled workers can build a wall in 30 days. If a team has 2 skilled, 6 semi-skilled and 5 unskilled workers, how long will it take to build the wall ?

(A) 20 days (B) 18 days

(C) 16 days (D) 15 days

MCQ 12.29 Given digits 2, 2, 3, 3, 3, 4, 4, 4, 4 how much distinct 4 digit numbers greater than 3000 can be formed ?

(A) 50 (B) 51

(C) 52 (D) 54

MCQ 12.30 Hari (H), Gita (G), Irfan (I) and Saira (S) are siblings (i.e. brothers and sisters.) All were born on 1st January. The age difference between any two successive siblings (that is born one after another) is less than 3 years. Given the following facts :

1. Hari's age + Gita's age > Irfan's age + Saira's age.

2. The age difference between Gita and Saira is 1 year. However, Gita is not the oldest and Saira is not the youngest.
3. There are no twins.

In what order were they born (oldest first) ?

(A) HSIG (B) SGHI

(C) IGSH (D) IHSG

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SOLUTION

YEAR 2012 ONE MARK

SOL 12.1 Option (A) is correct.
"Which" is used in a sentence when the person is unknown. But here the person means Suresh's dog is known and "that" is used in a sentence, when the person is known.
So, that will be used in this sentence.

SOL 12.2 Option (A) is correct.
Profit is given by,

$$P = \text{Selling price} - \text{Total cost of production}$$
$$= 50q - 5q^2$$

Using the principle of maxima – minima,

$$\frac{dP}{dq} = 50 - 10q = 0$$
$$q = \frac{50}{10} = 5$$

and $$\frac{d^2P}{dq^2} = -10 \text{ (maxima)}$$

So, for 5 units the profit is maximum.

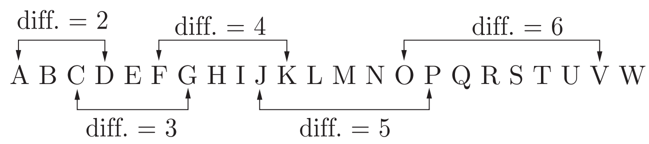
SOL 12.3 Option (B) is correct.
Despite several setbacks the mission succeeded in its attempt to resolve the conflict.

SOL 12.4 Option (A) is correct.
From the following options Diminish is the closest meaning to the Mitigate.

SOL 12.5 Option (A) is correct.
The grammatically incorrect sentence is :
(A) They gave us the money back less the service charges of three hundred rupees.

YEAR 2012 TWO MARKS

SOL 12.6 Option (A) is correct.



So, the next term is OV.

SOL 12.7 Option (D) is correct.

Not gender-discriminatory

Discriminatory involves the actual behaviors towards groups such as excluding or restricting members of one group from opportunities that are available to another group.

This given advertisement is not exclude or restrict Male or Female members from one another. Hence this is Not-gender discriminatory.

SOL 12.8 Option (B) is correct.

We have $$y = 2x - 0.1x^2 \qquad \text{...(i)}$$

Using the principle of maxima – minima,

$$\frac{dy}{dx} = 2 - 0.2x = 0$$

$$x = \frac{2}{0.2} = 10$$

And $$\frac{d^2y}{dx^2} = -0.2 \text{ (maxima)}$$

So, for maximum possible height, substitute $x = 10$ in equation (i),

$$y = 2 \times 10 - 0.1 \times (10)^2$$

$$= 20 - 10 = 10 \text{ meter}$$

SOL 12.9 Option (B) is correct.

Supplier X supplies 60% of shock absorbers, out of which 96% are reliable. So overall reliable fraction of shock absorbers from supplier X,

$$= 0.6 \times 0.96$$

$$= 0.576$$

And for supplier Y, suppliers 40% of shock absorbers, out of which 72% are reliable. So fraction of reliability $= 0.4 \times 0.72 = 0.288$.

Total fraction of reliability $= 0.576 + 0.288 = 0.864$

Hence the probability that is found to be reliable, is made by Y is,

$$= \frac{0.288}{0.288 + 0.576} = 0.334$$

SOL 12.10 Option (C) is correct.

For statement P, take three variables a, b, c

$$\text{Mean } (m) = \frac{a+b+c}{3}$$

Adding 7 to each entry,

$$m_1 = \frac{(a+7)+(b+7)(c+7)}{3}$$

$$m_1 = \frac{a+b+c}{3} + \frac{21}{3} = m+7$$

So, it is correct.

(Q) Standard deviation

$$\sigma = \sqrt{\frac{(a-m)^2+(b-m)^2+(c-m)^2}{3}}$$

Adding 7 to each entry,

$$\sigma_1 = \sqrt{(a-m+7)^2+(b-m+7)^2+(c-m+7)^2} \neq (\sigma+7)$$

It is wrong.

(R) By doubling each entry.

$$m_1 = \frac{2a+2b+2c}{3} = 2m \text{ (it is correct)}$$

(S) doubling each entry

$$\sigma_1 = \sqrt{\frac{(m-2a)^2+(m-2b)^2+(m-2c)^2}{3}} \neq (2\sigma)$$

Hence it is wrong.

SOL 12.11 Option (B) is correct.

Amalgamate means combine into a unified or integrated whole unit. The word split is nearly opposite in meaning to the Amalgamate.

SOL 12.12 Option (A) is correct.

Inexplicable means incapable of being explained or accounted. So, the best synonym here is incomprehensible.

SOL 12.13 Option (B) is correct.

We have

$$\log(P) = \frac{1}{2}\log(Q) = \frac{1}{3}\log(R)$$

or $$\log(P) = \log(Q)^{1/2} = \log(R)^{1/3} = \log C$$

Where $\log C$ is a constant.

or $P = C$, $Q = C^2$, $R = C^3$

Now From option (ii),

$$Q^2 = PR$$

$$(C^2)^2 = C \times C^3$$

$$C^4 = C^4$$

Equation (ii) satisfies.

SOL 12.14 Option (C) is correct.

The correct usage of contemplate is verb + ing form. It is a transitive verb. The most appropriate work is visiting.

SOL 12.15 Option (B) is correct.

The mean of the sentence indicates a word that is similar to understand is needed for the blank place.

Therefore, the best option is restrained which means controlled or reserved.

SOL 12.16 Option (D) is correct.

We know

Quantity of spirit left after n^{th} operation $= a \times \left(\frac{a-b}{a}\right)^n$

Where a = initial quantity of pure spirit

and b = quantity taken out and replaced every time

Now after three $(n = 3)$ operations,

Left quantity of spirit after 3^{rd} operation

$$= 10\left(\frac{10-1}{10}\right)^3 = 10\left(\frac{9}{10}\right)^3 = 7.29 \text{ litre}$$

SOL 12.17 Option (C) is correct.

This passage deals with how to deal with bereavement and grief. So, after the tragedy occurs and it is not about precautions. Thus option (C), what the leading causes of death are, not be included in a unit of bereavement. Rest all are important in dealing with grief.

SOL 12.18 Option (A) is correct.

Total cost = Variable cost + Fixed Cost

$$T.C. = V + F$$

$$= 4q + \frac{100}{q}$$

Not for minimize the total cost, using the options.

(A) For $q = 5$, $T.C. = 4 \times 5 + \frac{100}{5} = 40$

(B) For $q = 4$, $T.C. = 4 \times 4 + \frac{100}{4} = 41$

(C) For $q = 7$, $T.C. = 4 \times 7 + \frac{100}{7} = 42.28$

(D) For $q = 6$, $T.C. = 4 \times 6 + \frac{100}{6} = 40.66$

Hence, option (A) gives the minimum cost.

SOL 12.19 Option (D) is correct.

The danger of a microbe to human being will be directly proportional to potency and growth and inversely proportional to the toxicity.

So, level of dangerous $\propto \frac{\text{Potency} \times \text{growth}}{\text{Toxicity}}$

$$D = C\frac{PG}{T} \qquad \text{Where } C = \text{constant of proportionality}$$

For P, $$D_P = \frac{0.4 \times \pi \times (25)^2}{800} = 0.98$$

For Q, $$D_Q = \frac{0.5 \times \pi \times (20)^2}{600} = 1.047$$

For R, $$D_R = \frac{0.4 \times \pi \times (15)^2}{300} = 0.94$$

For S, $$D_S = \frac{0.8 \times \pi \times (10)^2}{200} = 1.25$$

Thus D_S is maximum and it is most dangerous among them and it is targeted in first attempt.

SOL 12.20 Option (C) is correct.

Let 'x' be the number of orders each day and y be the backlogs.

So, From the given conditions

$$4x + y = 4 \times 7 = 28$$

and $$10x + y = 3 \times 10 = 30$$

After solving these two equations, we get

$$x = \frac{1}{3}, \qquad y = \frac{80}{3}$$

Now determine the number of trucks, so that no pending order will be left end of the 5th day.

$$5x + y = 5n$$

Where n = Number of trucks

$$n = \frac{5 \times \frac{1}{3} + \frac{80}{3}}{5} = \frac{\frac{85}{3}}{5} = 5.56$$

Hence number of trucks have to be natural number,

$$n = 6$$

SOL 12.21 Option (B) is correct.

Circuitous means round about or not direct. Indirect is closest in meaning to this circuitous

(A) Cyclic : Recurring in nature

(B) Indirect : Not direct

(C) Confusing : lacking clarity of meaning

(D) Crooked : set at an angle; not straight

SOL 12.22 Option (B) is correct.

A worker may by unemployed. Like in same relation a sleeper may be unaware.

SOL 12.23 Option (D) is correct.

Here conserve is most appropriate word.

SOL 12.24 Option (C) is correct.

Betrayed means reveal unintentionally that is most appropriate.

SOL 12.25 Option (D) is correct.

Number of people who play hockey

$n(A) = 15$

Number of people who play football

$$n(B) = 17$$

Persons who play both hockey and football $n(A \cap B) = 10$

Persons who play either hockey or football or both :

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 15 + 17 - 10 = 22$$

Thus people who play neither hockey nor football $= 25 - 22 = 3$

SOL 12.26 Option (D) is correct.

SOL 12.27 Option (C) is correct.

Since $7 + 6 = 13$ but unit digit is 5 so base may be 8 as 5 is the remainder when 13 is divided by 8. Let us check.

$$\begin{array}{r} 137_8 \\ \underline{276_8} \\ 435 \end{array} \quad \text{Thus here base is 8. Now} \quad \begin{array}{r} 731_8 \\ \underline{672_8} \\ 1623 \end{array}$$

SOL 12.28 Option (D) is correct.

Let W be the total work.

Per day work of 5 skilled workers $= \frac{W}{20}$

Per day work of one skill worker $= \frac{W}{5 \times 20} = \frac{W}{100}$

Similarly per day work of 1 semi-skilled workers $= \frac{W}{8 \times 25} = \frac{W}{200}$

Similarly per day work of one semi-skill worker $= \frac{W}{10 \times 30} = \frac{W}{300}$

Thus total per day work of 2 skilled, 6 semi-skilled and 5 unskilled workers is $= \frac{2W}{100} + \frac{6W}{200} + \frac{5W}{300} = \frac{12W + 18W + 10W}{600} = \frac{W}{15}$

Therefore time to complete the work is 15 days.

SOL 12.29 Option (B) is correct.

As the number must be greater than 3000, it must be start with 3 or 4. Thus we have two case:

Case (1) If left most digit is 3 an other three digits are any of 2, 2, 3, 3, 4, 4, 4, 4.

(1) Using 2, 2, 3 we have 3223, 3232, 3322 i.e. $\frac{3!}{2!} = 3$ no.

(2) Using $2,2,4$ we have 3224, 3242, 3422 i.e. $\frac{3!}{2!} = 3$ no.

(3) Using $2,3,3$ we have 3233, 3323, 3332 i.e. $\frac{3!}{2!} = 3$ no.

(4) Using $2,3,4$ we have $3! = 6$ no.

(5) Using $2,4,4$ we have 3244, 3424, 3442 i.e. $\frac{3!}{2!} = 3$ no.

(6) Using $3,3,4$ we have 3334, 3343, 3433 i.e. $\frac{3!}{2!} = 3$ no.

(7) Using $3,4,4$ we have 3344, 3434, 3443 i.e. $\frac{3!}{2!} = 3$ no.

(8) Using $4,4,4$ we have 3444 i.e. $\frac{3!}{3!} = 1$ no.

Total 4 digit numbers in this case is

$1 + 3 + 3 + 3 + 6 + 3 + 3 + 3 + 1 = 25$

Case 2 : If left most is 4 and other three digits are any of 2, 2, 3, 3, 3, 4, 4, 4.

(1) Using $2,2,3$ we have 4223, 4232, 4322 i.e. . $\frac{3!}{2!} = 3$ no

(2) Using $2,2,4$ we have 4224, 4242, 4422 i.e. . $\frac{3!}{2!} = 3$ no

(3) Using $2,3,3$ we have 4233, 4323, 4332 i.e. . $\frac{3!}{2!} = 3$ no

(4) Using $2,3,4$ we have i.e. . $3! = 6$ no

(5) Using $2,4,4$ we have 4244, 4424, 4442 i.e. . $\frac{3!}{2!} = 3$ no

(6) Using $3,3,3$ we have 4333 i.e $\frac{3!}{3!} = 1$. no.

(7) Using $3,3,4$ we have 4334, 4343, 4433 i.e. . $\frac{3!}{2!} = 3$ no

(8) Using $3,4,4$ we have 4344, 4434, 4443 i.e. . $\frac{3!}{2!} = 3$ no

(9) Using $4,4,4$ we have 4444 i.e. $\frac{3!}{3!} = 1$. no

Total 4 digit numbers in 2nd case $= 3+3+3+6+3+3+1+3+1 = 26$
Thus total 4 digit numbers using case (1) and case (2) is $= 25+26 = 51$

SOL 12.30 Option (B) is correct.

Let H, G, S and I be ages of Hari, Gita, Saira and Irfan respectively.

Now from statement (1) we have $H+G > I+S$

Form statement (2) we get that $G-S = 1$ or $S-G = 1$

As G can't be oldest and S can't be youngest thus either GS or SG possible.
From statement (3) we get that there are no twins

(A) HSIG : There is I between S and G which is not possible

(B) SGHI : SG order is also here and $S > G > H > I$ and $G+H > S+I$ which is possible.

(C) IGSH : This gives $I > G$ and $S > H$ and adding these both inequalities we have $I+S > H+G$ which is not possible.

(D) IHSG : This gives $I > H$ and $S > G$ and adding these both inequalities we have $I+S > H+G$ which is not possible.

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