

1. Define conduction mode of heat transfer.

Ans. "Conduction" is the transfer of heat from one part of a substance to another part of the same substance, or from one substance to another in physical contact with it, without appreciable displacement of molecules forming the substance.

2. Which law is related to conduction heat transfer?

Explain it.

Ans. Fourier's law of heat conduction is related to conduction heat transfer.

"The rate of flow of heat through a simple homogeneous solid is directly proportional to the area of the section at right angles to the direction of heat flow, and to change of temperature with respect to the length of the path of the heat flow."

Mathematically,

$$Q \propto A \cdot \frac{dt}{dx}$$

where Q = Heat flow through a body per unit time (in watts). W.

A = surface area of heat flow (perpendicular to the direction of flow), m^2 .

dt = Temperature difference .

dx = Thickness of body in the direction of heat flow , m

Thus, $Q = -k \cdot A \frac{dt}{dx}$

where, k = constant of proportionality and is known as thermal conductivity of the body .

The temperature gradient $\frac{dt}{dx}$ is always negative along positive x -direction and, therefore, the value as Q becomes +ve .

3. write Laplace Equation .

Ans . In the absence of internal heat generation .

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = 0$$

$$\nabla^2 t = 0$$

Laplace equation

4. Write Poisson Equation.

Ans. Under this situations when temperature does not depend on time, the conduction then takes place in the steady state (i.e. $\frac{\partial t}{\partial \tau} = 0$).

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = 0$$

or, $\boxed{\nabla^2 t + \frac{q_g}{k} = 0}$ (Poisson's equation)

5. Write Fourier Equation.

Ans. When no internal source of heat generation is present.

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}$$

[Unsteady state ($\frac{\partial t}{\partial \tau} \neq 0$) heat flow with no internal heat generation]

or,

$$\boxed{\nabla^2 t = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}}$$

[Fourier's equation]

6. Define thermal diffusivity?

Ans. In case of homogeneous (in which properties e.g., specific heat, density, thermal conductivity etc. are same everywhere in the material) and isotropic (in which properties are independent of surface orientation) material, $k_x = k_y = k_z = k$ and diffusion equation.

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_v}{k} = \frac{\rho \cdot c}{k} \cdot \frac{\partial t}{\partial \tau} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}$$

where, $\alpha = \frac{k}{\rho \cdot c} = \frac{\text{Thermal conductivity}}{\text{Thermal capacity}}$.

The quantity, $\alpha = \frac{k}{\rho \cdot c}$ is known as thermal diffusivity.

7. Derive the general heat conduction equation in cartesian coordinates.

Ans. Consider an infinitesimal rectangular parallelepiped of sides dx , dy and dz parallel, respectively, to the three axes (x, y, z) in a medium in which temperature is varying with location and time.

Let

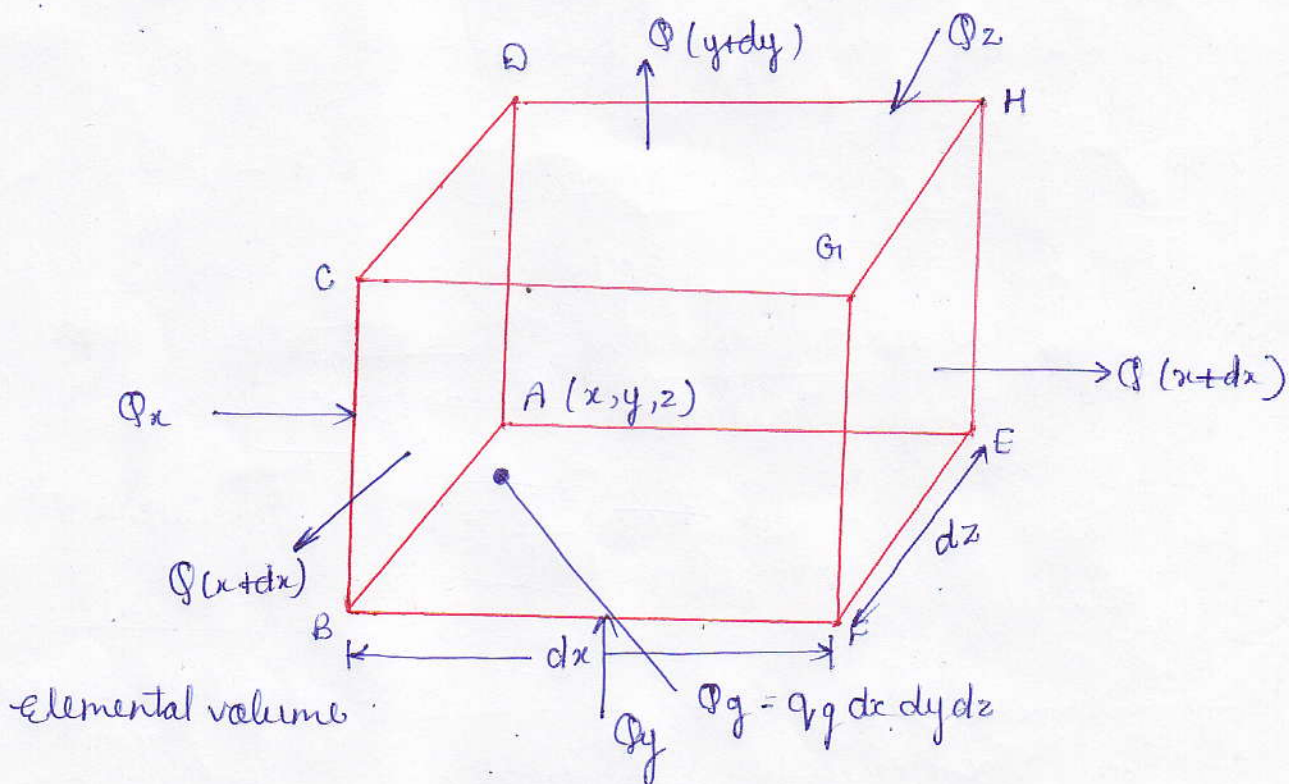
$$\frac{dt}{dx} = \text{Temperature changes and rate of change along } x\text{-direction.}$$

Then $\left(\frac{\partial t}{\partial x}\right) dx = \text{change of temperature through distance } dx \text{ and.}$

$t + \left(\frac{\partial t}{\partial x}\right) dx =$ temperature on the right face EFGH

let, $k_x, k_y, k_z =$ Thermal conductivity along x, y and z axes.

$q_g =$ Heat generated per unit volume per unit time



Energy balance / eqⁿ for volume element

Net heat accumulated in the element due to conduction of heat from all the coordinate directions considered (A) + heat generated within the element (B) = Energy stored in the element (C).

let, $Q =$ Rate of heat flow in a direction, and

$Q' = (Q \cdot dt) =$ Total heat flow (flux) in that direction (in time dt).

A. Heat influx, $Q'_x = -k_x (dy dz) \frac{\partial t}{\partial x} \cdot dx$

Heat efflux, $Q'(x+dx) = Q'_x + \frac{\partial}{\partial x} (Q'_x) dx$

Heat accumulated due to heat flow ~~in~~ in.

$$\begin{aligned}d\Phi'_x &= \Phi'_x - \left[\Phi'_x + \frac{\partial}{\partial x} (\Phi'_x) dx \right] \\&= -\frac{\partial}{\partial x} (\Phi'_x) dx \\&= -\frac{\partial}{\partial x} \left[-k_x (dy dz) \frac{\partial t}{\partial x} \right] dx \\&= \frac{\partial}{\partial x} \left[k_x \frac{\partial t}{\partial x} \right] dx \cdot dy \cdot dz \cdot dt\end{aligned}$$

Similarly by conduction,

$$d\Phi'_y = \frac{\partial}{\partial y} \left[k_y \frac{\partial t}{\partial y} \right] dx \cdot dy \cdot dz \cdot dt$$

$$d\Phi'_z = \frac{\partial}{\partial z} \left[k_z \frac{\partial t}{\partial z} \right] dx \cdot dy \cdot dz \cdot dt$$

Net heat accumulated

$$= \frac{\partial}{\partial x} \left[k_x \frac{\partial t}{\partial x} \right] dx \cdot dy \cdot dz \cdot dt + \frac{\partial}{\partial y} \left[k_y \frac{\partial t}{\partial y} \right] dx \cdot dy \cdot dz \cdot dt + \frac{\partial}{\partial z} \left[k_z \frac{\partial t}{\partial z} \right] dx \cdot dy \cdot dz \cdot dt$$

$$= \left[\frac{\partial}{\partial x} \left(k_x \frac{\partial t}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial t}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial t}{\partial z} \right) \right] dx \cdot dy \cdot dz \cdot dt$$

B. $\Phi_g' = \rho g (dx \cdot dy \cdot dz) dt$

C. Energy stored in the element

$$\rho (dx \cdot dy \cdot dz) c \cdot \frac{\partial t}{\partial t} \cdot dt$$

$$\text{Net} \rightarrow \frac{\partial}{\partial x} \left(k_x \frac{\partial t}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial t}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial t}{\partial z} \right) dx \cdot dy \cdot dz \cdot dt + \rho g (dx \cdot dy \cdot dz) dt = \rho (dx \cdot dy \cdot dz) c \cdot \frac{\partial t}{\partial t}$$

Dividing both sides by $dx \cdot dy \cdot dz \cdot dt$, we have

$$\frac{\partial}{\partial x} \left(k_x \frac{\partial t}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial t}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial t}{\partial z} \right) + q_g = \rho \cdot c \cdot \frac{\partial t}{\partial \tau}$$

or, using the vector operator ∇ , we get

$$\nabla \cdot (k \nabla t) + q_g = \rho \cdot c \cdot \frac{\partial t}{\partial \tau}$$

This is known as the general heat conduction eqn for 'non-homogeneous material', 'self heat generating' and 'unsteady three-dimensional heat flow.'

8. Derive the expression for heat conduction for plane wall.

Ans. Case I: Uniform thermal conductivity.

L = Thickness of the plane wall.

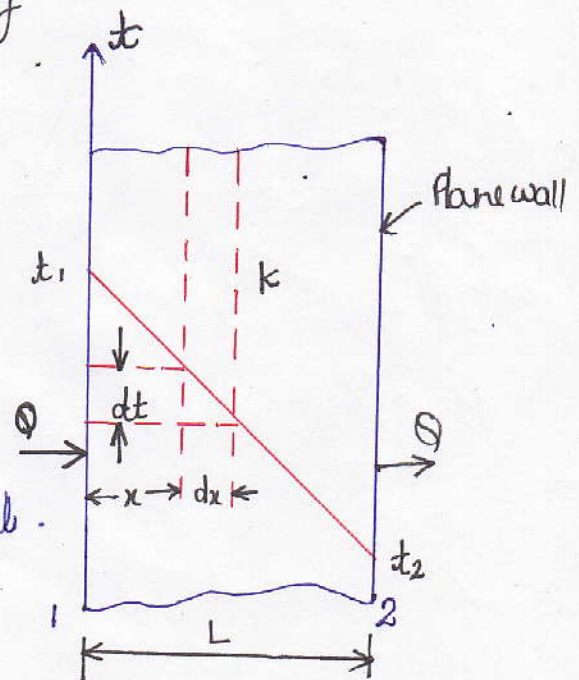
A = Cross-sectional area of the wall.

k = Thermal conductivity of the wall material, and

t_1, t_2 = Temperatures maintained at the two faces 1 and 2 of the wall.

The general heat conduction eqn in cartesian coordinates is given by

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}$$



$$(R_{th})_{cond} = \frac{L}{kA}$$

for steady state $\left(\frac{\partial t}{\partial t} = 0\right)$, one-dimensional $\left[\frac{\partial^2 t}{\partial y^2} = \frac{\partial^2 t}{\partial z^2} = 0\right]$
 and with no internal heat generation $\left(\frac{q_{\text{gen}}}{k} = 0\right)$ and the
 eqⁿ reduces to

$$\frac{\partial^2 t}{\partial x^2} = 0, \text{ or } \frac{d^2 t}{dx^2} = 0$$

By integrating the above differential twice,

$$\frac{dt}{dx} = C_1 \text{ and } t = C_1 x + C_2$$

Boundary conditions:

$$\text{At } x = 0$$

$$t = t_1$$

$$\text{At } x = L$$

$$t = t_2$$

Substituting the values in the eqⁿ, we get
 $t_1 = 0 + C_2$ and $t_2 = C_1 L + C_2$

After simplification, we have,

$$C_2 = t_1 \text{ and } C_1 = \frac{t_2 - t_1}{L}$$

$$t = \left(\frac{t_2 - t_1}{L}\right)x + t_1$$

Fourier's eqⁿ as follows:

$$Q = -kA \frac{dt}{dx}$$

$$\text{But, } \frac{dt}{dx} = \frac{d}{dx} \left[\left(\frac{t_2 - t_1}{L}\right)x + t_1 \right] = \frac{t_2 - t_1}{L}$$

$$\therefore Q = -kA \left(\frac{t_2 - t_1}{L}\right) = \frac{kA(t_1 - t_2)}{L}$$

$$Q = \frac{(t_1 - t_2)}{(L/kA)} = \frac{t_1 - t_2}{(R_{th})_{cond.}}$$

$$(R_{th})_{cond.} = \frac{L}{kA}$$

weight of the wall, $W = \rho AL$

$$W = \rho A \cdot (R_{th})_{cond.} \cdot kA = (\rho \cdot k) A^2 \cdot (R_{th})_{cond.}$$

Case II: Variable thermal conductivity

let the thermal conductivity vary with temperature according to the relation

$$k = k_0 (1 + \beta t)$$

k_0 = Thermal conductivity at zero temperature

The Fourier's eqn

$$Q = -kA \frac{dt}{dx}$$

$$Q = -k_0 (1 + \beta t) \frac{dt}{dx} \cdot A$$

$$Q/A \, dx = -k_0 (1 + \beta t) dt$$

$$\frac{Q}{A} \int_0^L dx = -k_0 \int_{t_1}^{t_2} (1 + \beta t) dt$$

$$\frac{QL}{A} = -k_0 \left[t + \frac{\beta}{2} t^2 \right]_{t_1}^{t_2}$$

$$\text{or. } \frac{QL}{A} = -k_0 \left[(t_2 - t_1) + \frac{\beta}{2} (t_2^2 - t_1^2) \right] \quad \text{--- (1)}$$

$$= k_0 \left[(t_1 - t_2) + \frac{\beta}{2} (t_1 - t_2)(t_1 + t_2) \right]$$

$$= k_0 \left[1 + \frac{\beta}{2} (t_1 + t_2) \right] (t_1 - t_2)$$

$$= k_0 (1 + \beta t_m) (t_1 - t_2)$$

$$Q = k_0 (1 + \beta t_m) \cdot \frac{A(t_1 - t_2)}{L} \quad \left[\text{where } t_m = \frac{t_1 + t_2}{2} \right]$$

t is replaced by t_m , then

$$k_m = k_0 (1 + \beta t_m)$$

$$Q = k_m A \left[\frac{t_1 - t_2}{L} \right]$$

k_m is known as mean thermal conductivity of the wall material.

Further, if t is the temperature of the surface at a dist. x from the left surface.

$$\frac{Qx}{A} = k_0 \left[(t - t_1) + \frac{\beta}{2} (t^2 - t_1^2) \right] \quad \text{--- (2)}$$

From eqn (1) & (2), we have

$$\left[(t_2 - t_1) + \frac{\beta}{2} (t_2^2 - t_1^2) \right] \cdot \frac{x}{L} = \left[(t - t_1) + \frac{\beta}{2} (t^2 - t_1^2) \right]$$

Solving the eqn for t ,

$$t = \frac{1}{\beta} \left[(1 + \beta t_1)^2 - \left\{ (1 + \beta t_1)^2 - (1 + \beta t_2)^2 \right\} \frac{x}{L} \right]^{1/2} - \frac{1}{\beta}$$

B. Temperature variation in terms of heat flux (Q):

Fourier's eqn

$$Q = -KA \frac{dt}{dx} = -k_0 (1 + \beta t) A \cdot \frac{dt}{dx}$$

$$\text{or } Q \cdot dx = -k_0 (1 + \beta t) A \cdot dt$$

Integrating both sides, we get.

$$Q \cdot x = -k_0 A \left(t + \frac{\beta}{2} t^2 \right) + C \quad \text{--- (i)}$$

To evaluate C , applying the condition: At $x=0$, $t=t_1$, we get

$$C = k_0 A \left(t_1 + \frac{\beta}{2} t_1^2 \right)$$

Substituting the values of the constant C in (i), we get

$$Q \cdot x = -k_0 A \left(t + \frac{\beta}{2} t^2 \right) + k_0 A \left(t_1 + \frac{\beta}{2} t_1^2 \right)$$

Dividing both sides by $k_0 A$ and rearranging,

$$\frac{\beta}{2} t^2 + t + \left[\frac{Q \cdot x}{k_0 A} - \left(t_1 + \frac{\beta}{2} t_1^2 \right) \right] = 0$$

By solving the above quadratic equⁿ, we have

$$t = \frac{-1 + \sqrt{1 - 4 \times \frac{\beta}{2} \left[\frac{Q \cdot x}{k_0 A} - \left(t_1 + \frac{\beta}{2} t_1^2 \right) \right]}}{2 \times \left(\frac{\beta}{2} \right)}$$

$$\text{or } t = -\frac{1}{\beta} + \left[\frac{1}{\beta^2} - \frac{2}{\beta} \left(\frac{Q \cdot x}{k_0 A} - t_1 - \frac{\beta}{2} t_1^2 \right) \right]^{1/2}$$

$$= -\frac{1}{\beta} + \left[\frac{1}{\beta^2} - \frac{2}{\beta} t_1 + t_1^2 - \frac{2Q \cdot x}{\beta k_0 A} \right]^{1/2}$$

$$= -\frac{1}{\beta} + \left[\left(t_1 + \frac{1}{\beta} \right)^2 - \frac{2Q \cdot x}{\beta k_0 A} \right]^{1/2}$$

Hence,

$$t = -\frac{1}{\beta} + \left[\left(t_1 + \frac{1}{\beta} \right)^2 - \frac{2Q \cdot x}{\beta k_0 A} \right]^{1/2}$$

If the variation of k with temperature is not linear then

$$k = k_0 f(t), \text{ and}$$

$$\frac{Q}{A} \int_0^L dx = - \int_{t_1}^{t_2} [k_0 f(t)] dt$$

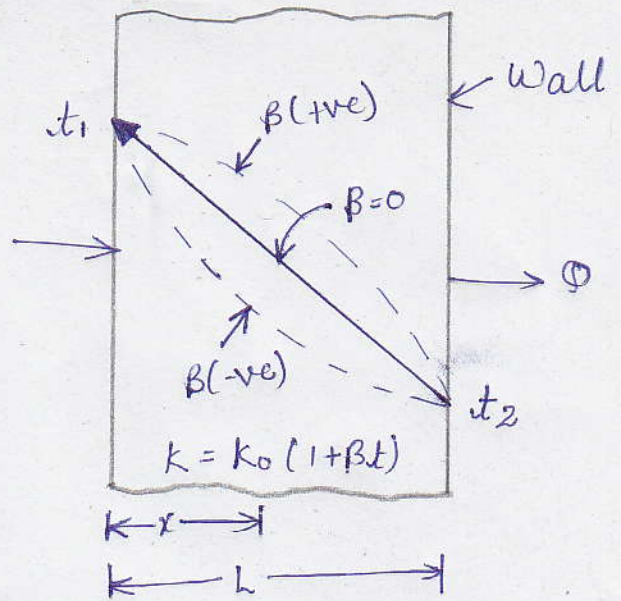
$$\text{or, } Q = \frac{A}{L} \left[- \int_{t_1}^{t_2} [k_0 f(t)] dt \right]$$

But, $Q = k_m A \left(\frac{t_1 - t_2}{L} \right)$

$$k_m = \frac{1}{(t_1 - t_2)} \left[\int_{t_1}^{t_2} [k_0 f(t)] dt \right]$$

$$= \frac{1}{(t_1 - t_2)} \left[\int_{t_2}^{t_1} [k_0 f(t)] dt \right]$$

The effect of $+B$ and $-B$ on temperature is depicted.



Q8: Derive the expression for heat conduction for composite wall.

Ans. 9: Consider the transmission of heat through a composite wall consisting of a number of slabs.

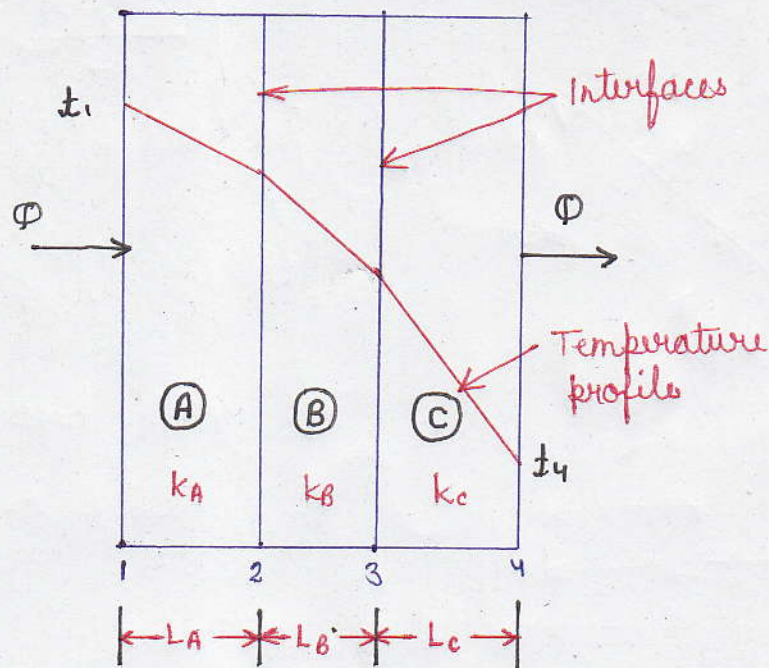
Let, L_A, L_B, L_C = Thicknesses of slabs A, B and C respectively
 k_A, k_B, k_C = Thermal conductivities of the slabs A, B & C respectively.

t_1, t_4 ($t_1 > t_4$) = Temperatures at the wall surfaces 1 and 4 respectively, and

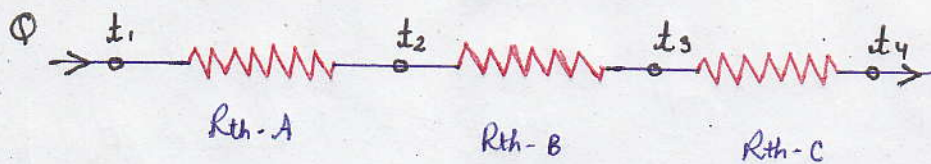
t_2, t_3 = Temperatures at the interfaces 2 and 3.

Since the quantity of heat transmitted per unit time through each slab/layer is same, we have -

$$Q = \frac{k_A \cdot A (t_1 - t_2)}{L_A} = \frac{k_B \cdot A (t_2 - t_3)}{L_B} = \frac{k_C \cdot A (t_3 - t_4)}{L_C}$$



(a)



$$R_{th} = \frac{L_A}{k_A \cdot A}, \quad R_{th-B} = \frac{L_B}{k_B \cdot A}, \quad R_{th-C} = \frac{L_C}{k_C \cdot A}$$

(b)

$$t_1 - t_2 = \frac{Q \cdot L_A}{k_A \cdot A} \quad \text{--- (i)}$$

$$t_2 - t_3 = \frac{Q \cdot L_B}{k_B \cdot A} \quad \text{--- (ii)}$$

$$t_3 - t_4 = \frac{Q \cdot L_C}{k_C \cdot A} \quad \text{--- (iii)}$$

Adding (i), (ii) & (iii), we have

$$(t_1 - t_4) = Q \left[\frac{L_A}{k_A \cdot A} + \frac{L_B}{k_B \cdot A} + \frac{L_C}{k_C \cdot A} \right]$$

$$Q = \frac{A (t_1 - t_4)}{\left[\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} \right]}$$

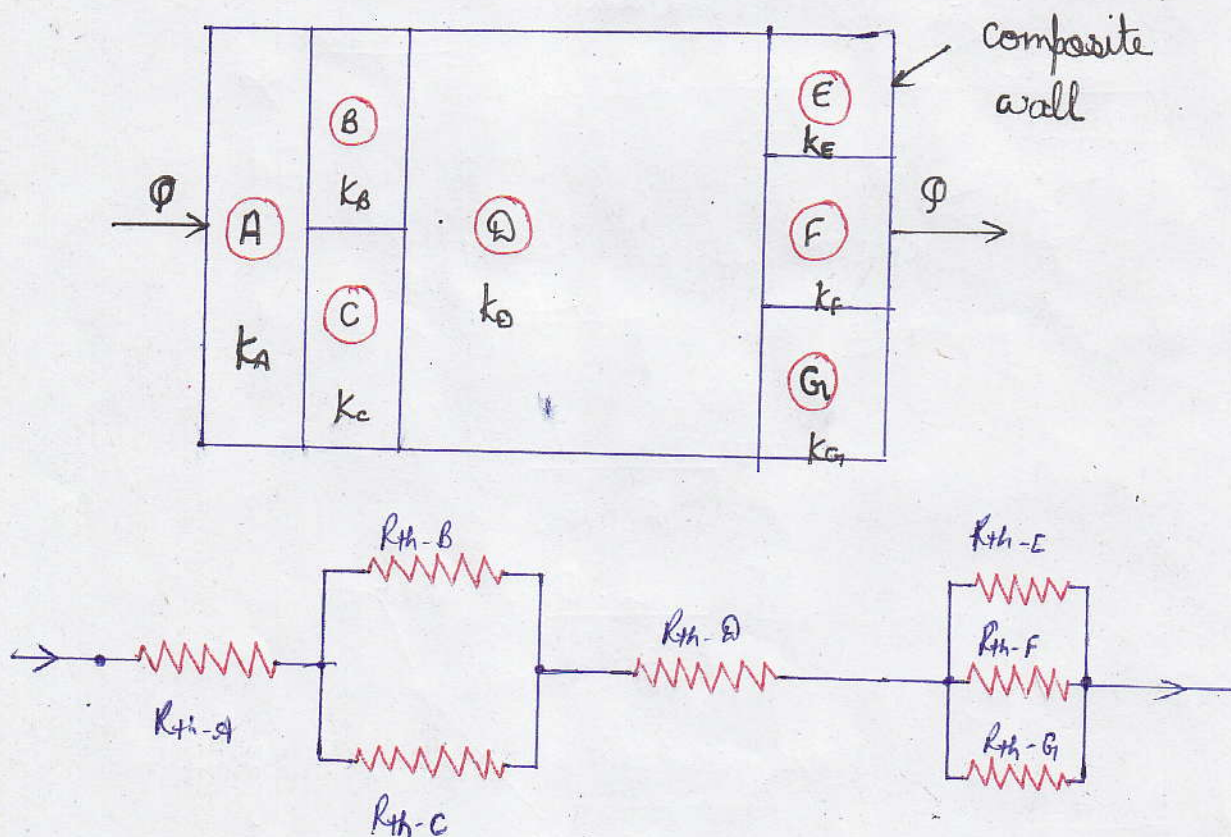
$$\text{Q. 1, } Q = \frac{(t_1 - t_4)}{\left[\frac{L_A}{k_A \cdot A} + \frac{L_B}{k_B \cdot A} + \frac{L_C}{k_C \cdot A} \right]} = \frac{(t_1 - t_4)}{[R_{th-A} + R_{th-B} + R_{th-C}]}$$

If the composite wall consists of n slabs/layers, then

$$Q = \frac{[t_1 - t_{(n+1)}]}{\sum_1 \frac{L}{k \cdot A}}$$

In order to solve more complex problems involving both series and parallel thermal resistances.

$$Q = \frac{\Delta t_{\text{overall}}}{\sum R_{th}}$$



Q10: Derive the expression for the overall Heat-Transfer coefficient

Q10: An overall transfer coefficient U which gives the heat transmitted per unit area per unit time per degree temperature difference between the bulk fluids on each side of the metal

Let L = Thickness of the metal wall

k = Thermal conductivity of the wall material

t_1 = Temperature of the surface-1

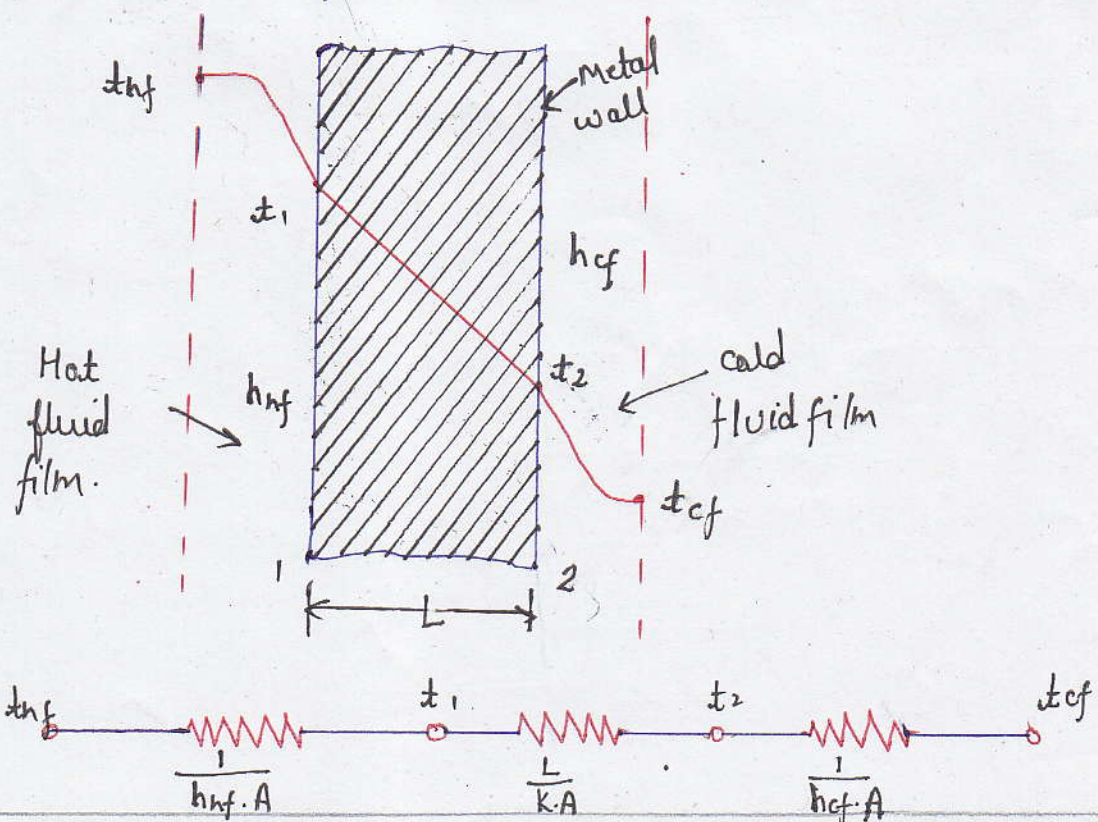
t_2 = Temperature of the surface-2

t_{hf} = Temperature of the hot fluid.

t_{cf} = Temperature of the cold fluid

h_{hf} = Heat transfer coefficient from hot fluid to metal surface, and

h_{cf} = Heat transfer coefficient from metal surface to cold fluid.



The eqns of heat flow through the fluid and the metal surface are given by

$$Q = h_{nf} \cdot A (t_{nf} - t_1) \quad \text{--- (1)}$$

$$Q = \frac{k \cdot A (t_1 - t_2)}{L} \quad \text{--- (2)}$$

$$Q = h_{cf} \cdot A (t_2 - t_{cf}) \quad \text{--- (3)}$$

By rearranging (i), (ii) & (iii), we get

$$t_{nf} - t_1 = \frac{Q}{h_{nf} \cdot A} \quad \text{--- (iv)}$$

$$t_1 - t_2 = \frac{Q L}{k \cdot A} \quad \text{--- (v)}$$

$$t_2 - t_{cf} = \frac{Q}{h_{cf} \cdot A} \quad \text{--- (vi)}$$

Adding (iv), (v) & (vi), we get

$$t_{nf} - t_{cf} = Q \left[\frac{1}{h_{nf} \cdot A} + \frac{L}{k \cdot A} + \frac{1}{h_{cf} \cdot A} \right]$$

$$Q = \frac{A (t_{nf} - t_{cf})}{\frac{1}{h_{nf}} + \frac{L}{k} + \frac{1}{h_{cf}}}$$

If U is the overall coefficient of heat transfer, then

$$Q = U \cdot A (t_{nf} - t_{cf}) = \frac{A (t_{nf} - t_{cf})}{\frac{1}{h_{nf}} + \frac{L}{k} + \frac{1}{h_{cf}}}$$

$$\text{or, } U = \frac{1}{\frac{1}{h_{nf}} + \frac{L}{k} + \frac{1}{h_{cf}}}$$

11. Discuss the effects of various parameters on the thermal conductivity of solid?
1. Chemical composition: Pure metals have very high thermal conductivity. Impurities or alloying elements reduce the thermal conductivity considerably.
 2. Mechanical forming: Forging, drawing and bending or heat treatment of metals cause considerable variation in thermal conductivity. eg: the thermal conductivity of hardened steel is lower than that of annealed state.
 3. Temperature rise: The value of k for most metals decreases with temperature rise since at elevated temperatures the thermal vibrations of the lattice become higher that retard the motion of free electrons.
 4. Non-metallic solids: Non-metallic solids have k much lower than that for metals. For many of the building materials (concrete, stone, brick, glass wool, cork etc) the thermal conductivity may vary from sample to sample due to variations in structure, composition, density and porosity.
 5. Presence of air: The thermal conductivity is reduced due to the presence of air filled pores or cavities.
 6. Dampness: Thermal conductivity of a damp material is considerably higher than that of dry material.
 7. Density: Thermal conductivity of insulating powder, asbestos etc. increases with density growth. Thermal conductivity of snow is also proportional to its density.

Q.12. The temperature at the inner and outer surfaces of a boiler wall made of 20mm thick steel and covered with an insulating material of 5mm thickness are 300°C and 50°C respectively. If the thermal conductivities of steel and insulating are 58 and $0.116 \text{ W/m}^{\circ}\text{C}$ resp. determine the rate of heat flow through the boiler wall.

Ans: \rightarrow

we have, $t_1 = 50$, $t_2 = 300$, $k_{\text{Steel}} = 58 \text{ W/m}^{\circ}\text{C}$, $k_{\text{insulat}} = 0.116 \text{ W/m}^{\circ}\text{C}$
 $l_s = 20\text{mm} = 0.020\text{m}$, $l_{\text{ins}} = 5\text{mm} = 0.005\text{m}$

$$\text{heat} = Q = \left(\frac{t_2 - t_1}{\frac{l_s}{k_s} + \frac{l_{\text{ins}}}{k_{\text{insul}}}} \right)$$

$$= \left(\frac{300 - 50}{\frac{0.020}{58} + \frac{0.005}{0.116}} \right)$$

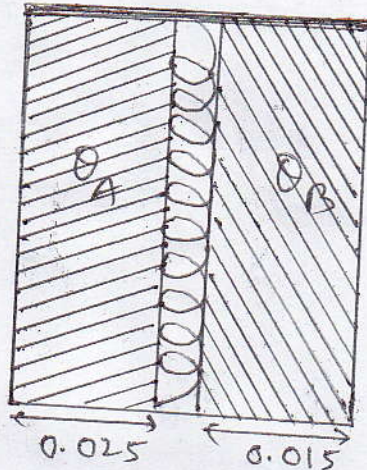
$$= \frac{250}{3.4482 \times 10^{-04} + 0.04310}$$

$$= \frac{250}{0.043448} \approx 5800 \text{ W/Sq m}$$

$$= 5.8 \text{ KW/Sq m}$$

Q.13 A membrane type electric heater of $20,000 \text{ W/Sq m}$ capacity is sandwiched between an insulation of 25 mm thickness with thermal conductivity of 0.029 W/mk & a metal plate with $k = 12.6 \text{ W/mk}$ of thickness 15 mm . The convection coefficient is 150.2 Sq m k . The surrounding are at 5°C . Determine the surface temperature of the heater.

Sol: \rightarrow



For steady heat flow, we have

$$\frac{Q}{A} = \text{Heat flow through slab A } (\theta_A) + \text{heat flow through slab B } (\theta_B)$$

$$= \frac{(t_{max} - t_a)}{\frac{L_A}{k_A} + \frac{1}{h_A}} + \frac{t_{max} - t_a}{\frac{L_B}{k_B} + \frac{1}{h_A}}$$

$$20000 = \frac{(t_{max} - 5)}{\frac{0.025}{0.029} + \frac{1}{150.2}} + \frac{(t_{max} - 5)}{\frac{0.015}{12.6} + \frac{1}{150.2}}$$

$$20000 = \frac{t_{max} - 5}{0.8620 + 0.00665} + \frac{t_{max} - 5}{0.00119 + 0.00665}$$

$$20000 = \frac{t_{max} - 5}{0.8686} + \frac{t_{max} - 5}{0.00784}$$

$$20000 = \frac{0.00784 t_{max} - 0.0392 + 0.8686 t_{max} - 4.343}{0.00681}$$

$$0.00681 \times 20000 = 0.8764 t_{max} - 4.382$$

$$0.8764 t_{max} = 140.75$$

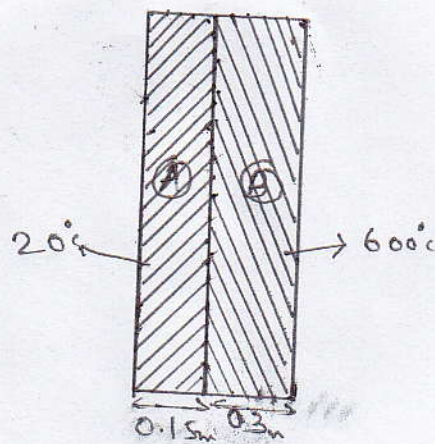
$$t_{max} = 160.59 \approx 160.7$$

$$\therefore \text{The surface temperature of the heater} = (160.7 - 5)^\circ\text{C}$$

$$= 155.7^\circ\text{C}$$

Q.14 A composite wall is made of two layers of 0.3m and 0.15m thickness with surfaces held at 600°C and 20°C respectively. If the conductivities are 20 & 50 W/mK, determine the heat conducted. In order to restrict the heat loss to $5\text{KW}/\text{sqm}$, another layer of 0.15m thickness is proposed. Determine the thermal conductivity of the material required.

Sol:



Heat

$$Q = A \frac{(T_B - T_A)}{\frac{L_A}{K_A} + \frac{L_B}{K_B}} = \frac{600 - 20}{\frac{0.3}{20} + \frac{0.15}{50}}$$

$$= \frac{580}{0.015 + 0.003} = \frac{580}{0.018} = 32222.2 \text{ watt}$$

$$= 32.22 \text{ kW}$$

If in order to restrict the heat loss to $5\text{KW}/\text{sqm}$, the another layer of 0.15m thickness is introduced, then

$$\frac{Q}{A} = \frac{600 - 20}{\frac{0.3}{20} + \frac{0.15}{50} + \frac{0.15}{K_C}} = \frac{580}{0.015 + 0.003 + \frac{0.15}{K_C}}$$

$$5000 = \frac{1}{0.018 + \frac{0.15}{K_C}} = \frac{K_C}{0.018K_C + 0.15}$$

$$580K_C = 750 + 90K_C, \quad K_C = 1.53 \text{ W/mK.}$$

$$490K_C = 750$$

Ans: